



Willingboro Public Schools

“Where Excellence is the Expectation”

Willingboro Public Schools WPS Mathematics Foundations of Algebra

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22-23 Curriculum WPS Mathematics Foundations of Algebra
(From [New Jersey Student Learning Standards](#))

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Standard	Mathematically Proficient Students ...
1. Make sense of problems and persevere in solving them	start by explaining to themselves the meaning of a problem and looking for entry points to its solution. check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
2. Reason abstractly and quantitatively	make sense of quantities and their relationships in problem situations.
3. Construct viable arguments and critique the reasoning of others	understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. Model with mathematics	apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
5. Use appropriate tools strategically	are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attend to precision	try to communicate precisely to others.

7. Look for and make use of structure	look closely to discern a pattern or structure
8. Look for and express regularity in repeated reasoning	notice if calculations are repeated, and look both for general methods and for shortcuts.

(NJ DOE, 2024)

**New Jersey Legislative Statutes and Administrative Code
Mathematics**

Amistad Law: <i>N.J.S.A. 18A 52:16A-88</i>
Radical Math : includes lessons on topics for social justice in mathematics African American Mathematician Bios

Holocaust Law: <i>N.J.S.A. 18A:35-28</i>
https://www.bbc.com/future/article/20191031-hilda-geiringer-mathematician-who-fled-the-nazis

LGBT and Disabilities Law: <i>N.J.S.A. 18A:35-4.35</i>
https://prideinstem.org/ LGBTQA+ Mathematician Bios https://www.ngpf.org/blog/math/math-monday-celebrating-disabled-mathematicians/ Mathematicians with Disabilities

Diversity & Inclusion: <i>N.J.S.A. 18A:35-4.36a</i>
http://www.ams.org/about-us/diversity

<https://mathematicallygiftedandblack.com/>

Standards in Action: *Climate Change*

New Jersey became the first state in the nation to include climate change across content areas with the adoption of the 2020 New Jersey Student Learning Standards (NJSLS). The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Suggestions for how to incorporate climate change examples into math instruction will be noted in **green text** throughout the units.

In Foundations of Algebra, instructional time should focus on three critical areas:

- (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
- (2) grasping the concept of a function and using functions to describe quantitative relationships;
- (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x -coordinate changes by an amount A , the output or y -coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y -intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Course Sequence/Table of Contents:

1	Unit 1: Exponents, Expressions, and Equations
2	Unit 2: Functions, Equations, and Solutions
3	Unit 3: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations
4	Unit 4: Statistics and Probability: Scatterplots and Association
5	Appendix A: Special Education Accommodations and Modifications
6	Appendix B: Exemplars and Explanations
7	Appendix C: Mathematics Classroom Guide

[Click here for the Grade 8 Pacing Guide.](#)

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 1 Exponents, Expressions, and Equations	<ul style="list-style-type: none"> ● 8.EE.A.1 ● 8.G.C.9 ● 8.EE.A.3 ● 8.EE.A.4 ● 8.EE.B.5 	<ul style="list-style-type: none"> ● Develop understanding of the properties of integer exponents and how they apply to simplifying expressions. ● Apply scientific notation to express and compare very large or small numbers in real-world contexts. ● Perform operations with numbers written in scientific notation, including problems involving measurement and unit conversions. Use proportional relationships to analyze and solve real-world problems involving linear functions. ● Apply concepts of volume to solve problems involving cylinders, cones, and spheres using geometric formulas. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically..</p>
<i>Unit 1: Suggested Open Educational Resources</i>	<p>8.EE.A.1 Extending the Definitions of Exponents</p> <p>8.G.C.9 A Canister of Tennis Balls</p> <p>8.EE.A.3 Ant and Elephant</p> <p>8.EE.A.4 Giantburgers</p> <p>8.EE.B.5 Who Has the Best Job?</p>		<p>MP.6 Attend to precision.</p>
Unit 2 Functions, Equations, and Solutions	<ul style="list-style-type: none"> ● 8.F.A.1 ● 8.F.A.2 ● 8.F.A.3 ● 8.F.B.4* ● 8.F.B.5 ● 8.EE.B.6. ● 8.NS.A.1. ● 8.NS.A.2. 	<ul style="list-style-type: none"> ● Deepen understanding of irrational numbers, including their decimal representations and how to approximate or compare them with rational numbers. ● Understand and apply properties of rational and irrational numbers, including how to recognize and categorize them. 	<p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning</p>

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
	<ul style="list-style-type: none"> ● 8.NS.A.3. ● 8.EE.C.7a. ● 8.EE.C.7b. 	<ul style="list-style-type: none"> ● Solve linear equations in one variable, including those with variables on both sides and solutions involving distribution. ● Use similar triangles and slope concepts to understand the meaning of slope and y-intercept in linear relationships. Define, compare, and interpret functions as rules that assign outputs to inputs and determine whether relationships are functions. Analyze and model linear functions from tables, graphs, and equations, and understand the real-world meaning of rate of change and initial value. ● Describe how functions behave (increasing, decreasing, constant) over intervals and represent them graphically. 	
<p><i>Suggested Open Educational Resources</i></p>	<p>8.F.A.1 Function Rules 8.F.A.2 Battery Charging 8.F.A.3 Introduction to Linear Functions 8.F.B.4 Chicken and Steak, Variation 1 8.F.B.4 Baseball Cards 8.EE.C.7 The Sign of Solutions 8.EE.C.7 Coupon versus discount</p>		
<p>Unit 3</p> <p>Geometry: Pythagorean Theorem, Congruence</p>	<ul style="list-style-type: none"> ● 8.EE.A.2 ● 8.EE.A.2a ● 8.EE.A.2b ● 8.G.B.6 ● 8.G.B.7 ● 8.G.B.8* ● 8.EE.C.8a. 	<ul style="list-style-type: none"> ● Explore and apply square roots and cube roots, including evaluating and estimating their values in mathematical and real-world contexts. ● Solve systems of linear equations using multiple methods (graphing, substitution, elimination) and interpret solutions contextually. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p>

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
<p>and Similarity Transformations</p>	<ul style="list-style-type: none"> ● 8.EE.8b. ● 8.EE.C.8c. ● 8.G.A.1 ● 8.G.A.1a. ● 8.G.A.1b. ● 8.G.A.1c. ● 8.G.A.2 ● 8.G.A.3 ● 8.G.A.4 ● 8.G.A.5 	<ul style="list-style-type: none"> ● Analyze the meaning of no solution, one solution, or infinitely many solutions in a system of equations. ● Understand and apply the properties of rigid transformations (translations, reflections, rotations) to explore congruence of figures. ● Use a sequence of transformations to determine if two figures are congruent. ● Investigate the effects of dilations and understand similarity in terms of transformations. ● Apply the Angle Relationships in Parallel Lines cut by a Transversal to solve for unknown angles. Prove and use the Pythagorean Theorem and its converse to find unknown side lengths in right triangles. <p>Solve real-world and mathematical problems using the Pythagorean Theorem in two and three dimensions.</p>	<p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically..</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p>
<p><i>Unit 3: Suggested Open Educational Resources</i></p>	<p>8.G.B.6 Converse of the Pythagorean Theorem 8.G.B.7 Running on the Football Field 8.G.B.8 Finding isosceles triangles 8.G.A.1 Reflections, Rotations, and Translations 8.G.A.2 Congruent Triangles 8.G.A.3 Effects of Dilations on Length, Area, and Angles 8.G.A.4 Are They Similar 8.G.A.5 Street Intersections 8.G.A.5 Similar Triangles II 8.G.A.5 Triangle's Interior Angles</p>	<p>MP.8 Look for and express regularity in repeated reasoning.</p>	

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 4 Statistics and Probability: Scatterplots and Association	<ul style="list-style-type: none"> ● 8.SP.A.1 ● 8.SP.A.2 ● 8.SP.A.3 ● 8.SP.A.4 ● 8.F.B.4* ● 8.G.B.8* ● 8.EE.C.8c* ● 8.G.B.7. 	<ul style="list-style-type: none"> ● Investigate and interpret patterns in bivariate data, including constructing and analyzing scatter plots. ● Use lines of best fit to model relationships between two variables and make predictions. ● Describe the strength and direction of associations between variables (positive, negative, linear, non-linear). ● Understand and interpret slope and y-intercept in real-world contexts as part of linear functions. ● Solve systems of equations that model real-world problems and explain the meaning of the solution in context. ● Apply the Pythagorean Theorem to solve problems involving distance on coordinate planes and real-world geometry. ● Use the theorem in two- and three-dimensional contexts to solve for unknown side lengths and distances. 	
<i>Unit 4: Suggested Open Educational Resources</i>	8.SP.A.1 Texting and Grades 1 8.SP.A.2 Animal Brains 8.SP.A.3 US Airports 8.SP.A.4 What's Your Favorite Subject 8.SP.A.4 Music and Sports 8.F.B.4 Delivering the Mail 8.G.B.8 Finding the distance between points 8.EE.C.8 Kimi and Jordan		

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations

Overview

In this unit, students deepen their understanding of exponents, scientific notation, and proportional relationships. They apply the properties of integer exponents to simplify expressions, convert between standard and scientific notation, and perform operations with numbers expressed in scientific notation. Students also explore proportional relationships in real-world and mathematical situations, interpreting unit rate as the slope of a line. These concepts lay the groundwork for comparing quantities, solving real-world problems involving scale and magnitude, and understanding linear relationships.

Essential Questions

Enduring Understandings

- How do the properties of exponents help simplify expressions and solve problems?
- Why is scientific notation a useful tool for representing large and small numbers?
- How do we perform operations with numbers written in scientific notation?
- What does the slope of a line represent in real-world contexts?
- How can we use graphs to compare proportional relationships?

- The rules of integer exponents (including zero and negative exponents) allow us to rewrite expressions in simpler or equivalent forms.
- Scientific notation is a powerful way to express and operate with very large or very small numbers, especially in scientific and technical settings.
- Arithmetic operations with numbers in scientific notation follow the same principles as operations with base-ten numbers, supported by properties of exponents.
- The slope of a line in a graph of a proportional relationship represents the constant rate of change or unit rate, and the graph passes through the origin.
- Proportional relationships can be represented and compared using tables, graphs, and equations.

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> ● 8.EE.A.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example,</i> $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ ● 8.G.C.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. 	<p>6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> ● write a numerical expression using whole-number exponents 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Exponents as simplified representation of repeated multiplication. <p>Students are able to:</p> <ul style="list-style-type: none"> ● apply properties of exponents to numerical expressions. ● generate equivalent numerical expressions using positive and negative integer exponents. ● find volume of cones, cylinders and spheres using to solve real world problems. <p>Learning Goal 1: Apply the properties of integer exponents to write equivalent numerical expressions; apply formulas to find the volume of a cone, a cylinder, or a sphere when solving real-world and mathematical problems.</p>
<ul style="list-style-type: none"> ● 8.EE.A.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 and determine</i> 	<p>5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p>We have learned to/that...</p>	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Very large and very small quantities can be approximated with numbers expressed in the form of a single digit times an integer power of 10. <p>Students are able to:</p> <ul style="list-style-type: none"> ● estimate very large and very small quantities with numbers expressed in the form of a single digit times an integer power of 10. ● compare numbers written in the form of a single digit times an integer power of 10 and express how many times as much one is than the other. <p>Learning Goal 2: Estimate and express the values of very large or very small numbers with numbers expressed in the form of a single digit times an</p>

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>that the world population is more than 20 times larger.</p>	<ul style="list-style-type: none"> denote powers of 10 by using whole number exponents 		<p>integer power of 10. Compare numbers expressed in this form, expressing how many times larger or smaller one is than the other.</p>
<ul style="list-style-type: none"> 8.EE.A.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. 	<p>7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will</i></p> <p style="text-align: center;">$\frac{1}{10}$</p> <p><i>make an additional</i> $\frac{1}{10}$ <i>of her salary an hour, or \$2.50, for a new salary of \$27.50. If you</i></p> <p style="text-align: right;">$9\frac{3}{4}$</p> <p><i>want to place a towel bar</i> $\frac{3}{4}$ <i>inches long in the center of a</i></p> <p style="text-align: center;">$27\frac{1}{2}$</p> <p><i>door that is</i> $\frac{1}{2}$ <i>inches</i></p>	<p>MP. 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s): No new concept(s) introduced Students are able to:</p> <ul style="list-style-type: none"> multiply and divide numbers expressed in scientific notation, including problems in which one number is in decimal form and one is in scientific notation. add and subtract numbers expressed in scientific notation, including problems in which one number is in decimal form and one is in scientific notation. use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. interpret scientific notation that has been generated by technology (e.g. recognize 4.1E-2 and 4.1e-2 as 4.1 x 10⁻²). <p>Learning Goal 3: Perform operations using numbers expressed in scientific notation, including problems where both decimals and scientific notation are used. In real-world problem-solving situations, choose units of appropriate size for measurement of very small and very large quantities and interpret scientific notation generated when technology has been used for calculations.</p>

