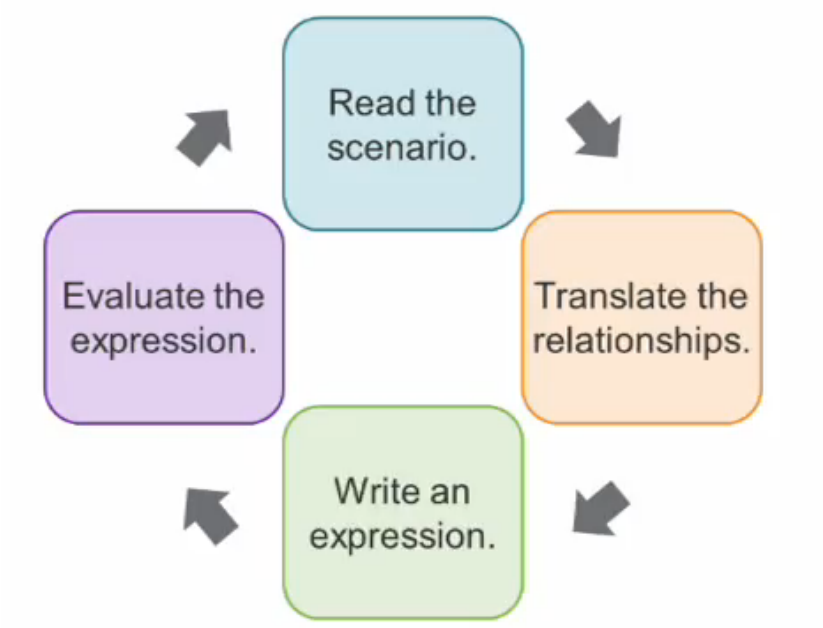
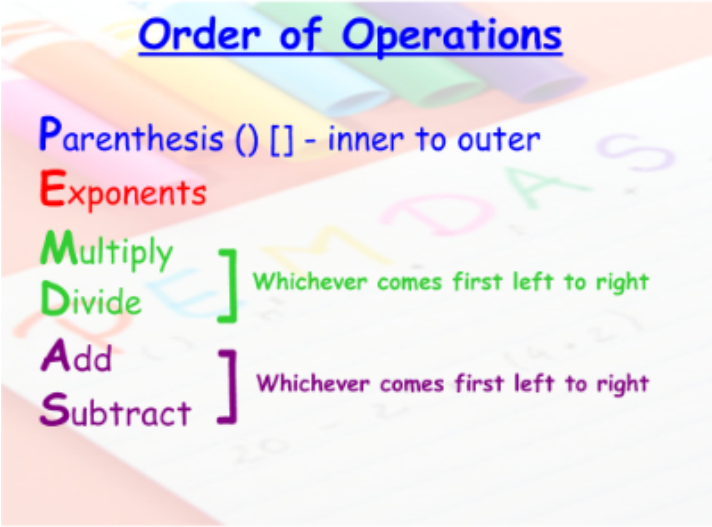
| |
|---|
| area = (length)(width) |
| perimeter = 2 (length) + 2 (width) |
| speed = distance/time |
| volume = (length)(width)(height) |




**Order of Operations**



**Absolute value** is the distance from 0 on a number line.

## Solutions of Absolute Value Equations

- Two solutions

$-|x| = -5$

$|x| = 5$

$x = -5$ or $x = 5$

- One solution ✓

$|x| = 0$

$x = 0$

- No solution ✓

$|x| = -5$

no solution

Absolute value equations represent a distance.

**Two Solutions**

$\left|\frac{3}{5}x - 4\right| = 15$

**One Solution**

$\left|7x - \frac{4}{37}\right| = 0$

**No Solution**

$\left|\frac{x}{2} + 11\right| = -3$

## Comparing Properties of Equality and Inequality

| Equations | Inequalities |
| --- | --- |
| $-2x = 6$ | $-2x < 6$ |
| $x = -3$ | $x > -3$ |

| Properties of equality | Properties of inequality |
| --- | --- |
| ▪ Adding or subtracting by the same value on both sides of an equation does not change the **solution set**. | ▪ Adding or subtracting the same value on both sides of an inequality does not change the solution set. |
| ▪ Multiplying or dividing by the same value on both sides of an equation does not change the solution set. | ▪ Multiplying or dividing by the same **positive value** on both sides of an inequality does not change the solution set. |
| | ▪ Multiplying or dividing by the same **negative value** on both sides of an inequality changes the sense of the inequality. |

## Solution Set of a One-Variable Inequality

| ▪ Number line | ▪ Set-builder notation | ▪ Interval notation |
| --- | --- | --- |
|   | $\{x \mid x \leq 3.65\}$ | $(-\infty, 3.65]$ |


## How to Solve Linear Equations

1. **Use** the distributive property to simplify, if necessary.
2. **Combine** like terms on either side of the equals sign.
3. **Isolate** the variable using the properties of equality.
4. **Check** the solution.

Algebraic equations contain variable terms with unknown values.

**One solution**

$4(x + 2) = 16$

$4x + 8 = 16$

$4x = 8$

$x = 2$

1 poss. soln

**Infinitely many solutions**

**Identity**

$2(x - 5) = 2x - 10$

$2x - 10 = 2x - 10$

$-2x \quad -2x$

$-10 = -10$ T

Any value for x

**No possible solutions**

$-3(x + 3) = -3x - 11$

$-3x - 9 = -3x - 11$

$+3x \quad +3x$

$-9 = -11$ F

No values of x

Distance = (Rate)(Time)

- A **literal equation** contains more than one variable.
- **Formulas** are literal equations.

The formula to convert °F to °C is $C = \frac{5}{9}(F - 32)$. Solve the formula for $F$.

$C = \frac{5}{9}(F - 32)$

$\left(\frac{9}{5}\right)\left(\frac{5}{9}\right) = \frac{45}{45} = 1$

$\left(\frac{9}{5}\right)C = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$\left(\frac{9}{5}\right)C = F - 32$

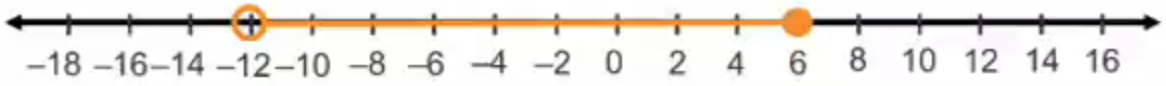
$\frac{9}{5}C + 32 = F$

$$C = \frac{5}{9}(F - 32)$$

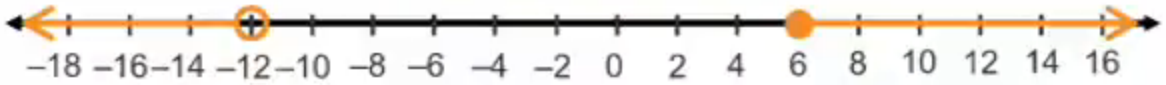
$$F = \frac{9}{5}C + 32$$

## What is a compound inequality and what does its solution look like?

**AND**
$-12 < x \leq 6$




**OR**
$x < -12$ or $x \geq 6$




**Rate of change** is the ratio of the change in the dependent value with respect to the change in the independent value.

