**Calculus 1 — Mock Test 1**
**MOCK TEST — 100 Points**
**ANSWER KEY**

No calculator • Show your work • Give exact answers

**1. (15 pts)** Complete the special-angle table. 1 point per blank.

| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin\theta$ | *0* | *1/2* | *√2/2* | *√3/2* | *1* |
| $\cos\theta$ | *1* | *√3/2* | *√2/2* | *1/2* | *0* |
| $\tan\theta$ | *0* | *√3/3* | *1* | *√3* | *undefined* |

**2. (9 pts)** Write the four Sum & Difference identities and the five Double-Angle identities below. 1 point each.

$\sin(A+B) =$ *sin A cos B + cos A sin B*

$\sin(A-B) =$ *sin A cos B − cos A sin B*

$\cos(A+B) =$ *cos A cos B − sin A sin B*

$\cos(A-B) =$ *cos A cos B + sin A sin B*

$\sin(2x) =$ *2sin x cos x*

$\cos(2x) =$ $\cos^2 x - \sin^2 x$

$\cos(2x) =$ $2\cos^2 x - 1$

$\cos(2x) =$ $1 - 2\sin^2 x$

$\tan(2x) =$ $2\tan x/(1-\tan^2 x)$

**3. (6 pts)** Find the exact value of $\sin 75°$.

*$\sin 75° = \sin(45°+30°) = (\sqrt{2}/2)(\sqrt{3}/2)+(\sqrt{2}/2)(1/2) = (\sqrt{6}+\sqrt{2})/4$.*

**4. (5 pts)** Find the exact value of $\cos^{-1}(-\sqrt{2}/2)$.

*$\cos^{-1}(-\sqrt{2}/2) = 3\pi/4$.*

**5. (6 pts)** Find the exact value of $\sin(\cos^{-1}(5/13))$.

*Let $\theta=\cos^{-1}(5/13)$. Then $\cos\theta=5/13$. A 5-12-13 triangle gives $\sin\theta=12/13$.*

**6. (8 pts)** Given $\sin x=5/13$ and $x$ is in QII, find $\sin(2x)$ and $\cos(2x)$.

*$\sin x=5/13$ in QII $\Rightarrow \cos x=-12/13$. $\sin(2x)=-120/169$; $\cos(2x)=119/169$.*

**7. (6 pts)** Use the provided identity $\cos(x/2)=\pm\sqrt{(1+\cos x)/2}$ to find the exact value of $\cos(\pi/8)$.

*Let $x/2=5\pi/8$, so $x=5\pi/4$. Use the positive sign. $\cos(\pi 5/8)= -\sqrt{(2-\sqrt{2})}/2$.*

**8. (8 pts)** Solve $2\sin^2 x + \cos x - 2 = 0$ for $0 \le x < 2\pi$.

*$2\sin^2 x+\cos x-2=0 \Rightarrow 2(1-\cos^2 x)+\cos x-2=0 \Rightarrow \cos x(1-2\cos x)=0$. $x=\pi/3, \pi/2, 3\pi/2, 5\pi/3$.*

**9. (8 pts)** Solve $\sin(2x)+\sin x=0$ for $0 \le x < 2\pi$.

*$\sin(2x)+\sin x=0 \Rightarrow \sin x(2\cos x+1)=0$. $x=0, 2\pi/3, \pi, 4\pi/3$.*

**10. (6 pts)** In $\triangle ABC$, $A=45°$, $B=30°$, and $a=6\sqrt{2}$. Find b. Give an exact answer.

*$b/\sin 30° = 6\sqrt{2}/\sin 45° \Rightarrow b=6$.*

b =

**11. (8 pts)** In $\triangle ABC$, $A=30°$, $a=2$, and $b=2\sqrt{2}$. Find all possible values of B. Determine how many triangles exist. Find all three angles of each triangle. Explain your conclusion.

*$\sin B/(2\sqrt{2})=\sin 30°/2 \Rightarrow \sin B=\sqrt{2}/2$. $B=45°$ or $135°$. Both work. Triangles: $(A,B,C)=(30°,45°,105°)$ and $(30°,135°,15°)$. Number of triangles=2.*

A = B = C = Number of triangles =

Explanation:

**12. (5 pts)** In $\triangle ABC$, $a=6$, $b=10$, and $C=60°$. Find c. Give an exact answer.

*$c^2=6^2+10^2-2(6)(10)\cos 60°=76 \Rightarrow c=2\sqrt{19}$.*

$c =$

**13. (5 pts)** In $\triangle ABC$, $a=5$, $b=5$, and $c=5\sqrt{2}$. Find C.

*$(5\sqrt{2})^2=5^2+5^2-2(5)(5)\cos C \Rightarrow \cos C=0 \Rightarrow C=90°$.*

$C =$

**14. (5 pts)** Given $\tan x=-5/12$ and x is in QII, find $\sin(2x)$ and $\cos(2x)$.

*$\tan x=-5/12$ in QII $\Rightarrow \sin x=5/13$, $\cos x=-12/13$. $\sin(2x)=-120/169$; $\cos(2x)=119/169$.*