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> convert between forms (fractions, decimals, and whole numbers) as appropriate to solve multi-step real life and mathematical problems with positive and negative rational numbers in any form 		
<ul style="list-style-type: none"> 8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i> 	<p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <p>7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a</p>	<p>MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> Quantitative relationships can be represented in different ways. <p>Students are able to:</p> <ul style="list-style-type: none"> graph proportional relationships. interpret unit rate as the slope of a graph. compare two different proportional relationships that are represented in different ways (table of values, equation, graph, verbal description). <p>Learning Goal 4: Graph proportional relationships, interpreting slope as unit rate, and compare two proportional relationships, each</p>

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>straight line through the origin. 7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> • decide whether two quantities show a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin • identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships • identify the constant of proportionality 		<p>represented in different ways.</p>

Unit 1 Grade 8 Mathematics: Exponents, Expressions, and Equations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	(unit rate) in tables, graphs, and diagrams		

Integrated Accommodations and Modifications		
Special Education Students	English Language Learners	At Risk
<ul style="list-style-type: none"> Utilize modifications & accommodations delineated in the student’s IEP Provide additional manipulatives to support instruction Allow for alternative strategies to solve algorithms or tasks Provide the steps needed to complete the task Model frequently Provide repetition and practice. Use visuals to demonstrate/model the processes Restate, reread, and clarify directions/questions Ask students to restate information, directions, and assignments. Provide copy of class notes Distribute study guide for classroom tests. Provide preferential seating to be mutually determined by the student and teacher Provide extra textbooks for home. Provide regular parent/ school communication 	<p>WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors</p> <ul style="list-style-type: none"> Modify Assignments Use testing and portfolio assessment Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) Repeat, rephrase, paraphrase key concepts and directions Allow for extended time for assignment completion as needed Highlight key vocabulary Define essential vocabulary in context Use graphic organizers, visuals, manipulatives and other concrete materials Use gestures, facial expressions and body language Read aloud Build on what students already know and prior experience 	<ul style="list-style-type: none"> Pair visual prompts with verbal presentations Ask students to restate information, directions, and assignments. Provide repetition and and practice Model skills / techniques to be mastered. Provide extended time to complete class work Provide copy of class notes Provide preferential seating to be mutually determined by the student and teacher Allow the use of a computer to complete assignments. Establish expectations for correct spelling on assignments Provide extra textbooks for home. Provide Peer Support Increase one on one time

<ul style="list-style-type: none"> ● Allow extended time to complete assignment ● Establish procedures for accommodations / modifications for assessments ● Allow student to take/complete tests in an alternate setting as needed <p>Appendix A: Special Education Accommodations and Modifications</p>		
Gifted and Talented Students		504 Plan
<ul style="list-style-type: none"> ● Utilize advanced, accelerated, or compacted content ● Provide assignments that emphasize higher- level thinking skills. ● Allow for individual student interest ● Gear assignments to development in areas of affect, creativity, cognition, and research skills ● Allow for a variety in types of resources ● Provide problem-based assignments with planned scope and sequence ● Utilize inquiry-based instruction ● Adjust the pace of lessons ● Utilize Choice Boards ● Provide Problem-Based Learning ● Establish flexible Grouping 	<ul style="list-style-type: none"> ● Pair visual prompts with verbal presentations ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Break long assignments into smaller parts ● Assist student in setting short term goals ● Allow for preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Model and reinforce organizational systems (i.e. color-coding) ● Write out homework assignments, check student's recording of assignments 	
Interdisciplinary Connections		Computer Science and Design Thinking
<p>ELA Standards</p> <ul style="list-style-type: none"> ● SL.PE.8.1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others’ ideas and expressing their own clearly. ● L.KL.8.2. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases. 	<p>Computer Science and Design Thinking Practices</p> <ul style="list-style-type: none"> ● Fostering an Inclusive Computing and Design Culture ● Collaborating Around Computing and Design ● Recognizing and Defining Computational Problems ● Developing and Using Abstractions ● Creating Computational Artifacts ● Testing and Refining Computational Artifacts 	

- W.RW.8.7. Write routinely over extended time frames (time for research, reflection, metacognition/self-correction, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Exponents come into play in terms of social media, the spread of infectious diseases, the efficacy of vaccinations, population growth, and radioactive decay. Additionally, scientific notation is used to help keep track of very large and very small numbers, distances, sizes, and populations.

Project Ideas:

- [g8 Exponents Zombie Project](#)

- Communicating About Computing and Design

Computer Science and Design Thinking Standards

- 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch).
- 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.

Computer Applications for Math Classes

- Zearn (access via Clever)
- Eureka Math in Sync videos (access via Clever)
- xtramath.org
- Google Sheets for Problem Solving

Mathematics Virtual Manipulatives:

- Algebra Virtual Manipulatives
[teachMathematics: Algebra Virtual Manipulatives](#)
- Simulations
 - Phet Simulations
<https://phet.colorado.edu/en/simulations/filter?subjects=math&type=html&sort=alpha>
 - Interactive Exponents Applet [Interactive Exponential Function Graph/Applet](#)
 - Exponent Pirate Game [Exponents Pirate Game](#)
 - Exponents Games:
[Exponents Games](#)
[Math Games on Exponents for Children in 4th to 7th Grade to Test Varied Math Skills Online.](#)
- Exponent Calculator with notes:
[Exponent Calculator](#)

Career Readiness, Life Literacies and Key Skills

Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity, increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards**9.1 Personal Financial Literacy**

- 9.1.8.CR.2: Compare various ways to give back through strengths, passions, goals, and other personal factors.
- 9.1.8.CP.1: Compare prices for the same goods or services.
- 9.1.8.CP.2: Analyze how spending habits affect one's ability to save.
- 9.1.8.CP.3: Explain the purpose of a credit score and credit record, the factors and impact of credit scores.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).
- 9.1.8.FP.5: Determine how spending, investing, and using credit wisely contributes to financial well-being.

9.2 Career Awareness, Exploration, Preparation, and Training

- 9.2.8.CAP.1: Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
- 9.2.8.CAP.2: Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.4 Life Literacies and Key Skills

21st Century Skills

- 9.4.8.CI.1: Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions (6.SP.B.5)
- 9.4.8.GCA.2: Demonstrate openness to diverse ideas and perspectives through active discussions to achieve a group goal.

Technology Integration

- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping. (6.SP.B.4)
- 9.4.8.IML.4: Ask insightful questions to organize different types of data and create meaningful visualizations.
- 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.

SEL Competencies
<ul style="list-style-type: none"> ● Self - Awareness ● Self - Management ● Social Awareness ● Responsible Decision Making ● Relationship Skills <p>https://www.nj.gov/education/safety/wellness/selearning/index.shtml</p>

District/School Formative Assessment Plan	District/School Summative Assessment Plan
<p><i>Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards.</i></p> <p>Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students’ mastery of content through a variety of methods:</p> <ul style="list-style-type: none"> ● Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom’s Taxonomy) ● Exit tickets, rotational activities (stations), quizzes, and small group activities ● Classwork, homework, group work (formative assessment) ● Pre-Assessment, teacher’s observation, class discussion, and journal 	<p><i>Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.</i></p> <p>Benchmark Assessments:</p> <ul style="list-style-type: none"> ● Assessment 8.1.1: Mid-Unit Assessment ● Assessment 8.1.2: End of Unit Assessment ● Assessment 8.1.3: End of Unit Performance Assessment <p>Standardized Assessments:</p> <ul style="list-style-type: none"> ● NJSLA <p>Other Summative Assessments: Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.</p>

Focus Mathematical Concepts		
Targeted Academic Vocabulary	Common Misconceptions	Fluency
<ul style="list-style-type: none"> ● Exponent ● Base ● Power ● Zero exponent ● Negative exponent 	<p>8.EE.A.1</p> <ul style="list-style-type: none"> ● Students may think a negative exponent makes the value negative (e.g., thinking $2^{-3} = -82^{-3} = -82^{-3} = -8$). ● Students may believe that any number to the 0 power is 0, rather than understanding it equals 1. 	<p>Math Fluency Support for Grades 6-8</p> <p>Targeted Fact reinforcement</p> <p>Fact Fluency</p>

<ul style="list-style-type: none"> ● Product rule ● Quotient rule ● Scientific notation ● Standard form ● Power of ten ● Significant digits ● Slope ● Proportional relationship ● Constant of proportionality ● Unit rate ● Coordinate plane ● Origin 	<ul style="list-style-type: none"> ● Students often confuse exponent rules, such as adding bases or multiplying exponents incorrectly (e.g., thinking $32 \cdot 34 = 383^2 \cdot 3^4 = 3^8 32 \cdot 34 = 38$). <p>8.EE.A.3</p> <ul style="list-style-type: none"> ● Students may think that larger exponents always mean larger numbers, even when the base (coefficient) is smaller. ● Students often struggle to convert between scientific and standard notation, especially when working with negative exponents. ● They may misplace the decimal point when rewriting scientific notation into standard form (e.g., $4.5 \times 10^{-34.5} \times 10^{-3}$ as 450 instead of 0.0045). <p>8.EE.A.4</p> <ul style="list-style-type: none"> ● Students may perform operations on exponents incorrectly (e.g., adding or subtracting exponents when multiplying or dividing coefficients). ● Students may forget to adjust exponents when performing operations, leading to errors in simplification. ● They may assume scientific notation can be added/subtracted without converting to the same exponent. <p>8.EE.B.5</p> <ul style="list-style-type: none"> ● Students may think all straight lines are proportional, even if they don't pass through the origin. ● Students may confuse the slope as just the “rise” or just the “run” rather than the ratio of the two. Students may not recognize that unit rate and slope are equivalent in proportional relationships. ● They may misinterpret the graph, thinking a steeper line always means a greater value, regardless of units. 	<p>Dr. Riccomini's Math Facts – Basic Facts Link Math Fact Practice http://www.xtramath.org Differentiation problems https://www.openmiddle.com/</p>
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District/School Tasks	District/School Primary and Supplementary Resources
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<ul style="list-style-type: none"> ● NJDOE Digital Item Library ● NJSLA Released Items in Mathematics ● NJSLA Mathematics Practice Tests ● NJSLA Mathematics Evidence Statements ● Progression Tables ● GUES Strategy + ECR problems 	<p><u>District-Mandated Resources</u></p> <ul style="list-style-type: none"> ● Eureka Mathematics ● EdGenuity/MyPath ● Zearn <p>Assessment Resources:</p> <ul style="list-style-type: none"> ● 8.EE.A.1 Extending the Definitions of Exponents ● 8.G.C.9 A Canister of Tennis Balls ● 8.EE.A.3 Ant and Elephant ● 8.EE.A.4 Giantburgers ● 8.NS.A.1 Converting Decimal Representations of Rational Numbers to Fraction Representations ● 8.NS.A.2 Irrational Numbers on the Number Line ● 8.EE.B.5 Who Has the Best Job? ● 8.EE.B.6 Slopes Between Points on a Line <p>Other Resources:</p> <ul style="list-style-type: none"> ● 100 Questions that Promote Mathematical Discourse ● Asking Effective Questions in Mathematics ● Amistad Law Resources ● Holocaust Law Resources ● LGBTQ+ and Disability Resources ● Climate Change Resources <p>Additional Mathematics Resources</p> <ul style="list-style-type: none"> ● EmbarcOnline [https://embarc.online/] ● Dan Meyer’s 3-Act Math Tasks [https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTOEsfqouLWNNC6Z4/edit#gid=0] ● OpenMiddle.com ● CPalms [https://www.cpalms.org/Public/], and ● Illustrative Mathematics [https://www.illustrativemathematics.org/] <p>Project Ideas:</p> <ul style="list-style-type: none"> ● g8 Exponents Zombie Project
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Instructional Best Practices and Exemplars

See Appendix B for Instructional Best Practices and Exemplars

Pacing Guide
24-25 WPS Pacing Guide Mathematics Foundations of Algebra

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Overview

In this unit, students develop a deep understanding of functions, equations, and the number system. They learn to distinguish functions from other types of relationships and explore the connections between input-output pairs, tables, graphs, and algebraic expressions. Students explore rational and irrational numbers, approximating their values and understanding their placement on the number line. They also learn to solve linear equations—including those with variables on both sides—and use geometric reasoning to explain the consistency of slope in linear functions. These concepts build toward fluency in analyzing patterns, solving real-world problems, and justifying reasoning with precision and structure.

Essential Questions

- What makes a relationship a function, and how can we represent it in different ways?
- How do we determine if an equation or graph represents a function?
- How do we solve linear equations, including those with variables on both sides or with multiple steps?
- How are rational and irrational numbers different, and how can we estimate or compare them?
- What does the slope of a line represent, and how can we prove it is constant?
- How can we identify and interpret the rate of change and initial value in real-world contexts?

Enduring Understandings

- A function assigns exactly one output for each input, and it can be represented using tables, graphs, equations, or verbal descriptions.
- The domain and range describe the set of inputs and outputs of a function, and these can vary depending on the context.
- The slope of a non-vertical line is constant and can be explained using similar triangles, forming the basis for the linear equation $y=mx+by = mx + by=mx+b$.
- Linear equations can be solved using properties of equality and require attention to structure and potential extraneous solutions.
- Rational numbers can be expressed as fractions or terminating/repeating decimals; irrational numbers cannot, but both types of numbers can be located approximately on the number line.
- Mathematical models and equations can be used to represent and solve real-world problems involving linear relationships and functions.