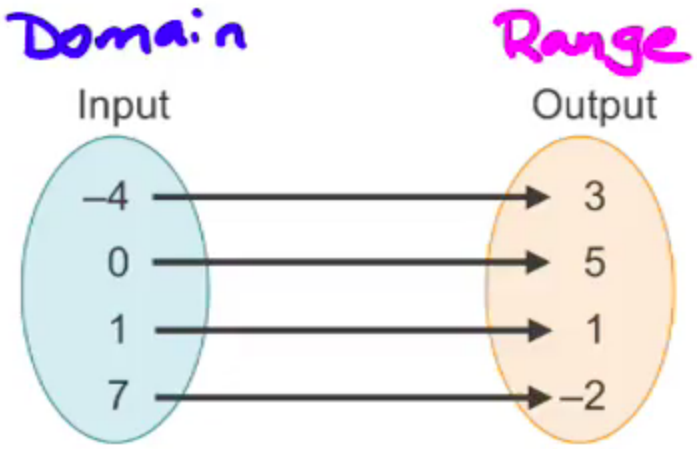
$$\frac{\text{Dep}}{\text{Ind}}$$

A **linear function** describes the relationship between two quantities having a constant additive rate of change. The graph of a linear function is a straight line.

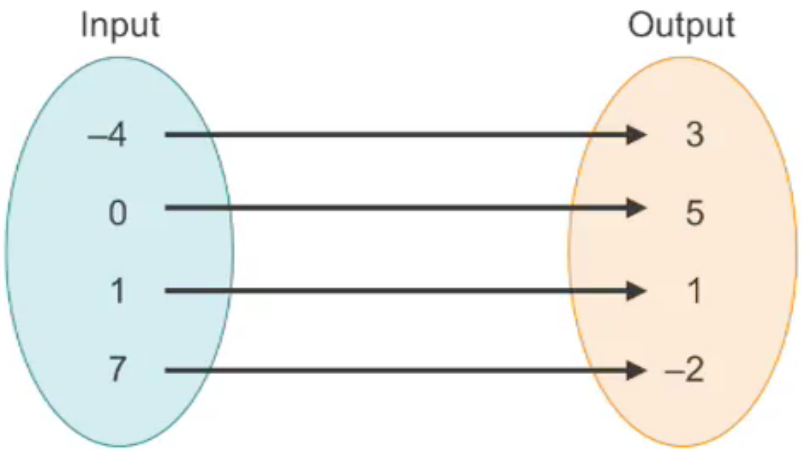
The **independent variable** represents the input values and the **dependent variable** represents the output values.

The **domain** is the set of input values for which the **relation** is defined. The **range** is the set of output values corresponding to the domain values.

| Domain | Range |
| --- | --- |
| values for the independent variable | output values |
| input values | values for the dependent variable |



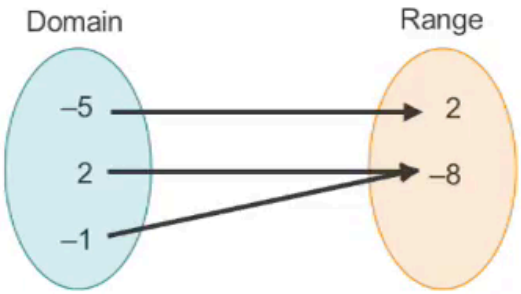

Mapping of a Function



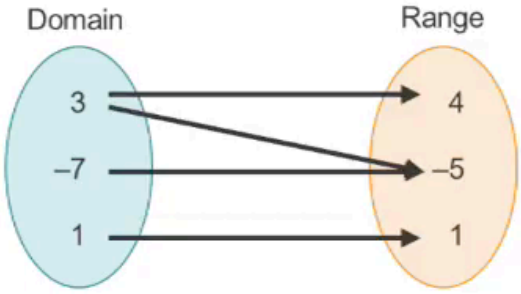
In a **function**, each value for the independent variable (input values) maps to exactly one value for the dependent variable (output values).

To determine if a relationship is a function, ask, "Does each element in the domain correspond to exactly one element in the range?"

**Function**



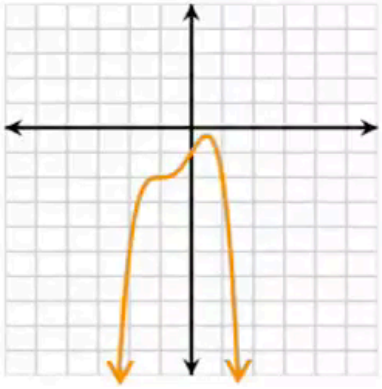

**Not a Function**



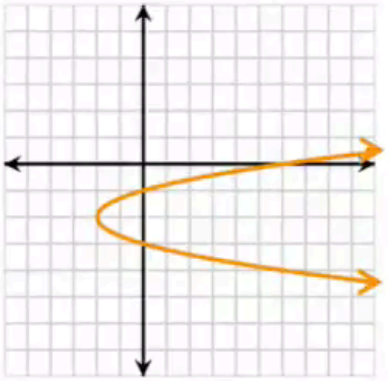

## The Vertical Line Test

If any vertical line passes through no more than one point, then the graph represents a function.

**Function**



**Not a Function**



$$f(x) = 3x - 4$$

Input — Output

## Function Notation

The equation $y = 3x - 4$ can be written using **function notation**.

$$f(x) = 3x - 4$$

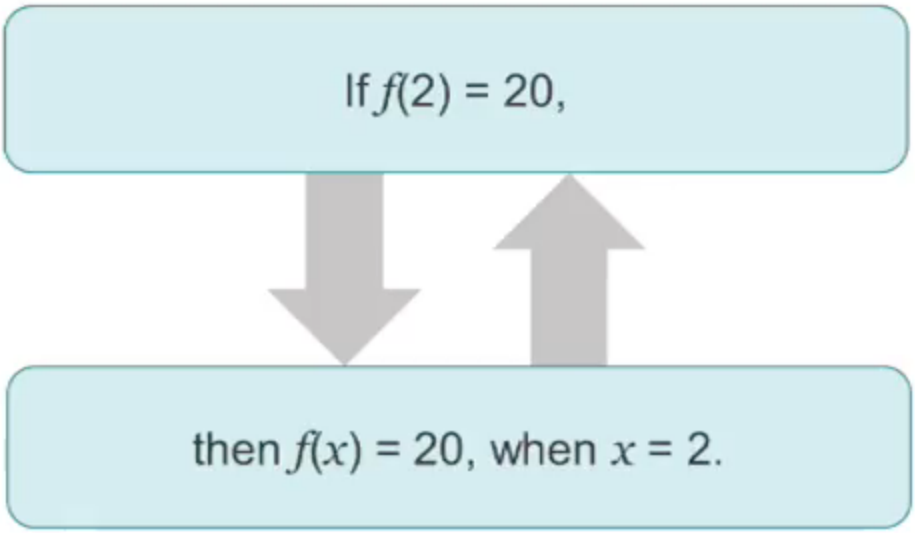
"f of x" — input — output

The tiles on the left contain functions written using function notation. Match each function with its input.

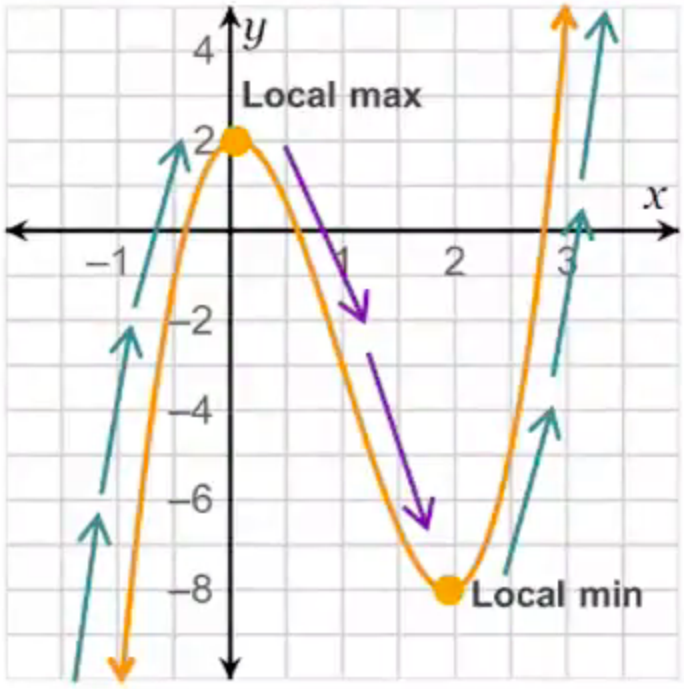
| | |
|---|---|
| h(g) = –4 + g | The input is f. |
| f(h) = h – 7 | The input is h. |
| g(f) = 2f | The input is g. |

