UNIT 1.6 | Polynomial Functions & End Behavior

TEACHER EXPLANATION & COMPLETE KEY Name: _____ Date: _____ Class: _____

TODAY I AM learning to describe what happens to polynomial graphs as x goes far left/right.
SO I CAN predict a graph's ends using its leading term.
SUCCESS: I can write both end limits, justify degree/sign, and sketch a matching graph.

Essential Question: How can one term predict the far-left and far-right behavior of a polynomial?

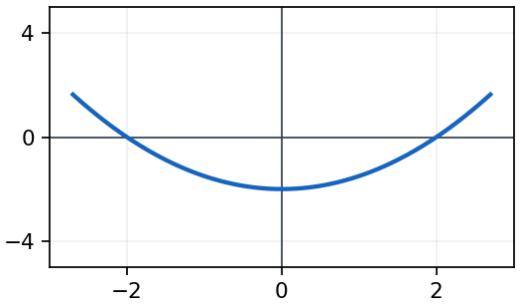
1 | Review the language of end behavior

Input means x; output means f(x) or y. "Without bound" means the values keep going with no finite endpoint.

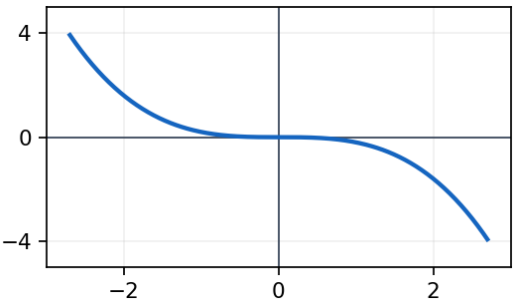
Inputs increase without bound → $x \to +\infty$ (RIGHT). Inputs decrease without bound → $x \to -\infty$ (LEFT). Outputs increase → $y \to +\infty$ (UP). Outputs decrease → $y \to -\infty$ (DOWN).

Complete formula: Left end = lim as $x \to -\infty$ of f(x). Right end = lim as $x \to +\infty$ of f(x).

Example A: both ends UP



Example B: left UP, right DOWN



CFU 1. Describe BOTH limits for Example A.

ANSWER / WHY: As $x \to -\infty$, $f(x) \to +\infty$; as $x \to +\infty$, $f(x) \to +\infty$. Both arrows rise.

CFU 2. Describe BOTH limits for Example B.

ANSWER / WHY: As $x \to -\infty$, $f(x) \to +\infty$; as $x \to +\infty$, $f(x) \to -\infty$. Left rises; right falls.

2 | The leading term decides the ends

A polynomial is a sum of nonnegative-integer powers of x. Its leading term has the highest exponent. At very large |x|, this term dominates all lower powers.

For $p(x)=7x^6-4x^5+8x^4+x^3-3x^2-9x+10$, leading term = $7x^6$; degree = 6 (even); leading coefficient = +7.

Degree / sign	LEFT end $x\to-\infty$	RIGHT end $x\to+\infty$
Even, positive	$+\infty$	$+\infty$
Even, negative	$-\infty$	$-\infty$
Odd, positive	$-\infty$	$+\infty$
Odd, negative	$+\infty$	$-\infty$

KEY IDEA: The leading coefficient controls the RIGHT end. Even degree → ends SAME; odd degree → ends OPPOSITE.

3 | I DO · WE DO · YOU DO

I DO 1. Determine end behavior of $f(x)=-3x^4+2x^2-7$.

ANSWER / WHY: Leading term $-3x^4$: even degree, negative coefficient. Both ends down. $\lim x\to-\infty f(x)=-\infty$ and $\lim x\to+\infty f(x)=-\infty$. Check using $x=\pm10$: $-3(10^4)$ dominates.

WE DO 2. Determine end behavior of $g(x)=5x^3+2x^2-7$. Leading term: _____ ; degree: _____ ; sign: _____.

ANSWER / WHY: Leading term $+5x^3$; degree 3 odd; coefficient positive. Left down: $\lim x\to-\infty g(x)=-\infty$. Right up: $\lim x\to+\infty g(x)=+\infty$.

YOU DO 3. Determine BOTH end limits of $h(x)=8x^2+4-x^5$.

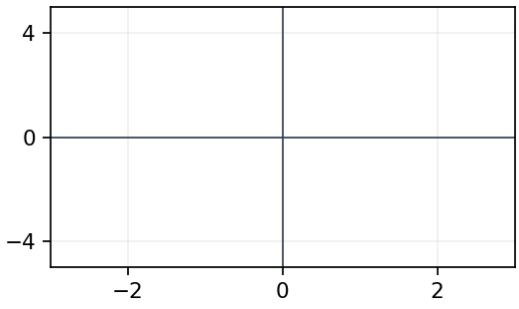
ANSWER / WHY: Rewrite as $-x^5+8x^2+4$; leading term $-x^5$ (odd, negative). Left $+\infty$; right $-\infty$.

YOU DO 4. Determine BOTH end limits of $m(x)=2x(x-1)(6-x)$.

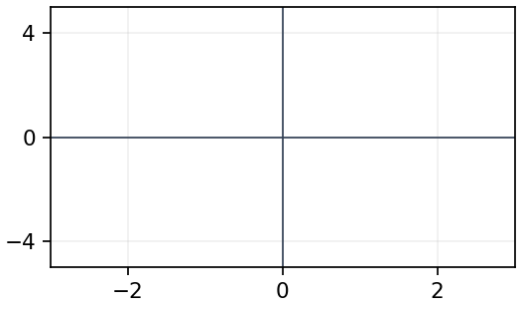
ANSWER / WHY: Highest-degree factors $2x\cdot x\cdot(-x)=-2x^3$: odd negative. Left $+\infty$; right $-\infty$.