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> ● 8.F.A.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Clarification: Function notation is not required in Grade 8) 	<p>5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> ● form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane 	<p>MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● A function is a rule. ● If a rule is a function, then for each input there is exactly one output. <p>Students are able to:</p> <ul style="list-style-type: none"> ● use function language. ● describe a function as providing a single output for each input. ● determine whether non-numerical relationships are functions. ● describe a function as a set of ordered pairs. ● read inputs and outputs from a graph. ● describe the ordered pairs as containing an input, and the corresponding output. <p>Learning Goal 1: Define a function as a rule that assigns one output to each input and determine if data represented as a graph or in a table is a function.</p>
<ul style="list-style-type: none"> ● 8.F.A.2. Compare properties (e.g. rate of 	<p>n/a</p>	<p>MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p>

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p>		<p>MP.8 Look for and express regularity in repeated reasoning.</p>	<ul style="list-style-type: none"> • Functions (quantitative relationships) can be represented in different ways. • Functions have properties; properties of linear functions. <p>Students are able to:</p> <ul style="list-style-type: none"> • analyze functions represented algebraically, as a table of values, and as a graph. • interpret functions represented by a verbal description. • given two functions, each represented in a different way, compare their properties. <p>Learning Goal 2: Compare two functions each represented in a different way (numerically, verbally, graphically, and algebraically) and draw conclusions about their properties (rate of change and intercepts).</p>
<ul style="list-style-type: none"> • 8.F.A.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), 	<p>6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the</p>	<p>MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> • A linear function is defined by the equation $y = mx + b$. • The graph of a linear function is a straight line. <p>Students are able to:</p> <ul style="list-style-type: none"> • analyze tables of values, graphs, and equations in order to classify a function as linear or non-linear. • determine if equations presented in forms other than $y = mx + b$ (for example $3y - 2x = 7$) define a linear function. • give examples of equations that are non-linear functions. • show that a function is not linear using pairs of points.

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>which are not on a straight line.</p>	<p>equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> two quantities which change in relationship to one another are expressed as independent and dependent variables write an equation using two quantities, an independent and a dependent variable, to represent a real-world problem 		<p>Learning Goal 3: Classify functions as linear or non-linear by analyzing equations, graphs, and tables of values; interpret the equation $y = mx + b$ as defining a linear function.</p>
<ul style="list-style-type: none"> 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two 	<p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <p>7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs,</p>	<p>MP.6 Attend to precision. MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> As with equations, two (x,y) values can be used to construct a function. <p>Students are able to:</p> <ul style="list-style-type: none"> determine the rate of change and initial value of a function from a description of a relationship.

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>(x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>	<p>equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships identify the constant of proportionality (unit rate) in tables, graphs, and diagrams <p>6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and</p>		<ul style="list-style-type: none"> determine the rate of change and initial value of a function from two (x, y) values by reading from a table of values. determine the rate of change and initial value of a function from two (x, y) values by reading these from a graph. construct a function in order to model a linear relationship. interpret the rate of change and initial value of a linear function in context. <p>Learning Goal 4: Model a linear relationship by constructing a function from two (x, y) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> two quantities which change in relationship to one another are expressed as independent and dependent variables write an equation using two quantities, an independent and a dependent variable, to represent a real-world problem 		
<ul style="list-style-type: none"> 8.F.B.5, Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is 	n/a	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p>	<p>Concept(s): No new concept(s) introduced</p> <p>Students are able to:</p> <ul style="list-style-type: none"> analyze a graph. provide qualitative descriptions of graphs (e.g. where increasing or decreasing, linear or non-linear).

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>		<p>MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.</p>	<ul style="list-style-type: none"> given a verbal description, sketch a graph of a function based on the qualitative features described. <p>Learning Goal 5: Sketch a graph of a function from a qualitative description and give a qualitative description of a graph of a function.</p>
<ul style="list-style-type: none"> 8.EE.C.7 Solve linear equations in one variable. 8.EE.C.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). 	<p>7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p>We have learned to/tha...</p> <ul style="list-style-type: none"> apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients <p>7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p>	<p>MP.5 Use appropriate tools strategically. MP.6 Attend to precision.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> Linear equations may have an infinite number of solutions. Linear equations may have no solution or a single solution. <p>Students are able to:</p> <ul style="list-style-type: none"> give examples of linear equations in one variable with one solution ($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$.) transform a given equation, using the properties of equality, into simpler forms. transform a given equation until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (<i>a and b are different numbers</i>). solve linear equations that have fractional coefficients; include equations requiring use of the distributive property and collecting like terms. <p>Learning Goal 6: Apply the distributive property and collect like terms to solve linear equations in one variable that contain rational numbers as coefficients. Use an equivalent equation of the form $x = a$, $a = a$, or $a = b$</p>

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>8.EE.C.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p>	<p>7.EE.B.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where P, q, and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> • solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers with accuracy and efficiency 		<p>(where a and b are different numbers) to describe the number of solutions.</p>

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> 8.EE.B.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. 	<p>7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>We have learned to/that:</p> <ul style="list-style-type: none"> use scale drawings to find actual lengths and distances. scale factors help us enlarge or reduce figures proportionally. create scale drawings and interpret scale as a ratio. <p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <p>We have learned to/that:</p> <ul style="list-style-type: none"> identify proportional relationships in tables, graphs, 	<p>MP1: Make sense of problems and persevere in solving them.</p> <p>MP3: Construct viable arguments and critique the reasoning of others.</p> <p>MP6: Attend to precision.</p> <p>MP7: Look for and make use of structure.</p> <p>MP8: Look for and express regularity in repeated reasoning.</p>	<p>Concepts</p> <ul style="list-style-type: none"> The slope of a line is constant and can be understood using similar triangles. Similar triangles demonstrate that the ratio of vertical change to horizontal change is the same between any two points on a line. The equation of a line in slope-intercept form $y = mx + by = mx + by = mx + b$ models linear relationships with slope m and y-intercept b. <p>Students Are Able To</p> <ul style="list-style-type: none"> Explain why slope is constant for a non-vertical line using properties of similar triangles. Identify and construct similar triangles on the coordinate plane between two points on a line. Derive and interpret the slope-intercept form equation $y = mx + by = mx + by = mx + b$ of a line. Use the equation $y = mx + by = mx + by = mx + b$ to describe and analyze linear relationships. Apply geometric reasoning to connect algebraic and graphical representations. <p>Learning Goal 7: use similar triangles to explain why the slope between any two points on a non-vertical line is always the same, derive the slope-intercept form equation $y = mx + by = mx + by = mx + b$, and interpret the slope and y-intercept to describe and analyze linear relationships both geometrically and algebraically.</p>

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>equations, and verbal descriptions.</p> <ul style="list-style-type: none"> the constant of proportionality (unit rate) defines the relationship. represent proportional relationships using equations and analyze them using graphs. <p>8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. We have learned to/that:</p>		

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<ul style="list-style-type: none"> • use angle relationships (complementary, supplementary, vertical, and adjacent) to solve for unknown angles. • when parallel lines are cut by a transversal, special angle pairs form predictable patterns. • justify angle relationships using informal reasoning and diagrams. 		
<ul style="list-style-type: none"> • 8.NS.A.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which 	<p>7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>We have learned to/that:</p> <ul style="list-style-type: none"> • add, subtract, multiply, and divide 	<p>MP2: Reason abstractly and quantitatively by understanding the difference between rational and irrational numbers.</p> <p>MP3: Construct viable arguments to explain why certain numbers are irrational.</p> <p>MP6: Attend to precision when distinguishing between types of numbers.</p>	<p>Concepts</p> <ul style="list-style-type: none"> • Numbers can be classified as rational (expressible as fractions) or irrational (cannot be expressed as fractions). Irrational numbers include non-terminating, non-repeating decimals like $2\sqrt{2}$ and π. • The real number system includes both rational and irrational numbers. <p>Students Are Able To</p> <ul style="list-style-type: none"> • Identify and classify numbers as rational or irrational. • Explain why certain numbers are irrational using examples.

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>repeats eventually into a rational number.</p>	<p>all rational numbers, including positive and negative fractions and decimals.</p> <ul style="list-style-type: none"> operations with negative numbers follow specific rules that extend the properties of operations with whole numbers. apply these operations in real-world contexts and solve problems involving rational numbers. 		<ul style="list-style-type: none"> Represent irrational numbers approximately on the number line. <p>Learning Goal 8: distinguish between rational and irrational numbers, explain why some numbers are irrational, and represent both types on a number line.</p>
<ul style="list-style-type: none"> 8.NS.A.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\sqrt{2}$). For example, by truncating the 	<p>N/A</p>	<p>MP1: Make sense of problems involving very large or very small numbers. MP6: Attend to precision when converting between scientific and decimal notation and performing operations. MP7: Look for and make use of structure when applying exponent rules in scientific notation.</p>	<p>Concepts</p> <ul style="list-style-type: none"> Scientific notation expresses very large or very small numbers as a product of a number between 1 and 10 and a power of 10. Operations (multiplication, division, addition, subtraction) can be performed with numbers in scientific notation. Conversions between scientific and decimal forms are essential for understanding scale and magnitude. <p>Students Are Able To</p> <ul style="list-style-type: none"> Convert numbers between decimal and scientific notation accurately.

Unit 2: Grade 8 Mathematics: Functions, Equations, and Solutions

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</p>			<ul style="list-style-type: none"> • Perform multiplication and division with numbers in scientific notation. • Solve real-world problems using operations with scientific notation. <p>Learning Goal 9: convert numbers between scientific and decimal notation and accurately perform operations with numbers written in scientific notation to solve problems involving very large or very small quantities.</p>
<ul style="list-style-type: none"> • 8.NS.A.3. Understand that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. 			

Integrated Accommodations and Modifications

Special Education Students	English Language Learners	At Risk
<ul style="list-style-type: none"> ● Utilize modifications & accommodations delineated in the student’s IEP ● Provide additional manipulatives to support instruction ● Allow for alternative strategies to solve algorithms or tasks ● Provide the steps needed to complete the task ● Model frequently ● Provide repetition and practice. ● Use visuals to demonstrate/model the processes ● Restate, reread, and clarify directions/questions ● Ask students to restate information, directions, and assignments. ● Provide copy of class notes ● Distribute study guide for classroom tests. ● Provide preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Provide regular parent/ school communication ● Allow extended time to complete assignment ● Establish procedures for accommodations / modifications for assessments ● Allow student to take/complete tests in an alternate setting as needed <p>Appendix A: Special Education Accommodations and Modifications</p>	<p>WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors</p> <ul style="list-style-type: none"> ● Modify Assignments ● Use testing and portfolio assessment ● Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) ● Repeat, rephrase, paraphrase key concepts and directions ● Allow for extended time for assignment completion as needed ● Highlight key vocabulary ● Define essential vocabulary in context ● Use graphic organizers, visuals, manipulatives and other concrete materials ● Use gestures, facial expressions and body language ● Read aloud ● Build on what students already know and prior experience 	<ul style="list-style-type: none"> ● Pair visual prompts with verbal presentations ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Provide preferential seating to be mutually determined by the student and teacher ● Allow the use of a computer to complete assignments. ● Establish expectations for correct spelling on assignments ● Provide extra textbooks for home. ● Provide Peer Support ● Increase one on one time

<p style="text-align: center;">Gifted and Talented Students</p>	<p style="text-align: center;">504 Plan</p>
<ul style="list-style-type: none"> ● Utilize advanced, accelerated, or compacted content ● Provide assignments that emphasize higher- level thinking skills. ● Allow for individual student interest ● Gear assignments to development in areas of affect, creativity, cognition, and research skills ● Allow for a variety in types of resources ● Provide problem-based assignments with planned scope and sequence ● Utilize inquiry-based instruction ● Adjust the pace of lessons ● Utilize Choice Boards ● Provide Problem-Based Learning ● Establish flexible Grouping 	<ul style="list-style-type: none"> ● Pair visual prompts with verbal presentations ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Break long assignments into smaller parts ● Assist student in setting short term goals ● Allow for preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Model and reinforce organizational systems (i.e. color-coding) ● Write out homework assignments, check student's recording of assignments
<p style="text-align: center;">Interdisciplinary Connections</p>	<p style="text-align: center;">Computer Science and Design Thinking</p>
<p>ELA Standards</p> <ul style="list-style-type: none"> ● SL.PE.8.1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others’ ideas and expressing their own clearly. ● L.KL.8.2. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases. ● W.RW.8.7. Write routinely over extended time frames (time for research, reflection, metacognition/self- correction, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. <p>The basic definition of functions can be somewhat confusing when looked at from a purely mathematical perspective. The assignment of a rule where each “input has exactly one output” is not clear until we look at applications. When</p>	<p>Computer Science and Design Thinking Practices</p> <ul style="list-style-type: none"> ● Fostering an Inclusive Computing and Design Culture ● Collaborating Around Computing and Design ● Recognizing and Defining Computational Problems ● Developing and Using Abstractions ● Creating Computational Artifacts ● Testing and Refining Computational Artifacts ● Communicating About Computing and Design <p>Computer Science and Design Thinking Standards</p> <ul style="list-style-type: none"> ● 8.2.8.ITH.2: Compare how technologies have influenced society over time. ● 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch).

you look at your bank account balance, you can have only one amount of money in your account. You cannot have both \$100 and \$500 in your account at the exact same time. You can, however, have the same amount in your account for multiple days. Similarly, the 8th grade could have 8 different graduation parties on the same night, but each person can only attend one of these parties at any one time.