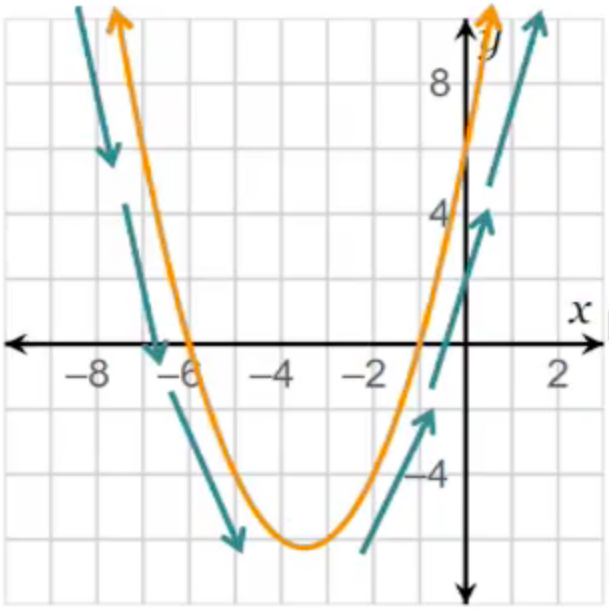
The meaning of $f(x)$ is "the output value of $f$ when the input value is $x$."

The meaning of $f(x)$ is **not** "$f$ times $x$."




| | | |
|---|---|---|
| $x$-intercept | ✔ | location on graph where input is zero |
| $f(x) > 0$ | ✔ | intervals of the domain where the graph is above the $x$-axis |
| $f(x) < 0$ | ✔ | intervals of the domain where the graph is below the $x$-axis |
| $y$-intercept | ✔ | location on graph where output is zero |





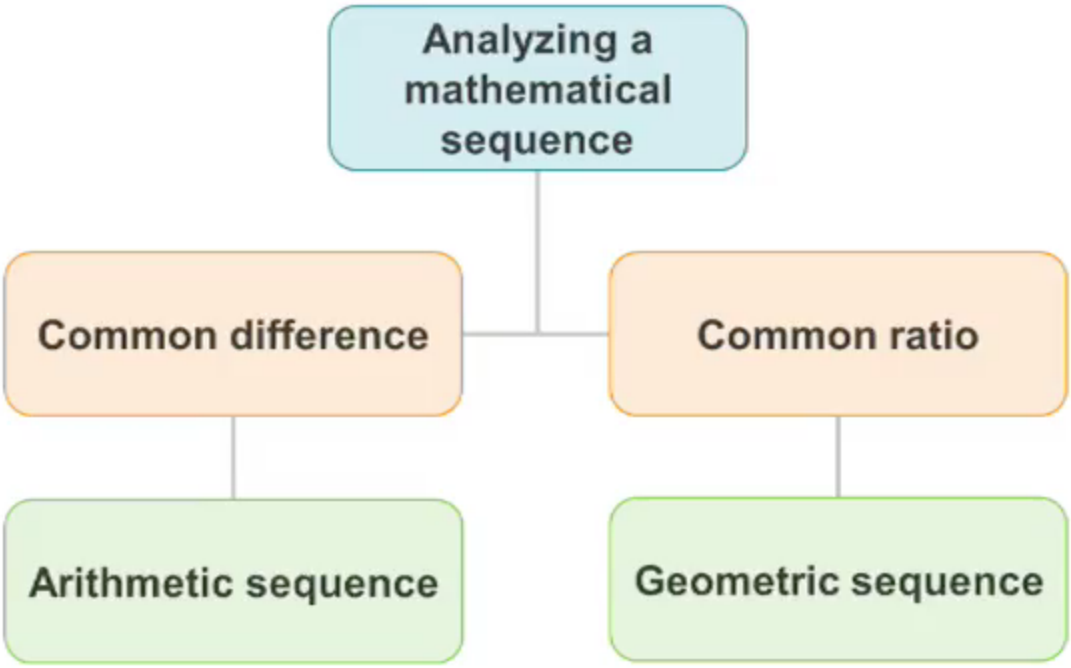

| | $x$ | $f(x)$ |
|---|---|---|
| Decreasing | −7.5 | 9.75 |
| Decreasing | −5.5 | −2.25 |
| Minimum | −3.5 | −6.25 |
| Increasing | −1.5 | −2.25 |
| Increasing | 1.5 | 18.75 |
| Increasing | 3.5 | 42.75 |

- A **sequence** is a set of ordered numbers.

- Finite sequence:
  - 4.5, 5.6, 6.7, 7.8
- Infinite sequence:
  - 4.5, 5.6, 6.7, 7.8, …

- A **term**, $a_n$, is the sequence value at a specific position, $n$.

| 4.5 | 5.6 | 6.7 | 7.8 |
|---|---|---|---|
| 1st term | 2nd term | 3rd term | 4th term |
| $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ |
| $a_1 = 4.5$ | $a_2 = 5.6$ | $a_3 = 6.7$ | $a_4 = 7.8$ |




- In a **recursive sequence**, any term is determined by a function of previous terms.
  - 1, 1, 2, 3, 5, 8, 13, 21, …
- A recursive sequence can be defined by a **recursive formula.**

In an **arithmetic sequence**, there is a common difference between any two consecutive terms.

6.9, 1.9, –3.1, –8.1, …

In a **geometric sequence**, there is a common ratio between any two consecutive terms.

- 2, –4, 8, –16 …

| Arithmetic | Geometric | Neither |
|---|---|---|
| ✓ 98.3, 94.1, 89.9, 85.7, … | ✓ 1.75, 3.5, 7, 14 | ✓ 1, 0, –1, 0, … |
| ✓ –12, –10.8, –9.6, –8.4 | ✓ –1, 1, –1, 1, … | |