4 | Sketching graphs from end limits

5. Sketch left DOWN / right UP (sample: $y=x^3$).



6. Sketch both ends DOWN (sample: $y=-x^4$).



5. Draw a continuous curve from bottom-left to top-right (e.g., $y=x^3$). 6. Draw a downward-opening even-degree curve (e.g., $y=-x^4$).

CFU 3. Can two polynomials share end behavior but have different turning points? Explain.

ANSWER / WHY: Yes. x^4 and x^4-5x^2 both have two upward ends, but different middle shapes. Leading terms match.

5 | Turning points and real zeros

For a degree n polynomial: at most n−1 turning points, and at most n distinct real zeros. A turning point changes from increasing to decreasing, or the reverse.

7. For $f(x)=-x^3+2x^2+x+1$, state degree, coefficient sign, both end limits, and maximum number of turning points and real zeros.

ANSWER / WHY: Degree 3, leading coefficient −1: left $+\infty$, right $-\infty$; at most 2 turning points and at most 3 distinct real zeros. "At most" is a bound, not a promise.

8. For $g(x)=x^4-3x^3+2x$, repeat Question 7.

ANSWER / WHY: Degree 4, leading coefficient +1: both ends $+\infty$; at most 3 turning points and 4 distinct real zeros.

9. AP reasoning: lim as $x\to-\infty$ of $p(x)=+\infty$ and p has an ODD number of turning points. What can you conclude about degree parity and leading coefficient?

ANSWER / WHY: For a polynomial with an odd number of turning points, degree must be EVEN: turning points alternate maxima/minima, and opposite-facing ends give an even number. An even-degree polynomial whose left end rises has POSITIVE leading coefficient. Degree at least 2.

6 | Important calculator comparison

Not every function is polynomial. For rational functions, the polynomial degree/sign shortcut may NOT determine the end behavior of the quotient.

10. Compare $p(x)=x^2-3x+1$, $r(x)=(2x-3)/(x+1)$, and $s(x)=-10/x^2$ as $x\to\pm\infty$.

ANSWER / WHY: p has both ends $+\infty$ (even/positive). $r(x)=2-5/(x+1)\to 2$ at both ends (horizontal asymptote $y=2$). $s(x)=-10/x^2\to 0$ from below at both ends. Do not use polynomial-only $\pm\infty$ rules on r or s.

7 | Practice: Basic → Application → AP-level

11. BASIC $p(x)=-8x^2-3x+10$: give both limits.

ANSWER / WHY: Even/negative leading term $-8x^2$: both limits $-\infty$.

12. BASIC $q(x)=4x^7-3x^4+x$: give both limits.

ANSWER / WHY: Odd/positive leading term $4x^7$: left $-\infty$, right $+\infty$.

13. APPLICATION $y=-2(x+3)^6$: give both limits.

ANSWER / WHY: Leading term $-2x^6$: even/negative, both $-\infty$; shift does not affect ends.

14. APPLICATION Write a polynomial with left $+\infty$, right $-\infty$.

ANSWER / WHY: Example $p(x)=-x^3$. Any odd-degree negative-leading-coefficient polynomial works.

15. AP CHALLENGE A degree-6 polynomial has 7 claimed turning points. Is that possible? Justify.

ANSWER / WHY: No. At most $6-1=5$ turning points; 7 exceeds the bound.

16. AP CHALLENGE Explain why $+2x^8-1000x^5$ and $+2x^8$ have the same end behavior.

ANSWER / WHY: As $|x|$ grows, x^8 dominates x^5. Degree 8 is even and +2 is positive, so both ends approach $+\infty$.

8 | Lesson wrap-up and exit ticket

WHAT I LEARNED: end behavior; leading term; degree; leading coefficient; limit notation. Right end uses coefficient sign; left end also uses parity.

EXIT 1. What does $\lim_{x\to-\infty} f(x)=-\infty$ mean?

ANSWER / WHY: As x moves endlessly left, f(x) moves endlessly down.

EXIT 2. Determine both end limits of $f(x)=-7x^5+3x^2-1$.

ANSWER / WHY: Leading term $-7x^5$, odd/negative: left $+\infty$ and right $-\infty$.

EXIT 3. A student says "the constant -500 changes the end behavior of $2x^4-500$." Explain the error.

ANSWER / WHY: The leading term $2x^4$ dominates for very large $|x|$; the constant affects height near the center but not the ends. Both ends $+\infty$.

Videos and College Board lesson resources

Video 1 (user supplied): https://www.youtube.com/watch?v=j_SKxwxsr68

Video 2 (user supplied): https://www.youtube.com/watch?v=LLRM_ISXeLk&t=9s

Video 3 (user supplied): https://www.youtube.com/watch?v=uThQ4_i9qJw

AP Classroom (login may be required): https://apclassroom.collegeboard.org/d/4vc4197yuh?sui=117.1

Teacher teaching tips / common errors

Ask students to point LEFT before writing $x \to -\infty$. Emphasize that $x \to -\infty$ does not automatically mean $f(x) \to -\infty$. Rewrite polynomials in descending degree; for factored form, multiply only leading factors. Do not confuse degree n with $n-1$ turning-point bound. Check the two ends independently.