When we talk about linear functions, we begin to look at proportional relationships, where we can compare their values. This leads to decisions about which company to use (such as taxis or phone companies), supply-demand curves, advertising dollars, etc. Similar skills are also used in navigation and military applications..

Project Ideas:

- [G8. Simultaneous Equation Project](#)
- [G8. The Progression Project](#)

- 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
- 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch).
- 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.

Computer Applications for Math Classes

- Zearn (access via Clever)
- Eureka Math in Sync videos (access via Clever)
- xtramath.org
- Google Sheets for Problem Solving

Mathematics Virtual Manipulatives:

- Algebra Tiles: <https://mathsbot.com/manipulatives/tiles>

Career Readiness, Life Literacies and Key Skills

Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards

9.1 Personal Financial Literacy

- 9.1.8.CR.2: Compare various ways to give back through strengths, passions, goals, and other personal factors.
- 9.1.8.CP.1: Compare prices for the same goods or services.
- 9.1.8.CP.2: Analyze how spending habits affect one’s ability to save.

- 9.1.8.CP.3: Explain the purpose of a credit score and credit record, the factors and impact of credit scores.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).
- 9.1.8.FP.5: Determine how spending, investing, and using credit wisely contributes to financial well-being.

9.2 Career Awareness, Exploration, Preparation, and Training

- 9.2.8.CAP.1: Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
- 9.2.8.CAP.2: Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.4 Life Literacies and Key Skills

21st Century Skills

- 9.4.8.CI.1: Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions (6.SP.B.5)
- 9.4.8.GCA.2: Demonstrate openness to diverse ideas and perspectives through active discussions to achieve a group goal.

Technology Integration

- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping. (6.SP.B.4)
- 9.4.8.IML.4: Ask insightful questions to organize different types of data and create meaningful visualizations.
- 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.

SEL Competencies

- **Self - Awareness**
- **Self - Management**
- **Social Awareness**
- **Responsible Decision Making**
- **Relationship Skills**

<https://www.nj.gov/education/safety/wellness/selearning/index.shtml>

District/School Formative Assessment Plan

Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards.

District/School Summative Assessment Plan

Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.

<p>Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods:</p> <ul style="list-style-type: none"> • Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) • Exit tickets, rotational activities (stations), quizzes, and small group activities • Classwork, homework, group work (formative assessment) • Pre-Assessment, teacher's observation, class discussion, and journal 	<p>Benchmark Assessments:</p> <ul style="list-style-type: none"> • Assessment 8.2.1: Mid-Unit Assessment • Assessment 8.2.2: End of Unit Assessment • Assessment 8.2.3: End of Unit Performance Assessment <p>Standardized Assessments:</p> <ul style="list-style-type: none"> • NJSLA <p>Other Summative Assessments: Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.</p>
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Focus Mathematical Concepts

Targeted Academic Vocabulary	Common Misconceptions	Fluency
<p>Vocabulary</p> <ul style="list-style-type: none"> • Slope • Similar triangles • Slope-intercept form • Linear equation • Y-intercept • Rational number • Irrational number • Scientific notation • Exponent • Base (of an exponent) • Power • Real numbers • Square root • Cube root • Function • Input • Output 	<p>8.EE.B.6</p> <ul style="list-style-type: none"> • Students may think slope changes between different points on the same line. • Confusing similar triangles with any triangles, missing the proportionality needed to justify constant slope. • Believing the slope formula applies only to positive slopes or specific line types. <p>8.EE.C.7a</p> <ul style="list-style-type: none"> • Misunderstanding that solutions to equations must be integers or rational numbers, overlooking irrational solutions. • Assuming all radical expressions can be simplified to whole numbers. • Confusing the steps to isolate variables when radicals are involved. <p>8.EE.C.7b</p>	<p>Math Fluency Support for Grades 6-8</p> <p>Targeted Fact reinforcement</p> <p>Fact Fluency Dr. Riccomini's Math Facts – Basic Facts Link Math Fact Practice http://www.xtramath.org Differentiation problems https://www.openmiddle.com/</p>

<ul style="list-style-type: none"> ● Domain ● Range ● Function rule ● Proportional relationship ● Constant of proportionality ● Linear function ● Graph of a function ● Rate of change ● Independent variable ● Dependent variable ● Coordinate plane ● Equation ● Variable ● Expression ● Operation 	<ul style="list-style-type: none"> ● Thinking that squaring both sides of an equation never introduces extraneous solutions. ● Ignoring or missing extraneous solutions after solving radical equations. ● Believing radical equations always have two solutions. <p>8.F.A.1</p> <ul style="list-style-type: none"> ● Misinterpreting a function as any relation, not recognizing the unique input-output rule. ● Thinking a function can assign multiple outputs to one input. ● Confusing the function notation $f(x)f(x)f(x)$ with multiplication. <p>8.F.A.2</p> <ul style="list-style-type: none"> ● Struggling to identify input (independent variable) versus output (dependent variable). ● Believing that the domain includes only positive numbers or whole numbers. ● Confusing domain and range or mixing up their definitions. <p>8.F.A.3</p> <ul style="list-style-type: none"> ● Not understanding how to determine the domain from a graph or table. ● Assuming the domain of a function is always all real numbers. ● Confusing continuous and discrete domains. <p>8.F.B.4</p> <ul style="list-style-type: none"> ● Thinking that a function must be linear or proportional to be a function. ● Struggling to write a function rule from a table of values. ● Ignoring or misinterpreting non-integer or negative outputs in function rules. <p>8.F.B.5</p>	
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	<ul style="list-style-type: none"> • Believing that every function rule must be written as $y=mx+by = mx + by=mx+b$. • Struggling to connect the rate of change from a table or graph to the slope in the equation. • Confusing rate of change with initial value or vice versa. 	
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District/School Tasks	District/School Primary and Supplementary Resources
<ul style="list-style-type: none"> • NJDOE Digital Item Library • NJSLA Released Items in Mathematics • NJSLA Mathematics Practice Tests • NJSLA Mathematics Evidence Statements • Progression Tables • GUES Strategy + ECR problems 	<p><u>District-Mandated Resources</u></p> <ul style="list-style-type: none"> • Eureka Mathematics • EdGenuity/MyPath • Zearn <p>Assessment Resources:</p> <ul style="list-style-type: none"> • 8.F.A.1 Function Rules • 8.F.A.2 Battery Charging • 8.F.A.3 Introduction to Linear Functions • 8.F.B.4 Chicken and Steak, Variation 1 • 8.F.B.4 Baseball Cards • 8.EE.C.7 The Sign of Solutions • 8.EE.C.7 Coupon versus discount • 8.EE.C.8a Intersection of Two Lines • 8.EE.C.8 How Many Solutions <p>Other Resources:</p> <ul style="list-style-type: none"> • 100 Questions that Promote Mathematical Discourse • Asking Effective Questions in Mathematics • Amistad Law Resources • Holocaust Law Resources • LGBTQ+ and Disability Resources • Climate Change Resources <p>Additional Mathematics Resources</p> <ul style="list-style-type: none"> • Embarc online [https://embarc.online/]

	<ul style="list-style-type: none"> • Dan Meyer’s 3-Act Math Tasks [https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTOEsfqouLWNNC6Z4/edit#gid=0] • OpenMiddle.com • CPalms [https://www.epalms.org/Public/], and • Illustrative Mathematics [https://www.illustrativemathematics.org/]. <p>Project Ideas:</p> <ul style="list-style-type: none"> • G8. Simultaneous Equation Project • G8.The Progression Project
Instructional Best Practices and Exemplars	
See Appendix B for Instructional Best Practices and Exemplars	
Pacing Guide	
22-23 WPS Pacing Guide Mathematics Foundations of Algebra	

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations	
Overview	
<p>This unit focuses on algebraic reasoning and geometry in the 8th grade. Students extend their understanding of equations by solving systems of linear equations algebraically and graphically. They apply these skills to model and interpret real-world situations. In geometry, students explore transformations, congruence, similarity, and the properties of angles and triangles. They also develop and apply formulas for the volume of cones, cylinders, and spheres, and use the Pythagorean Theorem in problem-solving.</p>	
Essential Questions	Enduring Understandings

<ul style="list-style-type: none"> • How can algebraic methods be used to represent and solve real-world problems involving systems of equations? • How do transformations help us understand congruence and similarity? What relationships exist between angles formed by intersecting lines or within geometric figures? • How can we derive and apply formulas to calculate volume for different 3D shapes? • How can we use the Pythagorean Theorem to solve real-world problems involving distance? 	<ul style="list-style-type: none"> • A system of equations represents a situation where two conditions must be met simultaneously; solutions can be found by graphing, substitution, or elimination. • Transformations such as translations, rotations, reflections, and dilations preserve specific properties and are useful for establishing congruence and similarity. • Angle relationships (complementary, supplementary, vertical, and adjacent) and triangle properties can be used to solve geometric problems. • The Pythagorean Theorem establishes a fundamental relationship between the sides of a right triangle and has applications in geometry and real-life contexts. • The volume of cones, cylinders, and spheres can be derived and applied in real-world scenarios using established formulas.
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Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> • 8.EE.A.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number. • 8.EE.A.2a. Evaluate square roots of small perfect squares and cube roots of small 	<p>5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p>	<p>MP1: Make sense of problems and persevere in solving them MP2: Reason abstractly and quantitatively MP3: Construct viable arguments and critique the reasoning of others MP6: Attend to precision MP7: Look for and make use of structure MP8: Look for and express</p>	<p>Concepts</p> <ul style="list-style-type: none"> • Square and cube roots are inverse operations of squaring and cubing. • Square roots of perfect squares and cube roots of perfect cubes can be evaluated exactly. • Some roots, like $2\sqrt{2}$, are irrational and cannot be written as fractions. • Solving equations involving roots requires understanding and applying inverse operations. • Equations like $x^2=px^2 = px^2=p$ have both positive and negative solutions. <p>Students Are Able To</p>

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>8.EE.A.2.b</p> <p>Recognize that squaring and taking the square root (or cubing and taking the cube root) are inverse operations. Use this understanding to solve equations and evaluate expressions involving radicals and exponents.</p>	<p>We have learned to...</p> <ul style="list-style-type: none"> Recognize and explain patterns when multiplying and dividing by powers of 10. Use exponents to represent powers of 10. <p>6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.</p> <p>We have learned to...</p> <ul style="list-style-type: none"> Write expressions using whole-number exponents. Evaluate powers like 2^{32} or 10^{21} accurately. <p>6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the</p>	<p>regularity in repeated reasoning</p>	<ul style="list-style-type: none"> Use square root and cube root symbols to represent solutions to equations. Solve equations of the form $x^2=px^2 = px^2=p$ and $x^3=px^3 = px^3=p$ where p is rational. Identify and evaluate perfect square and cube roots. Recognize irrational roots such as $2\sqrt{2}$. Explain and apply the inverse relationship between squaring and square rooting, and between cubing and cube rooting. <p>Learning Goal 1: understand and apply the inverse relationships between exponents and roots to solve equations, evaluate perfect roots, and distinguish between rational and irrational results in both mathematical and real-world contexts.</p>

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>plane with negative number coordinates. We have learned to...</p> <ul style="list-style-type: none"> • Identify rational numbers on a number line, including negatives. • Represent numbers and opposites as locations on the number line. <p>6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. We have learned to...</p> <ul style="list-style-type: none"> • Use coordinates to describe positions and distances. • Interpret absolute value as distance from zero. <p>7.NS.A.1–3 Apply and extend previous understandings of operations with fractions to</p>		

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>add, subtract, multiply, and divide rational numbers. We have learned to...</p> <ul style="list-style-type: none"> • Add, subtract, multiply, and divide positive and negative rational numbers. • Solve problems involving rational • number operations. <p>7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form. Apply properties of operations to simplify and evaluate expressions. We have learned to...</p> <ul style="list-style-type: none"> • Solve multi-step equations and expressions using rational numbers. • Use inverse operations and properties (like 		

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	distributive) to simplify.		
<p>8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.</p> <p>8.EE.C.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>8.EE.C.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</p> <p>8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given</p>	<p>6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true</p> <p>We have learned to...</p> <ul style="list-style-type: none"> Understand that solving an equation means finding values that make the equation true. Use substitution to test whether a number is a solution to an equation. 	<p>MP1: Make sense of problems and persevere in solving them MP2: Reason abstractly and quantitatively MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure MP8: Look for and express regularity in repeated reasoning</p>	<p>Concepts</p> <ul style="list-style-type: none"> A system of linear equations consists of two or more equations with the same variables. The solution to a system is the point(s) where the graphs of the equations intersect. Systems can be solved by graphing, substitution, or elimination. Some systems have one solution, no solution, or infinitely many solutions. Real-world problems can be modeled and solved using systems of equations. <p>Students Are Able To</p> <ul style="list-style-type: none"> Understand that the solution to a system of two linear equations is the point of intersection. Solve systems of equations algebraically (substitution and elimination) and verify by graphing. Solve real-world problems by representing and solving systems of equations. Analyze and interpret the solution of a system in the context of a problem. Identify whether a system has one, none, or infinitely many solutions. <p>Learning Goal 2: analyze, represent, and solve systems of linear equations using multiple methods (graphing, substitution, and elimination) and apply these methods to solve real-world problems, interpreting the meaning of the solutions in context.</p>

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</p>	<ul style="list-style-type: none"> Analyze solution sets within a given context. <p>8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p> <p>We have learned to...</p> <ul style="list-style-type: none"> Understand that slope is constant between any two points on a straight, non-vertical line. Use similar triangles to justify that slope is consistent. Derive the equation $y=mx+by = mx + by=mx+b$ using slope and intercept. 		