### Arithmetic Sequences

$f(n + 1) = f(n) + d,\ n \geq 1$

$f(n) = c$

- $\frac{4}{5}, 1, 1\frac{1}{5}, 1\frac{2}{5}$

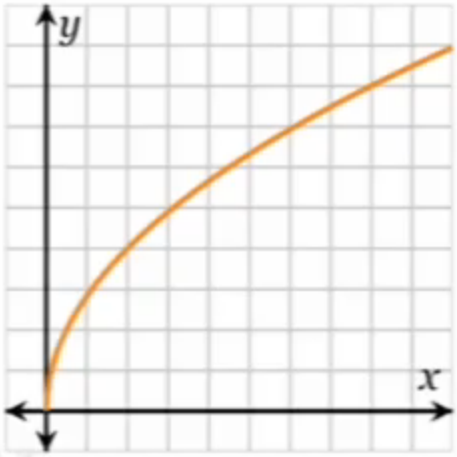
### Geometric Sequences

$f(n + 1) = rf(n),\ n \geq 1$

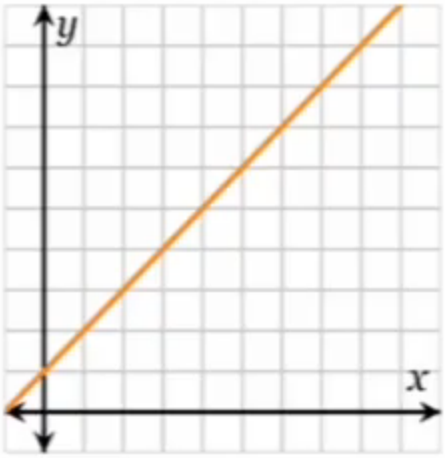
$f(n) = c$

- 2, –4, 8, –16 …

**Nonlinear function**




**Linear function**



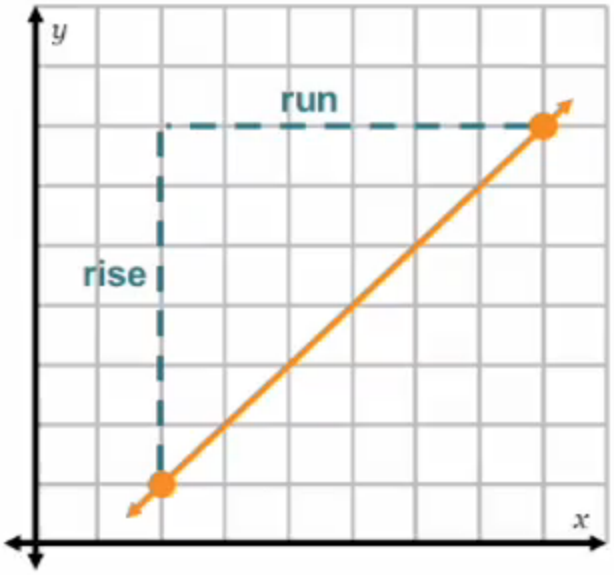

**Linear function** → **Has a constant additive rate of change**

**Nonlinear function** → **Does not have a constant additive rate of change**

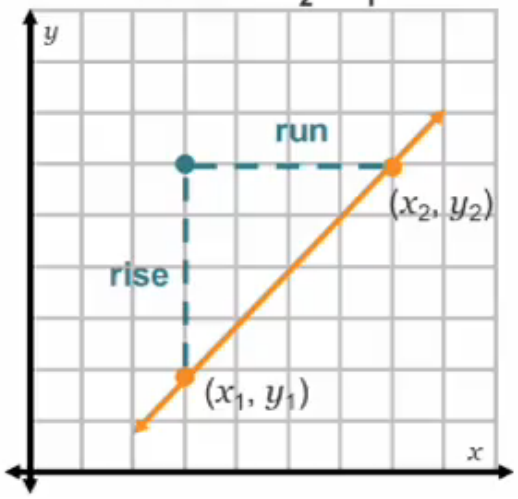
The rate of change is also called the **slope** of a line.

It is the ratio of the change in the dependent variable ($y$-values) with respect to the corresponding change in the independent variable ($x$-values).

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

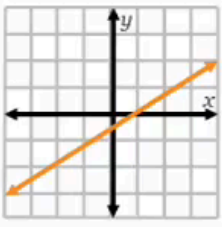



$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

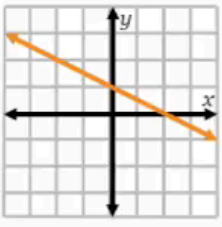



Lines can have a positive slope, negative slope, zero slope, or an undefined slope.

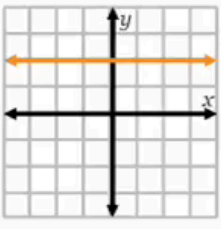
**Positive Slope**



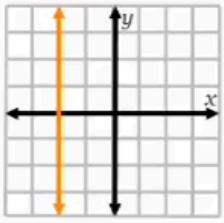

**Negative Slope**



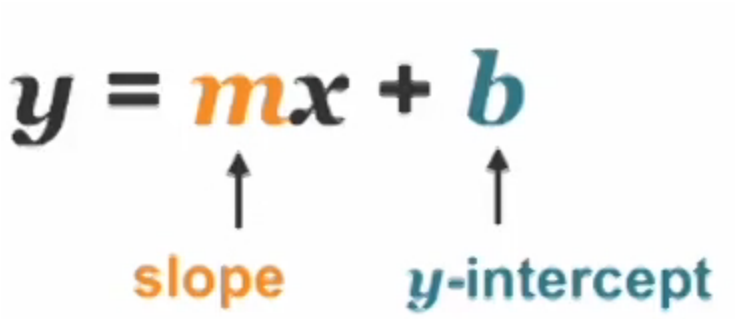

**Zero Slope**




**Undefined Slope**

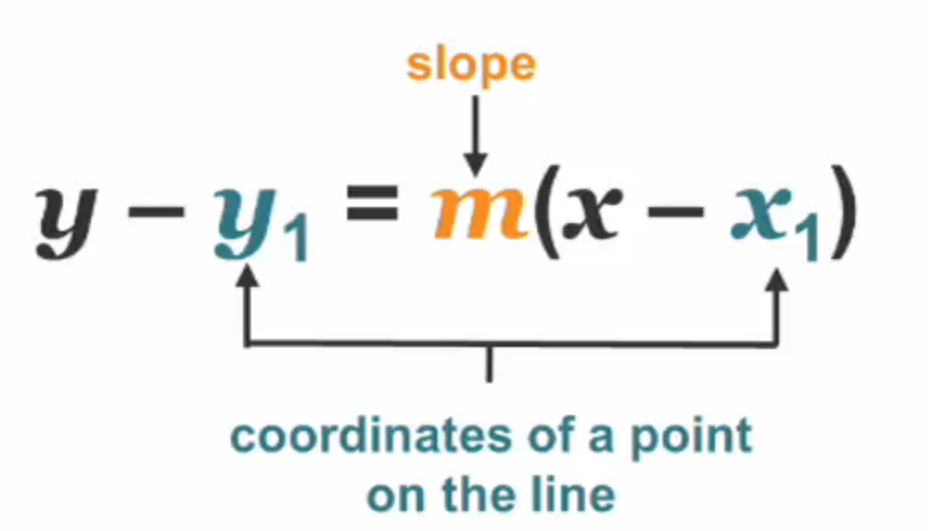


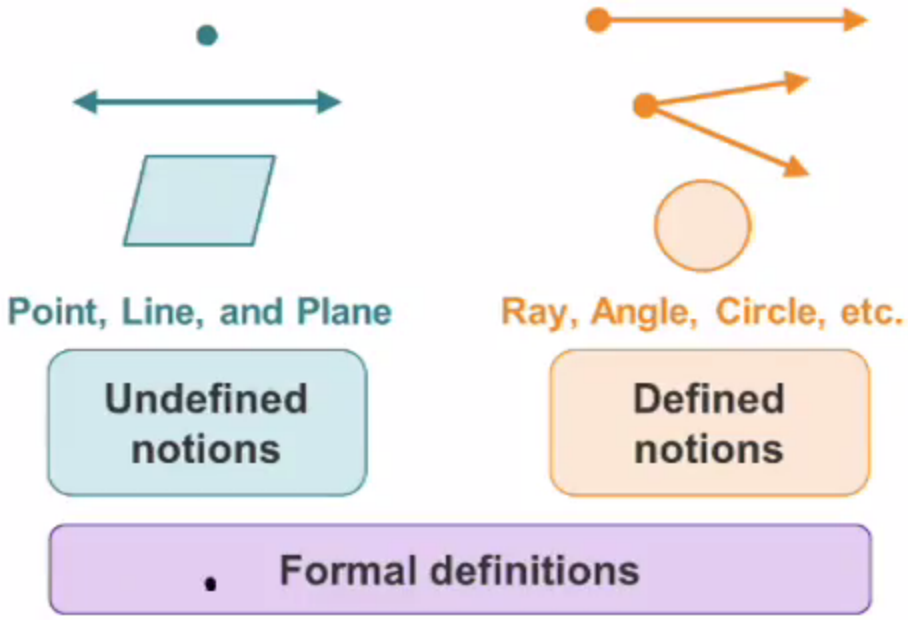

**Slope-Intercept form of a line**




The equation of a linear function in **slope-intercept form** is written as $y = mx + b$, where $m$ and $b$ are real numbers, $m$ is the slope, and $b$ is the **$y$-intercept** of the line.

