

Unit 8 Properties of Circles Geometry

Last Update: August 1, 2025

Archdiocesan Curriculum > Grade > Math-Geometry > Length of unit 20 to 23 days

Stage 1: Desired Results

General Information

This unit explores the relationships between angles, arcs, and segments in and around circles. Students will apply geometric theorems and algebraic reasoning to determine measures, prove relationships, and solve real-world problems involving circles. The unit culminates in the use of coordinate geometry and formulas for arc length, sector area, circumference, and area of circles.

Mathematical Practices:

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.

Enduring Understanding/Knowledge

Students will:

- Determine the measures of central angles, inscribed angles, and arcs of a circle.
- Use the properties of angles of quadrilaterals inscribed in a circle to prove theorems and solve problems.
- Prove theorems about tangents to a circle and use them to solve mathematical and real-world problems.
- Derive and write the equation of a circle with radius r and center (h, k).

Review/Assess

- Use proportional relationships in circles to prove the Chord-Chord, Secant-Secant, and Secant-Tangent Product Theorems. Apply the theorems to solve for segment lengths in mathematical and real-world problems.
- Determine the relationships that exist between secants, tangents, and chords in a circle and the angles and arcs formed by them. Prove and use theorems about these relationships to solve mathematical and real-world problems.

Review/Assess

- Justify and use the formulas for the circumference and area of a circle to solve real-world and mathematical problems.
- Use the arc length formula and apply it to real-world problems, and convert between degree and radian measure.
- Derive the formula for the area of a sector of a circle and use that formula to compute the area of

Essential Question(s)

- How do central and inscribed angles relate to the arcs they intercept?
- What properties can we prove about quadrilaterals inscribed in circles?
- How are tangent lines and angles formed by tangents and secants used in solving real-world problems?
- How do you represent a circle algebraically on the coordinate plane?
- What are effective ways to calculate arc lengths and sector areas using radians and degrees?

Vocabulary

New

- adjacent arcs
- arc
- central angle
- chord
- inscribed angle
- intercepted arc
- major arc
- minor arc
- semicircle
- congruent arcs
- congruent circles
- circumscribed angle
- exterior of a circle
- interior of a circle
- point of tangency
- tangent of a circle
- external secant segment
- secant
- secant segment
- tangent segment
- limit
- arc length
- concentric circles
- radian measure
- sector

Review

- circle
- diameter
- circumference
- arc

sectors of circles having different central angles and radii.

Review/ Assess

Connections to Catholic Identity / Other Subjects Differentiation

Religion/Catholic Identity:

- As you teach about the properties of a circle (like its circumference, radius, and diameter), you can reflect on how the circle's infinite symmetry mirrors the infinite and eternal nature of God. This idea can be tied to the understanding of God's perfection and His eternal love and presence.
- Have students reflect on how the Eucharist is central to Catholic life and how its circular form serves as a symbol of the completeness and unity found in Christ.

Other Subject Here:

- **ELA:** Students can analyze how authors use circle imagery to symbolize cycles and repetitive patterns in human life or nature.
- Some stories have circular structures, where events begin and end in similar places, reflecting a cycle or journey. For instance, hero's journey narratives often circle back to the protagonist's origins after completing their quest.
- Social Studies: Maps and Navigation: Circles are fundamental in the study of maps and navigation. especially in terms of latitude and longitude, as well as the Earth's rotation and the concept of the equator

Enrichment

- **Derive and write the equation of a circle** Have students explore derivations from geometric definitions and transform equations in different forms.
- Measure Arc Length and Use Radians -Introduce real-world applications such as gear ratios and engineering design challenges.
- **Compare Theoretical and Experimental Probability** – Have students design their own experiments, collect data, and then calculate both theoretical and experimental probabilities. Ask them to analyze discrepancies and explain possible causes.
- Determine relationships between segments in a **circle** – Let students create original geometric proofs using dynamic geometry software.
- Use the area and arc formulas Assign students projects where they must apply these formulas in real-world design tasks (e.g., blueprinting gardens or murals).

Support

- Identify and color-code parts of a circle Use visual diagrams and manipulatives to support vocabulary acquisition.
- Justify and use formulas Provide guided notes and step-by-step visual aids for using circumference and area formulas.
- Measure angles and arcs Offer scaffolded practice problems with hints or guided exploration tools (e.g., online angle tools).
- Write the equation of a circle Use templates and partially completed examples to support equation creation.
- **Apply theorems to solve problems** Use real-life scenarios and anchor charts that connect theorems to problem-solving contexts.

Standards & Benchmarks Angles and Segments in Circles:

G.G-GPE.A.1

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.G-GPE.B.4

Use coordinate geometry to prove simple geometric theorems. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.

G.G-CO.A.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.G-CO.D.13

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

G.G-C.A.3

Construct the inscribed and circumscribed circles of a triangle with technology, and investigate properties of a quadrilateral inscribed in a circle.

Relationships in Circles:

G.G-C.A.2

Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

CCSS.Math.Content.HSG-C.A.2

Identify and describe relationships among inscribed angles, radii, and chords.

G-C.2

Identify and describe relationships among inscribed angles, radii, and chords.

9.2.4.8

Identify, describe and solve situations using relationships among inscribed angles, circumscribed angles, radii and chords in circles. (MP5) ♣ ☼

C.M.GHS.29

Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Circumference and Area of a Circle:

G.G-C.B.5

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

CCSS.Math.Content.HSG-C.B.5

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G-C.5

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

C.M.GHS.30

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Teaching Ideas/Resources

Websites/Resources:

- Use this slideshow and these Student Notes to introduce parts of a circle
- Use these slideshows: <u>Part 1</u> and <u>Part 2</u>, along with these student exploration pages: <u>Part 1</u> and <u>Part 2</u>, to guide students through an exploration of Circle Theorems
- <u>Circle Theorem Student Notes</u> students complete once they have explored Circle Theorems
- Interactive sites to explore Circle theorems: <u>Geogebra Circle Theorems #1</u>, <u>Geogebra Circle Theorems #2</u>,
 <u>Interactive Maths Circle Theorems</u>, <u>Transum Circle Theorems</u>