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>We have learned to...</p> <ul style="list-style-type: none"> • Write equations and inequalities to represent real-world situations. • Solve equations and inequalities to find unknown values. • Interpret solutions in context and verify reasonableness. 		
<ul style="list-style-type: none"> • 8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse. 	<p>7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles,</p>	<p>MP.2 Reason abstractly and quantitatively.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> • Pythagorean Theorem • If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle (Pythagorean theorem converse). <p>Students are able to:</p>

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	quadrilaterals, polygons, cubes, and right prisms. We have learned to/that... <ul style="list-style-type: none"> solve real-world and mathematical problems involving area of two dimensional objects composed of triangles and quadrilaterals 		<ul style="list-style-type: none"> given a proof of the Pythagorean theorem, explain the proof. given a proof of the converse of the Pythagorean theorem, explain the proof. <p>Learning Goal 3: Explain a proof of the Pythagorean Theorem and its converse.</p>
<ul style="list-style-type: none"> 8.G.B.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 	n/a	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to: <ul style="list-style-type: none"> determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems involving two dimensional spaces. determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems involving three dimensional spaces. <p>Learning Goal 3: Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensional cases when solving real-world and mathematical problems.</p>
<ul style="list-style-type: none"> 8.G.B.8. Apply the Pythagorean Theorem to find the distance between 	6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to:

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
two points in a coordinate system.	length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. We have learned to/that... <ul style="list-style-type: none"> find the length of a side of a polygon using coordinates with the same first coordinate or the same second coordinate 		<ul style="list-style-type: none"> determine the distance between two points in a coordinate plane by drawing a right triangle and applying the Pythagorean Theorem. Learning Goal 4: Use the Pythagorean Theorem to determine the distance between two points in the coordinate plane.
<ul style="list-style-type: none"> 8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: <ul style="list-style-type: none"> 8.G.A.1a. Lines are transformed to lines, and line segments to line segments of the same length. 8.G.A.1b. Angles are transformed to angles of the same measure. 	n/a	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.8 Look for and express regularity in repeated reasoning.	Concept(s): <ul style="list-style-type: none"> A property of rigid motion transformations (rotation, reflection, and translation) is that the measure of a two-dimensional object under the transformation remains unchanged. Students are able to: <ul style="list-style-type: none"> show and explain that performing rotations, reflections, and translations on lines results in a line. show and explain that performing rotations, reflections, and translations on line segments results in a line segment and does not alter the length of the line segment. show and explain that performing rotations, reflections, and translations on angles results in an angle and does not alter the measure of the angle.

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>8.G.A.1c. Parallel lines are transformed to parallel lines.</p>			<ul style="list-style-type: none"> show and explain that performing rotations, reflections, and translations on parallel lines results in parallel lines. explain that a property of rigid motion transformations (rotation, reflection, and translation) is that the measure of a two-dimensional object under the transformation remains unchanged. <p>Learning Goal 5: Explain and model the properties of rotations, reflections, and translations with physical representations and/or geometry software using pre-images and resultant images of lines, line segments, and angles.</p>
<ul style="list-style-type: none"> 8.G.A.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. 	n/a	<p>MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. <p>Students are able to:</p> <ul style="list-style-type: none"> given two congruent figures, describe a transformation or sequence of transformations that shows the congruence between them. <p>Learning Goal 6: Describe and perform a sequence of rotations, reflections, and/or translations on a two dimensional figure in order to prove that two figures are congruent.</p>
<ul style="list-style-type: none"> 8.G.A.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 	6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second	<p>MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning. of others. MP.5 Use appropriate tools strategically.</p>	<p>Concept(s): No new concept(s) introduced Students are able to:</p> <ul style="list-style-type: none"> describe, using coordinates, the resulting two-dimensional figure after applying dilations with scale factor greater than, less than, and equal to 1.

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations			
Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> draw polygons in the coordinate plane given coordinates of the vertices find the length of a side of a polygon using coordinates with the same first coordinate or the same second coordinate 		<ul style="list-style-type: none"> describe, using coordinates, the resulting two-dimensional figure after applying translation, rotation, and reflection. <p>Learning Goal 7: Use the coordinate plane to locate images or pre-images of two-dimensional figures and determine the coordinates of a resultant image after applying dilations, rotations, reflections, and translations.</p>
<ul style="list-style-type: none"> 8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. 	n/a	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Congruent figures are also similar. <p>Students are able to:</p> <ul style="list-style-type: none"> describe a transformation or sequence of transformations that show the similarity between them given two similar two-dimensional figures. <p>Learning Goal 8: Apply an effective sequence of transformations to determine that figures are similar when corresponding angles are congruent and corresponding sides are proportional. Write similarity statements based on such transformations.</p>

Grade 8 Mathematics: Geometry: Pythagorean Theorem, Congruence and Similarity Transformations

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> 8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</i> 	n/a	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others.	Concept(s): No new concept(s) introduced Students are able to: <ul style="list-style-type: none"> give informal arguments to establish facts about the angle sum of triangles. give informal arguments to establish facts about exterior angles of triangles. give informal arguments to establish facts about the angles created when parallel lines are cut by a transversal. give informal arguments to establish the angle-angle criterion for similarity of triangles. Learning Goal 9: Give informal arguments to justify facts about the exterior angles of a triangle, the sum of the measures of the interior angles of a triangle, the angle-angle relationship used to determine similar triangles, and the angles created when parallel lines are cut by a transversal.

Integrated Accommodations and Modifications

Special Education Students	English Language Learners	At Risk
<ul style="list-style-type: none"> Utilize modifications & accommodations delineated in the student's IEP 	WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors	<ul style="list-style-type: none"> Pair visual prompts with verbal presentations

<ul style="list-style-type: none"> ● Provide additional manipulatives to support instruction ● Allow for alternative strategies to solve algorithms or tasks ● Provide the steps needed to complete the task ● Model frequently ● Provide repetition and practice. ● Use visuals to demonstrate/model the processes ● Restate, reread, and clarify directions/questions ● Ask students to restate information, directions, and assignments. ● Provide copy of class notes ● Distribute study guide for classroom tests. ● Provide preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Provide regular parent/ school communication ● Allow extended time to complete assignment ● Establish procedures for accommodations / modifications for assessments ● Allow student to take/complete tests in an alternate setting as needed <p>Appendix A: Special Education Accommodations and Modifications</p>	<ul style="list-style-type: none"> ● Modify Assignments ● Use testing and portfolio assessment ● Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) ● Repeat, rephrase, paraphrase key concepts and directions ● Allow for extended time for assignment completion as needed ● Highlight key vocabulary ● Define essential vocabulary in context ● Use graphic organizers, visuals, manipulatives and other concrete materials ● Use gestures, facial expressions and body language ● Read aloud ● Build on what students already know and prior experience 	<ul style="list-style-type: none"> ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Provide preferential seating to be mutually determined by the student and teacher ● Allow the use of a computer to complete assignments. ● Establish expectations for correct spelling on assignments ● Provide extra textbooks for home. ● Provide Peer Support ● Increase one on one time
<p>Gifted and Talented Students</p>	<p>504 Plan</p>	
<ul style="list-style-type: none"> ● Utilize advanced, accelerated, or compacted content ● Provide assignments that emphasize higher- level thinking skills. ● Allow for individual student interest 	<ul style="list-style-type: none"> ● Pair visual prompts with verbal presentations ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice 	

<ul style="list-style-type: none"> ● Gear assignments to development in areas of affect, creativity, cognition, and research skills ● Allow for a variety in types of resources ● Provide problem-based assignments with planned scope and sequence ● Utilize inquiry-based instruction ● Adjust the pace of lessons ● Utilize Choice Boards ● Provide Problem-Based Learning ● Establish flexible Grouping 	<ul style="list-style-type: none"> ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Break long assignments into smaller parts ● Assist student in setting short term goals ● Allow for preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Model and reinforce organizational systems (i.e. color-coding) ● Write out homework assignments, check student's recording of assignments
<p>Interdisciplinary Connections</p>	<p>Computer Science and Design Thinking</p>
<p>ELA Standards</p> <ul style="list-style-type: none"> ● SL.PE.8.1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others’ ideas and expressing their own clearly. ● L.KL.8.2. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases. ● W.RW.8.7. Write routinely over extended time frames (time for research, reflection, metacognition/self- correction, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. <p>The Pythagorean Theorem has applications in design, construction, engineering, and physics, as well as facial recognition algorithms. It is also foundational to the trigonometric functions students will study in Algebra 2 and beyond.</p> <p>While Pythagoras gets the credit for this rule, the Chinese proved it approximately 500 years before he was born, and the Babylonians knew of the relationship over 1000 years before he was born. Related to this is Fermat’s Last Theorem that says there are no integral solutions to the equation: $x^n + y^n = z^n$, for $n > 2$. Fermat, however, did not leave a proof, though. Instead, in the</p>	<p>Computer Science and Design Thinking Practices</p> <ul style="list-style-type: none"> ● Fostering an Inclusive Computing and Design Culture ● Collaborating Around Computing and Design ● Recognizing and Defining Computational Problems ● Developing and Using Abstractions ● Creating Computational Artifacts ● Testing and Refining Computational Artifacts ● Communicating About Computing and Design <p>Computer Science and Design Thinking Standards</p> <ul style="list-style-type: none"> ● 8.2.8.ITH.2: Compare how technologies have influenced society over time. ● 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch). ● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose. ● 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch). ● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.

margin of a textbook, he wrote that he knew that this relationship was not possible, but he did not have enough room on the page to write it down. Fermat’s Last Theorem was considered an unsolved problem that intrigued mathematicians for over 300 years, until it was finally proven true in 1993.

Transformations have applications in design and computer graphics, especially when it comes to animation and gaming.

Project Ideas:

- [gr 8 pythagoras project.pdf](#)
- [G8.Transformation Project](#)

Computer Applications for Math Classes

- Zearn (access via Clever)
- Eureka Math in Sync videos (access via Clever)
- [xtramath.org](#)
- Google Sheets for Problem Solving

Mathematics Virtual Manipulatives:

- [interactive graph paper](#)
- [Model Algebra \(equation help\)](#)
- Google Sheets

Career Readiness, Life Literacies and Key Skills

Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards

9.1 Personal Financial Literacy

- 9.1.8.CR.2: Compare various ways to give back through strengths, passions, goals, and other personal factors.
- 9.1.8.CP.1: Compare prices for the same goods or services.
- 9.1.8.CP.2: Analyze how spending habits affect one’s ability to save.
- 9.1.8.CP.3: Explain the purpose of a credit score and credit record, the factors and impact of credit scores.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).
- 9.1.8.FP.5: Determine how spending, investing, and using credit wisely contributes to financial well-being.

9.2 Career Awareness, Exploration, Preparation, and Training

- 9.2.8.CAP.1: Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
- 9.2.8.CAP.2: Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.4 Life Literacies and Key Skills

21st Century Skills

- 9.4.8.CI.1: Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions (6.SP.B.5)
- 9.4.8.GCA.2: Demonstrate openness to diverse ideas and perspectives through active discussions to achieve a group goal.

Technology Integration

- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping. (6.SP.B.4)
- 9.4.8.IML.4: Ask insightful questions to organize different types of data and create meaningful visualizations.
- 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.