**Point-Slope Formula of a Line**






Undefinable concepts can have identifying characteristics.

| Point | Line | Plane |
| --- | --- | --- |
| ▪ No dimensions<br>▪ Location on coordinate plane designated by an ordered pair $(x, y)$ | ▪ 1-dimesional set of all points<br>▪ Has no beginning or end | ▪ 2-dimensional set of all points<br>▪ Flat or level surface<br>▪ Has no beginning or end |

How can we define these mathematical terms?

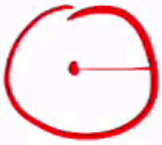
**Ray**

- A piece of a line with an endpoint that extends indefinitely in one direction

**Angle**

- Two rays with a common endpoint

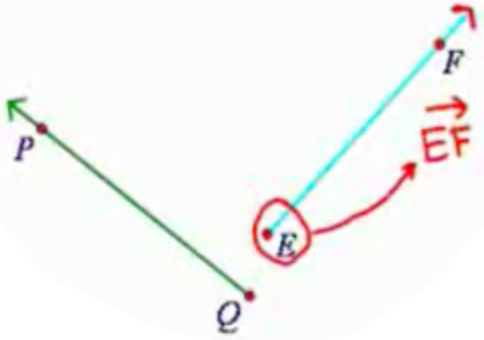
**Circle**



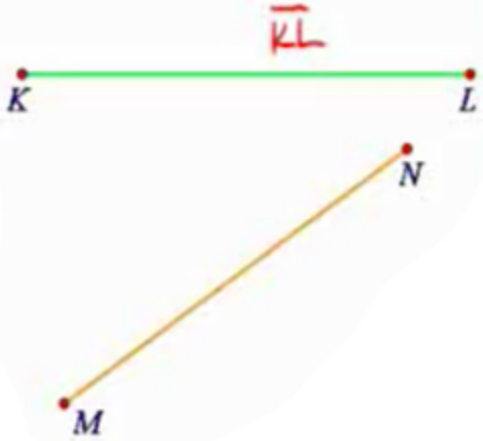
- All points in the same plane equidistant from a given point

- **Line**
  - Extends infinitely in both directions
  - No **endpoints**
  - No **length**

- **Ray**
  - Part of a line
  - Has one endpoint – the starting point
  - Extends indefinitely in one direction
  - Can't be measured




- **Line segment**
  - Two endpoints
  - Specific length



A line has no endpoints, extends infinitely in both directions, and has no length

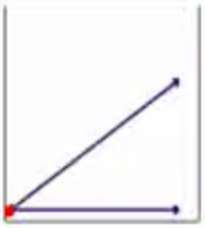
A ray has one endpoint, extends infinitely in one direction, and has no length

A line segment has two endpoints and is of a specific length

## Ruler Postulate

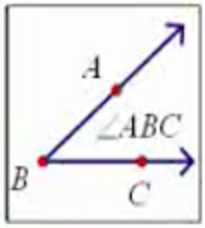
*The points of a line are in a one-to-one correspondence with the real number line such that the **distance between** any two points on the line is the absolute value of the difference of the **coordinates**.*

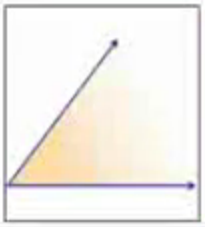

$$LM = |u_2 - u_1|$$



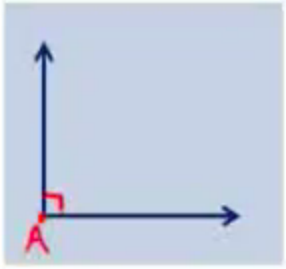
An angle is formed by two rays with a shared endpoint

Angles can be named in a number of ways



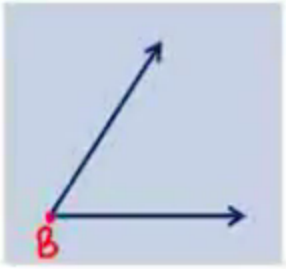



The interior of an angle is all points between the sides



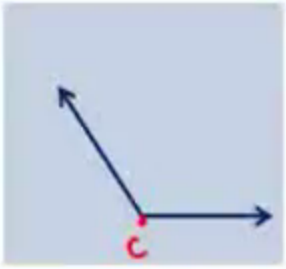

Right angle

$m\angle A = 90°$




Acute angle

$0° < m\angle B < 90°$

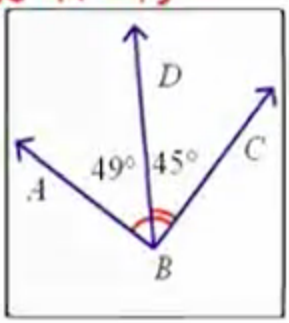



Obtuse angle

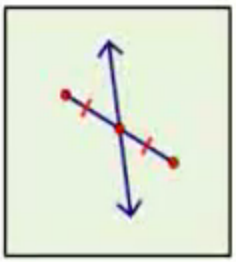
$90° < m\angle C < 180°$

Adjacent angles share a vertex, a side, and no interior points

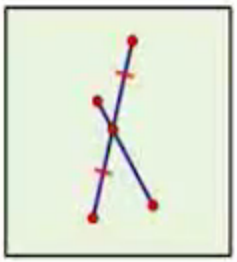
$m\angle ABC = 49° + 45°$



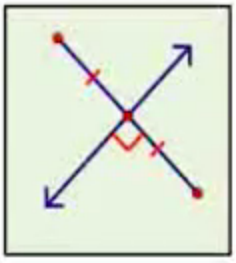

The sum of the measures of adjacent angles equals the measure of the larger angle they create



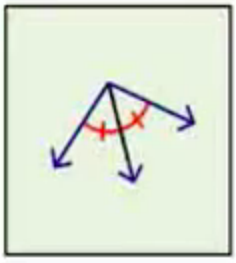
Bisector



Segment bisector



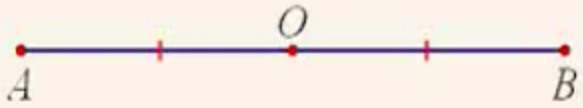
Perpendicular bisector



Angle bisector

## Midpoint Theorem

*If $O$ is the midpoint of $\overline{AB}$, then $\overline{AO} \cong \overline{OB}$.*




## Transformation

- A transformation moves an object from an original position to a new position
- The object in the new position is called the **image**, and the original figure prior to the transformation is called the **preimage**

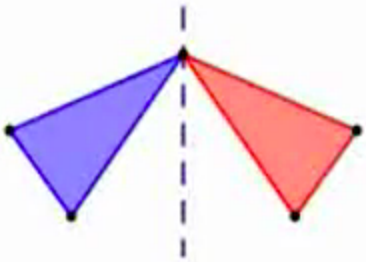
- **Rigid transformation**
  - Preserves the size, length, shape, lines, and angle measures of the figure
  - Also referred to as an **isometry**

Translation Reflection Rotation

**Translation** – a rigid transformation that moves every point of the **preimage** of an object the same distance and direction to create the **image**