SEL Competencies

- **Self - Awareness**
- **Self - Management**
- **Social Awareness**
- **Responsible Decision Making**
- **Relationship Skills**

<https://www.nj.gov/education/safety/wellness/selearning/index.shtml>

District/School Formative Assessment Plan	District/School Summative Assessment Plan
<p><i>Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards.</i></p> <p>Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students' mastery of content through a variety of methods:</p> <ul style="list-style-type: none"> • Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom's Taxonomy) • Exit tickets, rotational activities (stations), quizzes, and small group activities • Classwork, homework, group work (formative assessment) • Pre-Assessment, teacher's observation, class discussion, and journal 	<p><i>Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.</i></p> <p>Benchmark Assessments:</p> <ul style="list-style-type: none"> • Assessment 8.3.1: Mid-Unit Assessment • Assessment 8.3.2: End of Unit Assessment • Assessment 8.3.3: End of Unit Performance Assessment <p>Standardized Assessments:</p> <ul style="list-style-type: none"> • NJSLA <p>Other Summative Assessments: Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.</p>

Focus Mathematical Concepts

Targeted Academic Vocabulary	Common Misconceptions	Fluency
<ul style="list-style-type: none"> • System of equations • Solution (of a system) • Substitution method • Elimination method • Graphical solution • Linear equation • Parallel lines • Intersecting lines • Transformation • Translation • Rotation • Reflection • Dilation • Congruent • Similar 	<p>8.EE.A.2a</p> <ul style="list-style-type: none"> • think square roots only have one (positive) answer and not recognize that equations like $x^2=9x^2 = 9x^2=9$ have two solutions, $x=\pm 3x = \pm 3$. <p>8.EE.A.2b</p> <ul style="list-style-type: none"> • not understand that squaring and square rooting are inverse operations and might confuse when to apply each. <p>8.EE.C.8a</p> <ul style="list-style-type: none"> • believe the solution to a system of equations is always a point where the lines cross, failing to recognize systems with no or infinite solutions. <p>8.EE.C.8b</p>	<p>Math Fluency Support for Grades 6-8</p> <p>Targeted Fact reinforcement</p> <p>Fact Fluency</p> <p>Dr. Riccomini's Math Facts – Basic Facts Link</p> <p>Math Fact Practice</p> <p>http://www.xtramath.org</p> <p>Differentiation problems</p> <p>https://www.openmiddle.com/</p>

<ul style="list-style-type: none"> ● Angle relationships (vertical, complementary, supplementary) ● Triangle ● Interior angles ● Exterior angles ● Pythagorean Theorem ● Leg, hypotenuse ● Irrational number ● Distance formula ● Volume ● Cylinder ● Cone ● Sphere 	<ul style="list-style-type: none"> ● mix up substitution and elimination or apply the methods incorrectly, such as forgetting to solve for a single variable first. <p>8.EE.C.8</p> <ul style="list-style-type: none"> ● set up incorrect equations for real-world problems or misinterpret what the solution represents in context. <p>8.G.A.1a</p> <ul style="list-style-type: none"> ● think that transformations (e.g., rotations, reflections) change the size or shape of figures rather than preserving congruence. <p>8.G.A.1b</p> <ul style="list-style-type: none"> ● incorrectly identify figures as congruent without verifying that all corresponding parts match under transformations. <p>8.G.A.1c</p> <ul style="list-style-type: none"> ● believe that all figures with the same area or perimeter are congruent, confusing measurement with congruence. <p>8.G.A.2</p> <ul style="list-style-type: none"> ● struggle to justify congruence using transformations or think that only side lengths need to match for congruence. <p>8.G.A.3</p> <ul style="list-style-type: none"> ● confuse dilation with resizing arbitrarily, not realizing that dilations involve a scale factor and a center. <p>8.G.A.4</p> <ul style="list-style-type: none"> ● not understand that similar figures must have proportional sides and equal angles, confusing similarity with congruence. <p>8.G.A.5</p> <ul style="list-style-type: none"> ● misapply angle relationships (e.g., alternate interior, vertical angles) or incorrectly label them on diagrams. <p>8.G.B.6</p>	
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	<ul style="list-style-type: none"> misapply the Pythagorean Theorem to non-right triangles or incorrectly substitute side lengths. <p>8.G.B.7</p> <ul style="list-style-type: none"> forget to take the square root after finding the sum of squares, or misidentify which sides are legs or hypotenuse. <p>8.G.B.8</p> <ul style="list-style-type: none"> misapply the Pythagorean Theorem to coordinate distances or forget to calculate horizontal and vertical distances first. 	
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District/School Tasks	District/School Primary and Supplementary Resources
<ul style="list-style-type: none"> NJDOE Digital Item Library NJSLA Released Items in Mathematics NJSLA Mathematics Practice Tests NJSLA Mathematics Evidence Statements Progression Tables GUES Strategy + ECR problems 	<p><u>District-Mandated Resources</u></p> <ul style="list-style-type: none"> Eureka Mathematics EdGenuity/MyPath Zearn <p>Assessment Resources:</p> <ul style="list-style-type: none"> 8.G.B.6 Converse of the Pythagorean Theorem 8.G.B.7 Running on the Football Field 8.G.B.8 Finding isosceles triangles 8.G.A.1 Reflections, Rotations, and Translations 8.G.A.2 Congruent Triangles 8.G.A.3 Effects of Dilations on Length, Area, and Angles 8.G.A.4 Are They Similar 8.G.A.5 Street Intersections 8.G.A.5 Similar Triangles II 8.G.A.5 Triangle's Interior Angles <p>Other Resources:</p> <ul style="list-style-type: none"> 100 Questions that Promote Mathematical Discourse Asking Effective Questions in Mathematics Amistad Law Resources

	<ul style="list-style-type: none"> • Holocaust Law Resources • LGBTQ+ and Disability Resources • Climate Change Resources <p>Additional Mathematics Resources</p> <ul style="list-style-type: none"> • Embarc online [https://embarc.online/] • Dan Meyer’s 3-Act Math Tasks [https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0] • OpenMiddle.com • CPalms [https://www.cpalms.org/Public/], and • Illustrative Mathematics [https://www.illustrativemathematics.org/] <p>Project Ideas:</p> <ul style="list-style-type: none"> • gr 8 pythagoras project.pdf • G8.Transformation Project
Instructional Best Practices and Exemplars	
See Appendix B for Instructional Best Practices and Exemplars	
Pacing Guide	
24-25 WPS Pacing Guide Mathematics Foundations of Algebra	

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Overview

Unit 4 provides a continuation of the analysis of linear models as they pertain to bivariate data. Learners investigate patterns of association in bivariate data using scatter plots and two-way tables, including informally fitting and assessing the fit of a linear model for a scatter plot, interpreting the slope and intercept of a linear model in the context of bivariate data, and using joint and relative frequencies of a two-way table to describe possible association between two variables. This unit revisits previously taught standards from Unit 2 (8.F.B.4, 8.EE.C.8.) and Unit 3 (8.G.B.7, 8.G.B.8) regarding creating linear functions, analyzing systems of simultaneous linear equations, and applying the Pythagorean Theorem to find distance in the context of real world data.

Essential Questions

- How can you represent the relationship between paired data and use the representation to make predictions?
- How can you use functions to model linear relationships?
- What do the slope and y-intercept of the graph mean in context of the independent and dependent variables the data represents?

Enduring Understandings

- Scatter plots allow us to investigate patterns of association between two quantities such as: positive or negative association, linear association, and nonlinear association.
- For scatter plots that suggest a linear association, straight lines are used to model the relationship between variables.
- For scatter plots that suggest a linear association, the closeness of the data points to the model line (e.g. line of best fit) can be used to judge the strength of the model.
- Slope and y-intercept can be interpreted to solve real world problems in the context of bivariate measurement data.
- Two-way tables can be used to show patterns of association in categorical data.

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> • 8.SP.A.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns 	6.SP.B.5 Summarize numerical data sets in relation to their context, such as by:	MP.3 Construct viable arguments and critique the reasoning of others.	Concept(s): <ul style="list-style-type: none"> • Association in data (bivariate measurement data) Students are able to: <ul style="list-style-type: none"> • construct and interpret scatter plots.

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p>	<p>6.SP.B.5a Reporting the number of observations. 6.SP.B.5b Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. 6.SP.B.5c Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> summarize numerical data sets in relation to their context, such as by reporting the number of observations and describing how it was measured and the 	<p>MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.</p>	<ul style="list-style-type: none"> analyze patterns of association between the two quantities represented in a scatter plot. describe clustering, outliers, positive or negative association, linear or non-linear association when explaining patterns of association in a scatter plot. <p>Learning Goal 1: Construct and interpret scatter plots for bivariate measurement data and describe visual patterns of association (clusters, outliers, positive or negative association, linear association and nonlinear association, strong, weak, and no association).</p>

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>units for the measurement</p> <ul style="list-style-type: none"> describe overall patterns and any striking deviations from a data set by giving the measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) with reference to the context with which the data was collected <p>6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>6.NS.C.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find</p>		

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	and position pairs of integers and other rational numbers on a coordinate plane. We have learned to/that... <ul style="list-style-type: none"> • find and plot pairs of integers and other rational numbers on the coordinate plane 		
<ul style="list-style-type: none"> • 8.SP.A.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line. 	n/a	MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): <ul style="list-style-type: none"> • Straight lines are used to model <i>approximately</i> linear relationships between quantitative variables. Students are able to: <ul style="list-style-type: none"> • informally fit a line (of best fit) to a scatter plot that suggests a linear association. • informally assess the model’s fit by judging the closeness of the data points to the line (line of best fit). <p style="text-align: center;">Learning Goal 2: For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model’s fit.</p>
<ul style="list-style-type: none"> • 8.SP.A.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and 	n/a	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: <ul style="list-style-type: none"> • given the equation for a linear model (line of best fit), interpret the slope and intercept. • given the equation for a linear model, solve problems in the context of measurement data.

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</p>		<p>MP.7 Look for and make use of structure.</p>	<p>Learning Goal 3: Use a linear model (equation) representing measurement data to solve problems, interpreting the slope and intercept in the context of the situation.</p>
<ul style="list-style-type: none"> 8.SP.A.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights 	<p>n/a</p>	<p>MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> Categorical data: patterns of association can also be observed in bivariate categorical data through analyzing two-way tables containing frequencies or relative frequencies. <p>Students are able to:</p> <ul style="list-style-type: none"> construct and interpret a two-way frequency table containing data on two categorical variables. construct and interpret a two-way relative frequency table containing data on two categorical variables. describe any association between the two categorical variables using relative frequencies calculated for rows or columns. <p>Learning Goal 4: Construct two-way frequency tables and two-way relative frequency tables, and describe possible associations between two variables.</p>

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</p>			
<ul style="list-style-type: none"> ● 8.F.B.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 	<p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <p>7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> ● identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships ● identify the constant of proportionality (unit rate) in tables, graphs, and diagrams 	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● As with equations, two (x,y) values can be used to construct a function. <p>Students are able to:</p> <ul style="list-style-type: none"> ● construct a function in order to model a linear relationship. ● interpret the rate of change and initial value of a linear function in context. <p>Learning Goal 5: Model a linear relationship by constructing a function from two (x,y) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<p>6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i></p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> two quantities which change in relationship to one another are expressed as independent and dependent variables 		

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
	<ul style="list-style-type: none"> write an equation using two quantities, an independent and a dependent variable, to represent a real-world problem 		
<ul style="list-style-type: none"> 8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. 	<p>6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> find the length of a side of a polygon using coordinates with the same first coordinate or the same second coordinate 	<p>MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.</p>	<p>Concept(s): No new concept(s) introduced Students are able to:</p> <ul style="list-style-type: none"> determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems in two and three dimensions. determine the distance between two points in a coordinate plane by applying the Pythagorean Theorem. <p>Learning Goal 6: Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.</p>
<ul style="list-style-type: none"> 8.EE.C.8 Analyze and solve pairs of simultaneous linear equations. 	<p>6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a</p>	<p>MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> Simultaneous linear equations may have an infinite number of solutions.

Unit 4 Grade 8 Mathematics: Statistics and Probability: Scatterplots and Association

Content Standards	Prerequisite Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<p>8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables. <i>For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</i></p>	<p>specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p> <p>We have learned to/that...</p> <ul style="list-style-type: none"> determine if a given number from a specified set is a solution to an equation or an inequality using substitution 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.7 Look for and make use of structure.</p>	<ul style="list-style-type: none"> Simultaneous linear equations may have no solution or a single solution. Solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs. <p>Students will be able to:</p> <ul style="list-style-type: none"> solve systems of two linear equations in two variables algebraically. estimate solutions of a linear system of two equations by graphing. solve simple cases of a linear system of two equations by inspection. solve real-world and mathematical problems leading to two linear equations in two variables. <p>Learning Goal 7: Solve real world and mathematical problems leading to two linear equations in two variables, interpreting solutions in context.</p>

Integrated Accommodations and Modifications

Special Education Students	English Language Learners	At Risk
<ul style="list-style-type: none"> Utilize modifications & accommodations delineated in the student’s IEP Provide additional manipulatives to support instruction Allow for alternative strategies to solve algorithms or tasks 	<p>WIDA Can Do Descriptors https://wida.wisc.edu/teach/can-do/descriptors</p> <ul style="list-style-type: none"> Modify Assignments Use testing and portfolio assessment 	<ul style="list-style-type: none"> Pair visual prompts with verbal presentations Ask students to restate information, directions, and assignments. Provide repetition and and practice Model skills / techniques to be mastered.