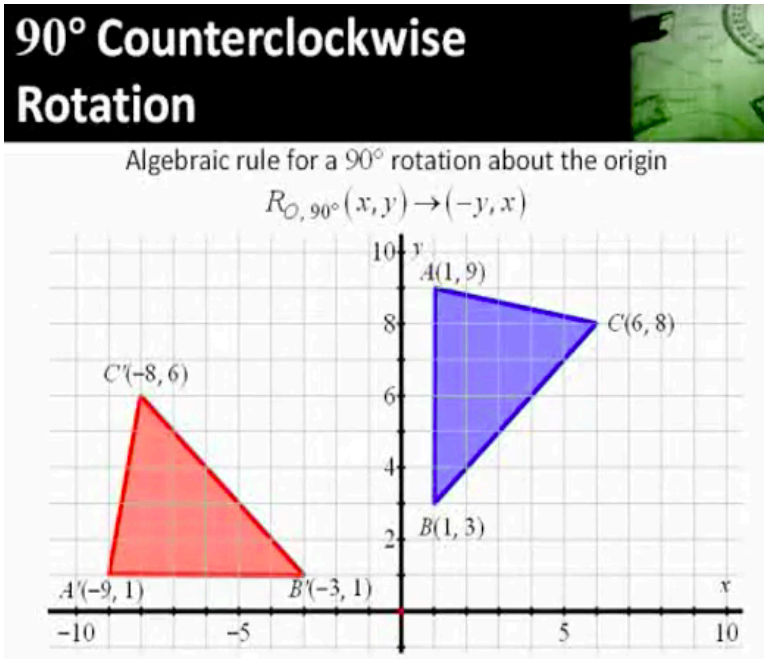
## Reflection

- In a plane, a transformation such that relative to a given **line of reflection**, a point on the preimage and the corresponding point on the image can be connected by a segment for which the line of reflection is the perpendicular bisector
- If a point on the preimage lies on the line of reflection, the image of that point is the same as the preimage



## Rotation

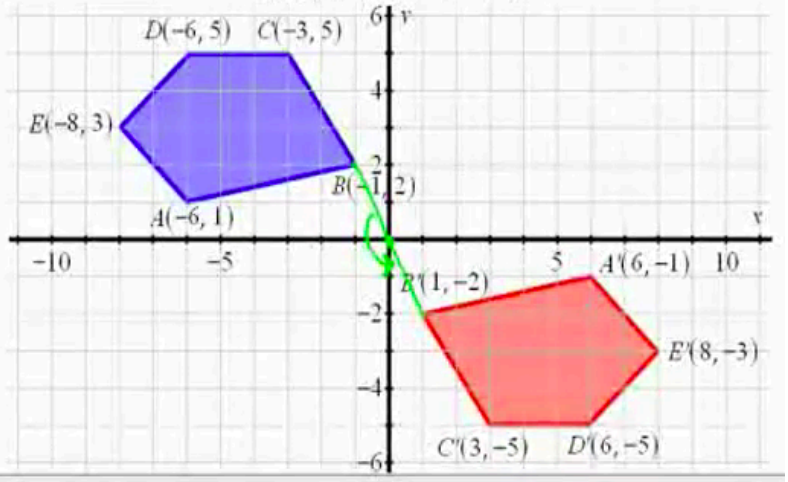
- In a plane, a transformation in which each point on a figure is turned through a given angle and direction around a given point called the **center of rotation**
- The distance between each point on the preimage and the center of rotation is equal to the distance between the center of rotation and the corresponding point on the image



# 180° Counterclockwise Rotation

Algebraic rule for a $180°$ rotation about the origin

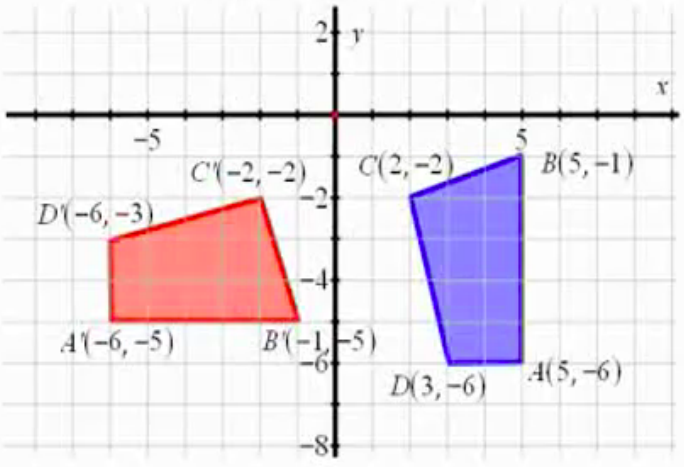
$$R_{O,\,180°}(x, y) \rightarrow (-x, -y)$$




# 270° Counterclockwise Rotation

Algebraic rule for a $270°$ rotation about the origin

$$R_{O,\,270°}(x, y) \rightarrow (y, -x)$$



- ✓ $R_{O,\,90^\circ}(x, y) \rightarrow (-y, x)$
- ✓ $R_{O,\,180^\circ}(x, y) \rightarrow (-x, -y)$
- ✓ $R_{O,\,270^\circ}(x, y) \rightarrow (y, -x)$

- ✓ Congruent figures have the same size and shape
- ✓ Two polygons are congruent if their corresponding parts are congruent
- ✓ Congruence statements must list vertices, sides, and angles in corresponding order

**Reflections**

- $r_{x\text{-axis}}(x, y) \rightarrow (x, -y)$
- $r_{y\text{-axis}}(x, y) \rightarrow (-x, y)$
- $r_{y=x}(x, y) \rightarrow (y, x)$
- $r_{y=-x}(x, y) \rightarrow (-y, -x)$

**Translations**

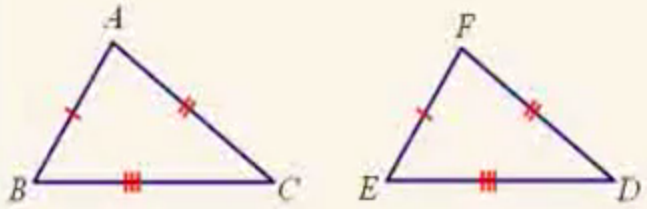
- $T_{a,b}(x, y) \rightarrow (x + a, y + b)$

**Rotations**

- $R_{O,\,90^\circ}(x, y) \rightarrow (-y, x)$
- $R_{O,\,180^\circ}(x, y) \rightarrow (-x, -y)$
- $R_{O,\,270^\circ}(x, y) \rightarrow (y, -x)$

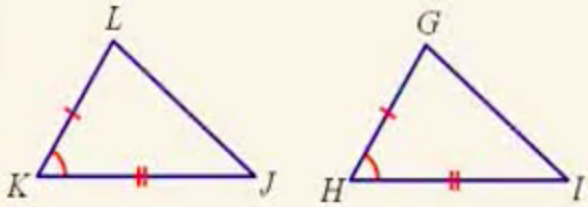
## Side-Side-Side (SSS) Postulate

*If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.*



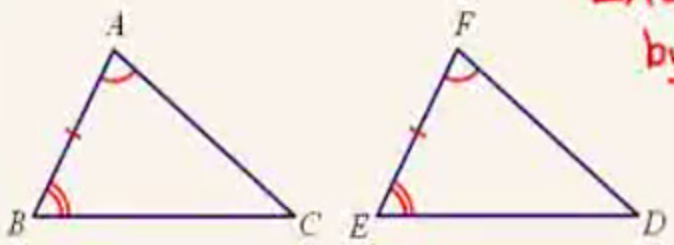

## Side-Angle-Side (SAS) Postulate

*If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.*



## Angle-Side-Angle (ASA) Postulate

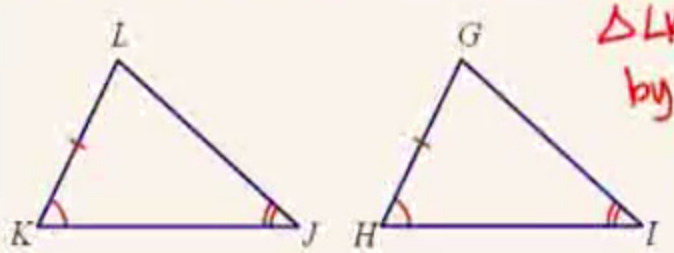
*If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.*




$\triangle ABC \cong \triangle FED$ by ASA

## Angle-Angle-Side (AAS) Theorem

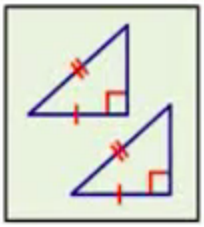
*If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the two triangles are congruent.*




$\triangle LKJ \cong \triangle GHI$ by AAS

Hypotenuse

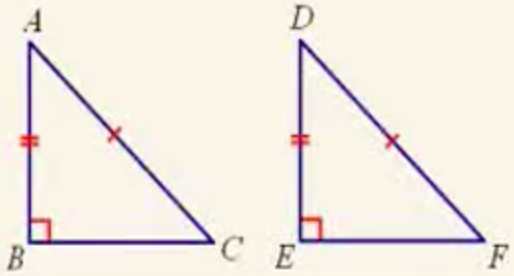
Legs



Hypotenuse-leg theorem

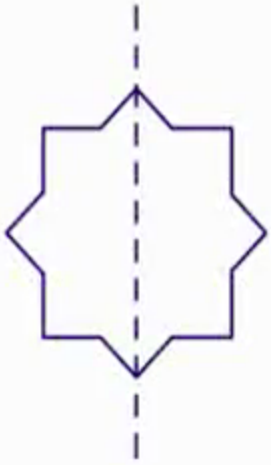
## Hypotenuse-Leg (HL) theorem

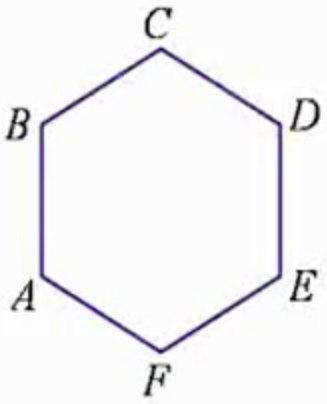
*If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.*

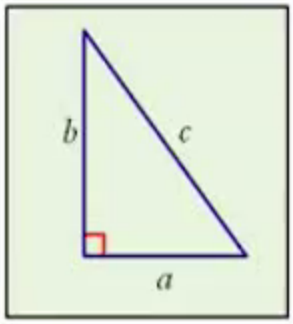



- If two triangles are proven congruent, then the corresponding parts of the congruent triangles are congruent
- This is referred to as **CPCTC**

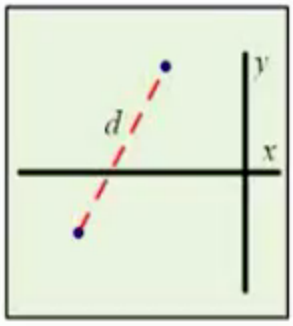
An object has **symmetry** if there is an isometric transformation that maps the object onto itself





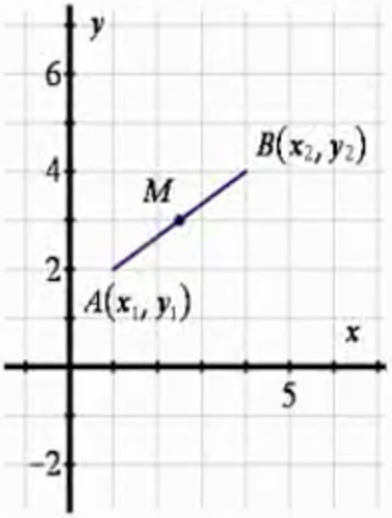

The distance formula is derived from the Pythagorean theorem



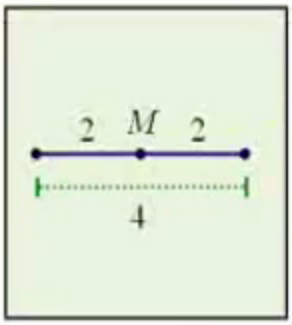

The distance formula:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

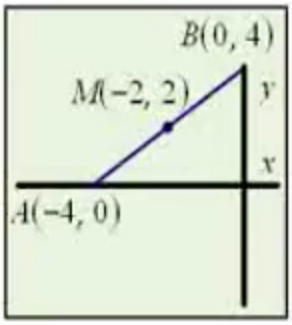
**Midpoint formula** – a formula used to find the point that divides a given line segment into two congruent segments




$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



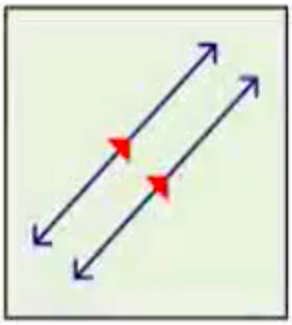

Use averages to find the midpoint of a line segment



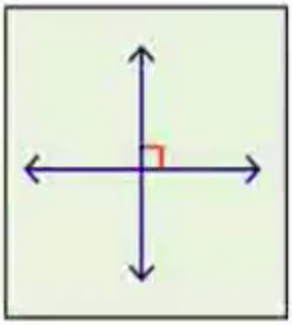

The midpoint formula:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

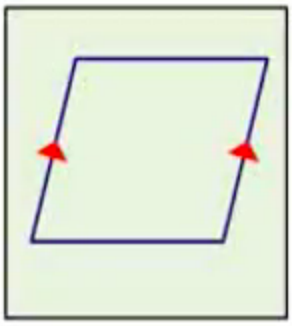
- The slope of a line is the ratio of its vertical change to its horizontal change
- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- The slope of a horizontal line is $0$
- The slope of a vertical line is undefined



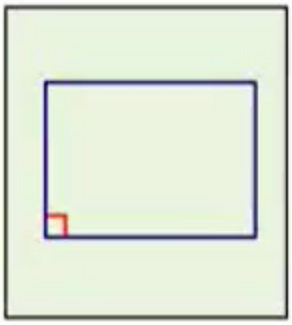
Parallel lines have the same slope



Perpendicular lines have slopes that are negative reciprocals of each other



If the slopes of two sides of a figure are equal, then those sides are parallel



If the slopes of two sides of a figure are negative reciprocals of each other, then those sides are perpendicular

Standard form of a line is $Ax + By = C$

Slope-intercept form of a line is $y = mx + b$

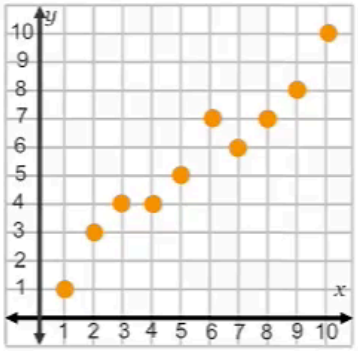
Point-slope form of a line is $y - y_1 = m(x - x_1)$

The $y$-intercept is $(0, b)$

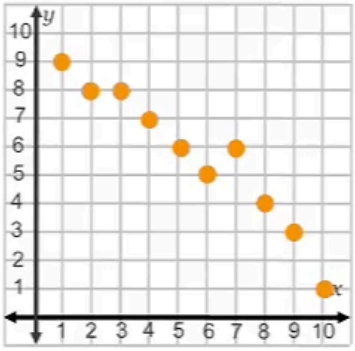
**Correlation** is the degree to which two variables vary together linearly.

- Positive correlation – shows as one variable increases, the other variable tends to increase
- Negative correlation – shows as one variable increases, the other variable tends to decrease
- No correlation – shows that as one variable increases, the other variable does not follow a specific pattern of increasing or decreasing

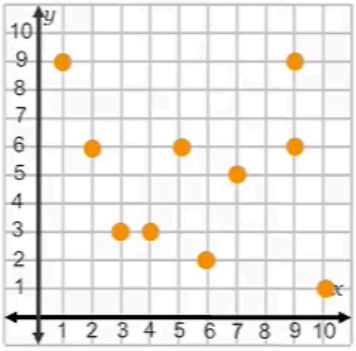
Positive Correlation



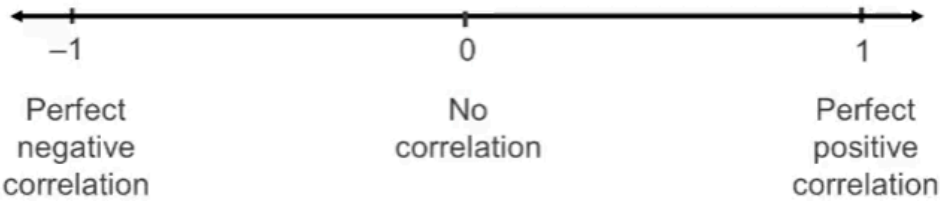

Negative Correlation




No Correlation




The **correlation coefficient**, $r$, is a value between –1 and 1 that measures the strength of the linear association between two variables.