<ul style="list-style-type: none"> ● Provide the steps needed to complete the task ● Model frequently ● Provide repetition and practice. ● Use visuals to demonstrate/model the processes ● Restate, reread, and clarify directions/questions ● Ask students to restate information, directions, and assignments. ● Provide copy of class notes ● Distribute study guide for classroom tests. ● Provide preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Provide regular parent/ school communication ● Allow extended time to complete assignment ● Establish procedures for accommodations / modifications for assessments ● Allow student to take/complete tests in an alternate setting as needed <p>Appendix A: Special Education Accommodations and Modifications</p>	<ul style="list-style-type: none"> ● Utilize Native Language Translation (peer, online assistive technology, translation device, bilingual dictionary) ● Repeat, rephrase, paraphrase key concepts and directions ● Allow for extended time for assignment completion as needed ● Highlight key vocabulary ● Define essential vocabulary in context ● Use graphic organizers, visuals, manipulatives and other concrete materials ● Use gestures, facial expressions and body language ● Read aloud ● Build on what students already know and prior experience 	<ul style="list-style-type: none"> ● Provide extended time to complete class work ● Provide copy of class notes ● Provide preferential seating to be mutually determined by the student and teacher ● Allow the use of a computer to complete assignments. ● Establish expectations for correct spelling on assignments ● Provide extra textbooks for home. ● Provide Peer Support ● Increase one on one time
<p>Gifted and Talented Students</p>	<p>504 Plan</p>	
<ul style="list-style-type: none"> ● Utilize advanced, accelerated, or compacted content ● Provide assignments that emphasize higher- level thinking skills. ● Allow for individual student interest ● Gear assignments to development in areas of affect, creativity, cognition, and research skills ● Allow for a variety in types of resources ● Provide problem-based assignments with planned scope and sequence 	<ul style="list-style-type: none"> ● Pair visual prompts with verbal presentations ● Ask students to restate information, directions, and assignments. ● Provide repetition and and practice ● Model skills / techniques to be mastered. ● Provide extended time to complete class work ● Provide copy of class notes ● Break long assignments into smaller parts 	

<ul style="list-style-type: none"> ● Utilize inquiry-based instruction ● Adjust the pace of lessons ● Utilize Choice Boards ● Provide Problem-Based Learning ● Establish flexible Grouping 	<ul style="list-style-type: none"> ● Assist student in setting short term goals ● Allow for preferential seating to be mutually determined by the student and teacher ● Provide extra textbooks for home. ● Model and reinforce organizational systems (i.e. color-coding) ● Write out homework assignments, check student's recording of assignments
<p>Interdisciplinary Connections</p>	<p>Computer Science and Design Thinking</p>
<p>ELA Standards</p> <ul style="list-style-type: none"> ● SL.PE.8.1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others’ ideas and expressing their own clearly. ● L.KL.8.2. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases. ● W.RW.8.7. Write routinely over extended time frames (time for research, reflection, metacognition/self- correction, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. <p>Statistics may be the topic from Grade 8 mathematics that is one of the most widely used outside of math class. Because theoretical probability is impossible to calculate for most human events, experimental probability, derived from past performance and population tendencies, is used as a basis for determining potential future events.</p> <p>Applications in weather forecasting, business, advertising, sports, science, economics, and any other industry that works to anticipate future events rely on statistical analysis. Data is used as a basis for many decisions in many industries.</p> <p>Project Ideas:</p>	<p>Computer Science and Design Thinking Practices</p> <ul style="list-style-type: none"> ● Fostering an Inclusive Computing and Design Culture ● Collaborating Around Computing and Design ● Recognizing and Defining Computational Problems ● Developing and Using Abstractions ● Creating Computational Artifacts ● Testing and Refining Computational Artifacts ● Communicating About Computing and Design <p>Computer Science and Design Thinking Standards</p> <ul style="list-style-type: none"> ● 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch). ● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose. <p>Computer Applications for Math Classes</p> <ul style="list-style-type: none"> ● Zearn (access via Clever) ● Eureka Math in Sync videos (access via Clever) ● xtramath.org ● Google Sheets for Problem Solving <p><u>Mathematics Virtual Manipulatives:</u></p> <ul style="list-style-type: none"> ● interactive graph paper ● Model Algebra (equation help) ● Google Sheets

- [G8. Statistics Project](#)
- [G8 Statistics Project II](#)
- [G8.Statistics Project III](#)

Career Readiness, Life Literacies and Key Skills

Career Readiness, Life Literacies and Key Skills Practices

- Act as a responsible and contributing community members and employee.
- Attend to financial well-being.
- Consider the environmental, social and economic impacts of decisions.
- Demonstrate creativity and innovation.
- Utilize critical thinking to make sense of problems and persevere in solving them.
- Model integrity, ethical leadership and effective management.
- Plan education and career paths aligned to personal goals.
- Use technology to enhance productivity increase collaboration and communicate effectively.
- Work productively in teams while using cultural/global competence.

Career Readiness, Life Literacies and Key Skills Standards

9.1 Personal Financial Literacy

- 9.1.8.CR.2: Compare various ways to give back through strengths, passions, goals, and other personal factors.
- 9.1.8.CP.1: Compare prices for the same goods or services.
- 9.1.8.CP.2: Analyze how spending habits affect one’s ability to save.
- 9.1.8.CP.3: Explain the purpose of a credit score and credit record, the factors and impact of credit scores.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).
- 9.1.8.FP.5: Determine how spending, investing, and using credit wisely contributes to financial well-being.

9.2 Career Awareness, Exploration, Preparation, and Training

- 9.2.8.CAP.1: Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
- 9.2.8.CAP.2: Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.4 Life Literacies and Key Skills

21st Century Skills

<ul style="list-style-type: none"> ● 9.4.8.CI.1: Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions (6.SP.B.5) ● 9.4.8.GCA.2: Demonstrate openness to diverse ideas and perspectives through active discussions to achieve a group goal. <p>Technology Integration</p> <ul style="list-style-type: none"> ● 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping. (6.SP.B.4) ● 9.4.8.IML.4: Ask insightful questions to organize different types of data and create meaningful visualizations. ● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.
SEL Competencies
<ul style="list-style-type: none"> ● Self - Awareness ● Self - Management ● Social Awareness ● Responsible Decision Making ● Relationship Skills <p>https://www.nj.gov/education/safety/wellness/selearning/index.shtml</p>

District/School Formative Assessment Plan	District/School Summative Assessment Plan
<p><i>Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards.</i></p> <p>Teachers are encouraged to incorporate Formative Assessments into all lessons. During instruction, teachers will collect ongoing information on students’ mastery of content through a variety of methods:</p> <ul style="list-style-type: none"> ● Questioning: using Socratic method, probing questions, a hierarchical system in complexity (Bloom’s Taxonomy) ● Exit tickets, rotational activities (stations), quizzes, and small group activities ● Classwork, homework, group work (formative assessment) ● Pre-Assessment, teacher’s observation, class discussion, and journal 	<p><i>Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.</i></p> <p>Benchmark Assessments:</p> <ul style="list-style-type: none"> ● Assessment 8.4.1: Mid-Unit Assessment ● Assessment 8.4.2: End of Unit Assessment ● Assessment 8.4.3: End of Unit Performance Assessment <p>Standardized Assessments:</p> <ul style="list-style-type: none"> ● NJSLA <p>Other Summative Assessments: Teachers are encouraged to design and their own assessments (topic/module tests and quizzes) individually and/or with their department or grade-level partners, as per Uniform Grading Profile.</p>

Focus Mathematical Concepts		
Targeted Academic Vocabulary	Common Misconceptions	Fluency
<p>New or Recently Introduced Terms</p> <ul style="list-style-type: none"> ● Relative frequency ● joint frequency ● marginal frequency ● Scatter plot ● Bivariate data ● Measurement data ● Positive association ● Negative association ● Line of best fit ● Linear association ● Nonlinear association ● Quantitative ● Categorical data ● Outlier ● Two-way table ● Causation ● Correlation ● (Causal) Factors <p>Familiar Terms and Symbols</p> <ul style="list-style-type: none"> ● Analyze ● Interpret ● Rate of change ● Initial value ● Slope ● Y-intercept ● Linear function ● Right triangle ● Legs ● Hypotenuse ● Square root 	<p>Students may:</p> <ul style="list-style-type: none"> ● believe bivariate data is only displayed in scatter plots. ● mistakenly think their lines of best fit for the same set of data will be exactly the same. Because students are informally drawing lines of best fit, the lines will vary slightly. To obtain the exact line of best fit, students would use technology to find the line of regression. ● infer a cause and effect between independent and dependent variables, but this is often not the case. ● graph incorrectly because they don't understand that x usually represents the independent variable and y represents the dependent variable. ● confuse one-variable and two-variable equations 	<p>8.EE.C.7 Fluently solve linear equations and inequalities in one variable.</p> <p>Math Fluency Support for Grades 6-8</p> <p>Targeted Fact reinforcement</p> <p>Fact Fluency Dr. Riccomini's Math Facts – Basic Facts Link Math Fact Practice http://www.xtramath.org Differentiation problems https://www.openmiddle.com/</p>

<ul style="list-style-type: none"> • Theorem • Converse • Proof • System of equations • Solution • Linear equation • Approximate • Graphically • Infinite solutions • No solution • parallel lines • coincident lines • point of intersection 		
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District/School Tasks	District/School Primary and Supplementary Resources
<ul style="list-style-type: none"> • NJDOE Digital Item Library • NJSLA Released Items in Mathematics • NJSLA Mathematics Practice Tests • NJSLA Mathematics Evidence Statements • Progression Tables • GUES Strategy + ECR problems 	<p><u>District-Mandated Resources</u></p> <ul style="list-style-type: none"> • Eureka Mathematics • EdGenuity/MyPath • Zearn <p>Assessment Resources:</p> <ul style="list-style-type: none"> • 8.SP.A.1 Texting and Grades 1 • 8.SP.A.2 Animal Brains • 8.SP.A.3 US Airports • 8.SP.A.4 What's Your Favorite Subject • 8.SP.A.4 Music and Sports • 8.F.B.4 Delivering the Mail • 8.G.B.8 Finding the distance between points • 8.EE.C.8 Kimi and Jordan <p>Other Resources:</p> <ul style="list-style-type: none"> • 100 Questions that Promote Mathematical Discourse • Asking Effective Questions in Mathematics • Amistad Law Resources • Holocaust Law Resources

	<ul style="list-style-type: none"> ● LGBTQ+ and Disability Resources ● Climate Change Resources <p>Additional Mathematics Resources</p> <ul style="list-style-type: none"> ● Embarc online [https://embarc.online/] ● Dan Meyer’s 3-Act Math Tasks [https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTOEsfqouLWNNC6Z4/edit#gid=0] ● OpenMiddle.com ● CPalms [https://www.cpalms.org/Public/], and ● Illustrative Mathematics [https://www.illustrativemathematics.org/] <p>Project Ideas:</p> <ul style="list-style-type: none"> ● G8. Statistics Project ● G8 Statistics Project II ● G8.Statistics Project III
Instructional Best Practices and Exemplars	
See Appendix B for Instructional Best Practices and Exemplars	
Pacing Guide	
Pacing Guide Mathematics Foundations of Algebra	

Appendix A: Accommodations and Modifications

Appendix A: Accommodations and Modifications: Unit 1

Accomodation:

- **Provide Visual Supports**
Use charts, graphic organizers, or anchor charts showing examples of exponential forms, scientific notation, and the Pythagorean Theorem for reference during activities.
- **Flexible Grouping**
Pair or group students strategically, mixing different ability levels so students can support each other, especially during collaborative

activities like the relay race and scavenger hunt.

- **Extended Time**

Allow extra time to complete tasks such as the scavenger hunt or cityscape design, ensuring students don't feel rushed and can process concepts thoroughly.

- **Use of Calculators or Technology**

Permit calculators or apps for computations involving powers of ten, square roots, or distances during activities, to reduce calculation errors and focus on conceptual understanding.

- **Step-by-Step Instructions**

Provide written and verbal step-by-step instructions for complex tasks like the Pythagorean Street Map Challenge or creating function tables to help students stay organized.

- **Chunking Tasks**

Break larger activities into smaller, manageable parts with check-ins, so students can focus on one step at a time (e.g., first design the map grid, then calculate distances).

Modifications:

- **Simplify Numerical Values**

Use smaller or whole number bases and exponents in the exponent card sort and scavenger hunt to reduce cognitive load and focus on concept acquisition.

- **Limit Scope of Tasks**

For students needing more support, reduce the number of cards to sort, limit the number of buildings in the cityscape, or shorten the number of distances to calculate on the map.

- **Provide Partially Completed Work**

Offer partially completed function tables or map grids for students to fill in, reducing the amount of initial problem-solving required.

- **Focus on One Representation**

Instead of requiring students to work with multiple forms of expressions (expanded, standard, exponential), have them focus on just one or two formats.

- **Use Concrete Manipulatives**

Replace abstract tasks with hands-on tools (e.g., base-ten blocks or algebra tiles) to illustrate powers and exponents for students who struggle with abstract representations.

- **Alternate Output Formats**

Allow students to demonstrate understanding through oral explanations, drawings, or video presentations instead of written work, especially for design-based activities like the cityscape or street map.

Appendix A: Accommodations and Modification: Unit 2

Accommodations:

- **Provide Visual Aids and Examples**

Offer reference sheets with examples of functions vs. non-functions, linear vs. nonlinear graphs, and step-by-step equation solving strategies to support student understanding.

- **Use Technology Tools**

Allow students to use graphing calculators or apps to plot points and check graphs during activities like “Graph That Story” and “Linear vs. Nonlinear Mystery Graphs.”

- **Flexible Grouping and Peer Support**

Organize students in pairs or small groups, mixing skill levels so that peers can support each other, especially during discussions and sorting activities.

- **Extended Time and Breaks**

Give students additional time to complete tasks, especially complex ones like the Equation Escape Room, and allow short breaks to maintain focus.

- **Step-by-Step Written Instructions**

Provide clear, written, and verbal instructions broken down into manageable steps for multi-part activities, such as the Escape Room or number line estimations.

- **Use Manipulatives or Number Lines**

Provide physical or digital number lines and manipulatives to help students visualize irrational numbers and their placement.