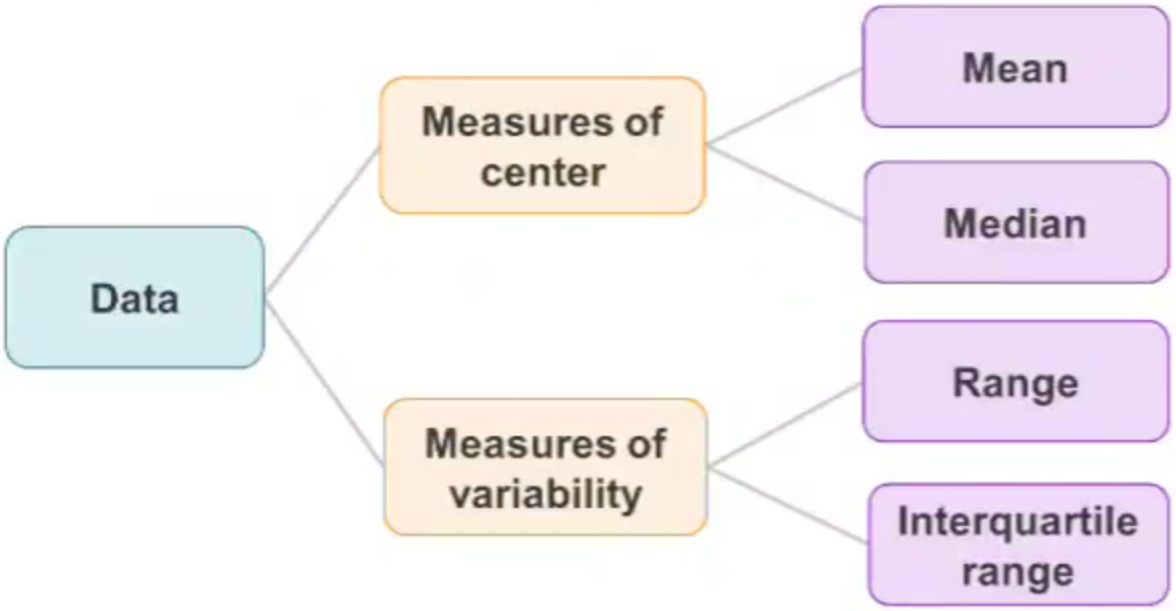



An **outlier** is a value in a data set that varies significantly from the rest of the data.

An **outlier** is identified as any data point that is greater than the upper quartile by more than 1.5 times the interquartile range, or less than the lower quartile by more than 1.5 times the interquartile range. Determine if the data sets have an outlier.

Which measure of center more accurately describes the data?

- If a data set contains an outlier, use the median.
- If a data set does not contain an outlier, use the mean.



An exponential function is of the form:

$$f(x) = ab^x$$

- Initial value = $a$
- Base = $b$, where $b \neq 1$ and is positive
  - **Rate of change**
  - Growth factor when $b > 1$
- Exponent = $x$
- Domain: {x| x is a real number}

Example: $f(x) = 2^x$

| $x$ | $f(x)$ |
|---|---|
| -2 | 0.25 |
| -1 | 0.5 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

General form of an **exponential decay** function: $f(x) = ab^x$

- The initial amount is $a$, with $a \neq 0$.
- The base, $b$, is a value between 0 and 1.
  - The base represents the constant multiplicative rate of change.
- The domain is all real numbers.

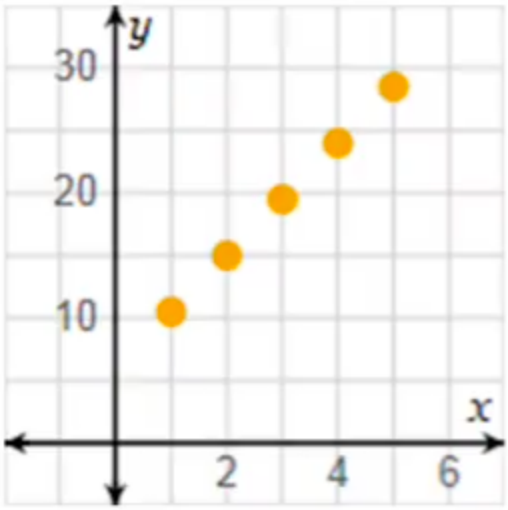
Example: $f(x) = 10\left(\frac{1}{2}\right)^x$

| $x$ | $f(x)$ |
|---|---|
| −2 | 40 |
| −1 | 20 |
| 0 | 10 |
| 1 | 5 |
| 2 | 2.5 |

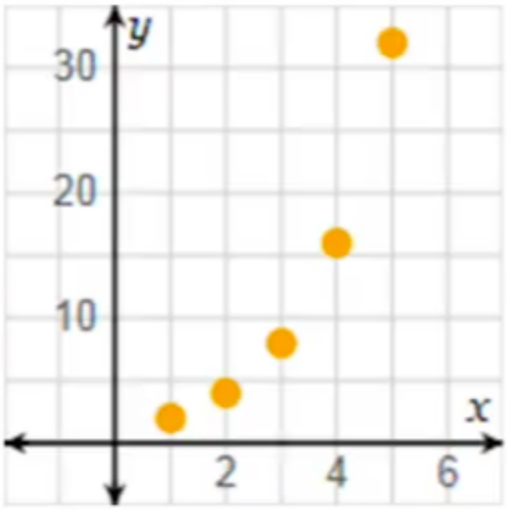
Exponential functions are of the form $f(x) = ab^x$. The value $a$ is the **initial value** of the function and the value $b$ is the base of the function.

- When $0 < b < 1$, $b$ is a decay factor, and the relationship is called **exponential decay**.
- When $b > 1$, $b$ is a growth factor, and the relationship is called **exponential growth**.

## Arithmetic Sequence




## Geometric Sequence

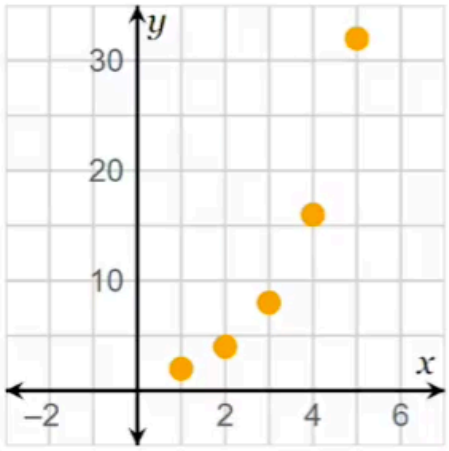



A **geometric sequence** is a sequence in which the ratio between any two consecutive terms is constant.

| $x$ | $f(x)$ |
|---|---|
| 1 | 100 |
| 2 | 90 |
| 3 | 81 |
| 4 | 72.9 |

- The **recursive formula** for a geometric sequence has the form $f(x + 1) = rf(x)$, where $r$ is the **common ratio**.

Determine the 10th term of the geometric sequence represented by the graph.




An **explicit formula** for a geometric sequence has the form $f(x) = f(1)r^{x-1}$, where $r$ is the common ratio.