Modifications:

- **Simplify Numbers and Equations**
Use simpler numbers or equations with fewer steps for students who need additional support, such as basic one-step equations instead of multi-step ones in the Escape Room.
- **Limit the Number of Examples**
Reduce the quantity of tables, graphs, or equations in sorting activities so students can focus on quality over quantity.
- **Focus on One Representation at a Time**
Have students work on just one form (e.g., only tables or only graphs) when deciding if a relation is a function to reduce cognitive load.
- **Provide Partially Completed Tasks**
Offer partially completed graphs, tables, or equations to scaffold learning and allow students to focus on key concepts.
- **Alternate Response Formats**
Allow students to explain their reasoning orally or through drawings instead of written justification, particularly for the sorting challenge or graph descriptions.
- **Use Number Line with Marked Intervals**
For irrational number estimation, provide number lines with key points (like $2\sqrt{2}$ or π) already marked, so students can place other numbers relative to those benchmarks.

Appendix A: Accommodations and Modification: Unit 3**Accommodations:**

- **Provide Visual Reference Materials**
Supply charts and posters summarizing the Pythagorean Theorem, types of transformations, angle relationships, and steps for solving systems of equations to support students during activities.
- **Flexible Grouping and Peer Assistance**
Pair students strategically to ensure peer support, especially for multi-step tasks like the System of Equations Partner Match and Angle Sorting Station.

- **Use Manipulatives and Tools**
Provide physical triangle models, protractors, graph paper, and calculators to help students visualize and accurately complete tasks like the Pythagorean Puzzle Challenge and Transformations Grid Game.
- **Clear, Step-by-Step Instructions**
Give written and verbal stepwise guidance for complex activities (e.g., scavenger hunt steps, solving systems of equations) and check for understanding before starting.
- **Extended Time and Breaks**
Allow additional time to complete multi-station activities and provide breaks as needed to maintain focus and reduce frustration.
- **Technology Integration**
Allow use of graphing calculators or geometry software for plotting transformations and solving systems to reduce manual errors and enhance conceptual understanding.

Modifications:

- **Simplify Numerical Values and Tasks**
Use smaller, whole numbers and limit the number of steps in problems, such as fewer stations in the Square Root Walkabout or simpler systems of equations with whole-number coefficients.
- **Reduce Number of Stations or Problems**
For students needing support, limit the number of rotations or puzzles to focus on mastery of key concepts rather than completing all activities.
- **Provide Partially Completed Work**
Offer partially filled graphs, angle diagrams with some measurements provided, or templates for transformations to scaffold student work.
- **Focus on Single Concepts at a Time**
Instead of combining multiple angle relationships in the Angle Sorting Station, focus on just one type (e.g., only alternate interior angles) per session.
- **Alternate Response Formats**
Allow students to demonstrate understanding through oral explanations, drawings, or verbal problem-solving instead of fully written work, especially for the scavenger hunt and puzzle activities.
- **Use Concrete Examples**
Provide physical objects or digital applets to demonstrate transformations and Pythagorean relationships, rather than relying solely on abstract diagrams.

Appendix A: Accommodations and Modification: Unit 4**Accommodations:**

- **Provide Visual Guides and Examples**
Offer charts and sample dot plots, scatterplots, and system-solving steps to support students as they work through data interpretation and transformations.
- **Use Technology Tools**
Allow students to use graphing calculators, tablets, or software to plot data points, draw lines of best fit, and perform transformations digitally for easier manipulation and visualization.
- **Flexible Grouping**
Pair or group students strategically to encourage peer support during activities, especially collaborative tasks like data collection and solving systems.
- **Step-by-Step Instructions**
Provide clear, written and verbal stepwise instructions for multi-part tasks such as plotting scatterplots or solving systems of equations, checking for understanding before moving on.
- **Extended Time and Breaks**
Allow extra time for students to complete complex tasks like line of best fit exploration and transformations, and provide breaks as needed to maintain focus.
- **Use Manipulatives and Tools**
Supply rulers, protractors, and graph paper for measuring lengths and angles in the similarity investigation to help with accuracy.

Modifications:

- **Simplify Data Sets and Problems**
Use smaller, more manageable data sets and simpler real-world scenarios with whole numbers for data collection, scatterplots, and system solving.
- **Limit Scope of Tasks**
Reduce the number of points to plot or transformations to perform, focusing on key concepts rather than extensive practice.

- **Provide Partially Completed Work**
Offer partially filled dot plots, scatterplots, or system tables so students can focus on interpretation rather than data entry.
- **Focus on One Representation**
Have students work with only one form of data (e.g., only dot plots or only scatterplots) before progressing to multiple types.
- **Alternate Output Formats**
Allow students to explain their findings orally or through drawings instead of written reports, especially for describing data trends and similarity conclusions.
- **Concrete Demonstrations**
Use physical models or interactive software to demonstrate transformations and similarity rather than abstract diagrams.

Appendix B: Instructional Best Practices and Exemplars:

Appendix B: Instructional Best Practices and Exemplars: Unit 1

Sample Activities:

- **Exponent Card Sort**
Students work in pairs to match cards with numerical expressions written in exponential form, standard form, and expanded form. After sorting, they create their own sets to challenge classmates.
- **Real-World Powers of Ten Scavenger Hunt**
Students search magazines, books, and online sources for real-world examples involving very large or very small numbers (e.g., population, bacteria size, space distances). They express these quantities using powers of ten and compare them.
- **Function Table Relay Race**
In teams, students solve linear equations and fill out corresponding function tables. Each correct table unlocks the next problem in the relay. The goal is to understand constant rate of change and linear relationships through collaboration.
- **Scientific Notation Cityscape**
Students design a city skyline where each building's height represents a number written in scientific notation. They label buildings with both scientific and standard forms and order them from shortest to tallest on a class mural.

- **Pythagorean Street Map Challenge**

Students design a map of a fictional town using a grid. They must calculate distances between key locations using the Pythagorean Theorem and create real-life scenarios (e.g., shortest walking paths between buildings).

Sample Exemplar:

- **Convert and Compare**

Given two very large or very small numbers in different forms (e.g., 3.2×10^{53} and 3.2×10^5 and 450,000), students convert both to standard form and scientific notation, then determine which is greater and explain why.

- **Ladder Length Problem**

Students are given the height of a wall and a distance from the base of the wall. They use the Pythagorean Theorem to calculate the length of a ladder that safely reaches the top of the wall.

- **Population Growth Analysis**

Students analyze population data given in exponential form (e.g., bacteria doubles every hour) and compare two populations using properties of integer exponents and powers of ten.

- **Phone Plan Pricing**

Two linear phone plans are represented with tables and graphs. Students interpret the rate of change and initial value of each plan and determine under what conditions one plan is more cost-effective than the other.

- **Equivalent Expressions Investigation**

Students explore and justify whether expressions like $23 \cdot 242^3 \cdot 2^4$ and $23 + 42^{\{3+4\}}$, or 5^7 and $5^3 \cdot 5^4$ and $57 - 35^{\{7-3\}}$, are equivalent, using the properties of exponents to support their reasoning.

Appendix B: Instructional Exemplars and Explanations: Unit 2

Sample Activities:

- **Function or Not? Sorting Challenge**

Students examine a variety of tables, graphs, equations, and mapping diagrams to decide whether each represents a function. They sort the examples into “Function” and “Not a Function” categories and justify their reasoning in small groups.

- **Estimating Irrational Numbers on a Number Line**

Students place irrational numbers like $2\sqrt{2}$, π , and non-terminating decimals on a number line between two consecutive integers. Then, they

explain their estimation strategies and refine their placements through discussion.

- **Graph That Story**

Students listen to or read short stories involving real-world scenarios (e.g., a person walking to a store, stopping, and returning home). They graph the relationship between time and distance, then describe and compare the features of the graphs.

- **Linear vs. Nonlinear Mystery Graphs**

Given unlabeled graphs and equations, students must determine whether each represents a linear or nonlinear relationship. They use slope, rate of change, and patterns in the data to support their classifications.

- **Equation Escape Room**

Students solve multi-step equations with variables on one or both sides to unlock clues and progress through themed "escape room" challenges. Each correct solution gets them closer to escaping the room by uncovering a final code.

Sample Exemplar:

- **Function Rule Match-Up**

Students are given a set of input-output tables, graphs, and equations. They must match each table to its corresponding graph and equation, determine whether it represents a function, and explain their reasoning.

- **Irrational Number Estimate**

Students approximate $50\sqrt{50}$, 50π , and similar irrational numbers to the nearest tenth and hundredth using a number line. They compare these to nearby rational numbers and explain how they know the numbers are irrational.

- **Compare Cell Phone Plans**

Students are given two phone plans, each represented with a linear equation. They graph the functions, interpret the slope and y-intercept, and determine which plan is better under different usage conditions.

- **Solve and Justify**

Students solve a multi-step equation with variables on both sides, such as $3(x-2)=2x+53(x-2)=2x+53$, and provide a written explanation of each step, justifying the properties used to isolate the variable.

- **Temperature Conversion Formula**

Using a table of values, students derive the linear equation that converts temperatures from Celsius to Fahrenheit. They interpret the slope and y-intercept in context, and determine whether the relationship is proportional.

Appendix B: Instructional Best Practices and Exemplars: Unit 3**Sample Activities:****• Square Root Walkabout**

Students complete a scavenger hunt around the classroom or hallway with cards that include perfect and non-perfect squares. They solve square root problems at each station and match their answers to hidden clues that lead them to the next location.

• Pythagorean Puzzle Challenge

Using triangle pieces of different side lengths, students assemble right triangles and apply the Pythagorean Theorem to verify relationships. They explore how the theorem works visually and numerically.

• Transformations Grid Game

Students use graph paper to perform a series of transformations (translations, reflections, rotations) on shapes. They predict the outcome of each move, then check by plotting, identifying congruence and orientation throughout.

• System of Equations Partner Match

Each student is given either a graph, table, or equation representing a linear function. They must find a classmate whose representation intersects with theirs to form a system, then collaboratively solve it using substitution or elimination.

• Angle Sorting Station

At rotating stations, students classify and solve for unknown angles in diagrams involving parallel lines cut by transversals, angle relationships in triangles, and exterior angles. They use protractors and logic to justify their classifications.

Sample Exemplar:**• Estimate and Classify Square Roots**

Students estimate the value of irrational square roots (e.g., $18\sqrt{18}$, $45\sqrt{45}$) to the nearest tenth and identify whether the square root is rational or irrational. They justify their answers using perfect square benchmarks.

• Find the Missing Side

Students are given real-world right triangle scenarios (e.g., the height of a ramp and the base length) and use the Pythagorean Theorem to solve for the unknown side, showing all steps and interpreting their solution in context.

• Solve the System Two Ways

Given a system such as

$$y=2x+3y = 2x + 3y=2x+3$$

$$y=-x+9y = -x + 9y=-x+9,$$

students solve the system both algebraically (by substitution or elimination) and graphically, then explain why both methods result in the same solution.

- **Transform and Compare**

Students draw a triangle on a coordinate plane and apply a series of transformations (rotation, reflection, translation, dilation). They describe each transformation and analyze whether the figures are congruent or similar.

- **Angle Maze**

Students navigate through a "maze" of geometric diagrams involving parallel lines, triangles, and exterior angles. At each stop, they solve for unknown angles using relationships such as vertical angles, alternate interior angles, and triangle angle sums.

Appendix B: Instructional Best Practices and Exemplars: Unit 4

Sample Activities:

- **Data Collection and Dot Plot Creation**

Students collect data from classmates on a simple question (e.g., number of pets, favorite sport). They organize the data and create dot plots, then analyze the distribution and identify clusters or gaps.

- **Scatterplot Detective**

Students are given scatterplots with different types of correlations (positive, negative, none). They interpret the relationship between variables, describe trends, and predict outcomes based on the data patterns.

- **Line of Best Fit Exploration**

Using technology or graph paper, students plot data points and draw lines that best fit the trend. They discuss how well the line represents the data and how it can be used to make predictions.

- **Solving Real-World Systems**

Students work through scenarios where two linear relationships intersect (e.g., two phone plans or mixing solutions). They solve systems of equations to find points of intersection and interpret the meaning in context.

- **Transformations and Similarity Investigation**

Students perform transformations on shapes and analyze resulting figures for congruence or similarity. They measure side lengths and angles to support their conclusions.

Sample Exemplar:

- **Collect and Analyze Data**

Students gather data from a real-world context, organize it into a frequency table, and create a dot plot. They describe the shape, center, and spread of

the data distribution in writing.

- **Interpret Scatterplots**

Given scatterplots showing various types of associations (positive, negative, none), students identify the type of correlation and write a summary explaining the relationship between the variables.

- **Calculate and Use Lines of Best Fit**

Students use a set of data points to draw a line of best fit by hand or using technology. They then use the line to predict values and assess the accuracy of their predictions.

- **Solve Systems from Context**

Students solve a system of linear equations that models a real-life problem, such as comparing costs from two different service plans, and explain what the solution means in context.

- **Similarity through Transformations**

Students apply sequences of transformations to geometric figures and determine whether the figures are similar or congruent by comparing side lengths and angles. They justify their conclusions with evidence.