

Simplified Description of the Lotka-Volterra Equations

The Lotka-Volterra equations, also known as the predator-prey equations, are a pair of differential equations used to describe the dynamics of biological systems in which two species interact: one as a predator and the other as prey.

The equations are an example of a mathematical model.

Basic Concepts:

1. **Prey Population** (e.g., Rabbits):

- The prey population grows exponentially in the absence of predators, because they have an unlimited food supply.
- However, the prey population decreases when predators are present because they are being hunted.

2. **Predator Population** (e.g., Foxes):

- The predator population depends on the availability of prey for food. If there are more prey, the predator population can grow.
- In the absence of prey, the predator population declines due to a lack of food.

The Equations:

Let $x(t)$ represent the prey population at time t .

Let $y(t)$ represent the predator population at time t .

The Lotka-Volterra equations are:

1. **Prey Equation:**

$$\frac{dx}{dt} = \alpha x - \beta xy$$

- α is the growth rate of the prey in the absence of predators.
- β is the rate at which predators destroy the prey.

2. **Predator Equation:**

$$\frac{dy}{dt} = \delta xy - \gamma y$$

- δ is the rate at which predators increase by consuming prey.
- γ is the death rate of predators in the absence of prey.

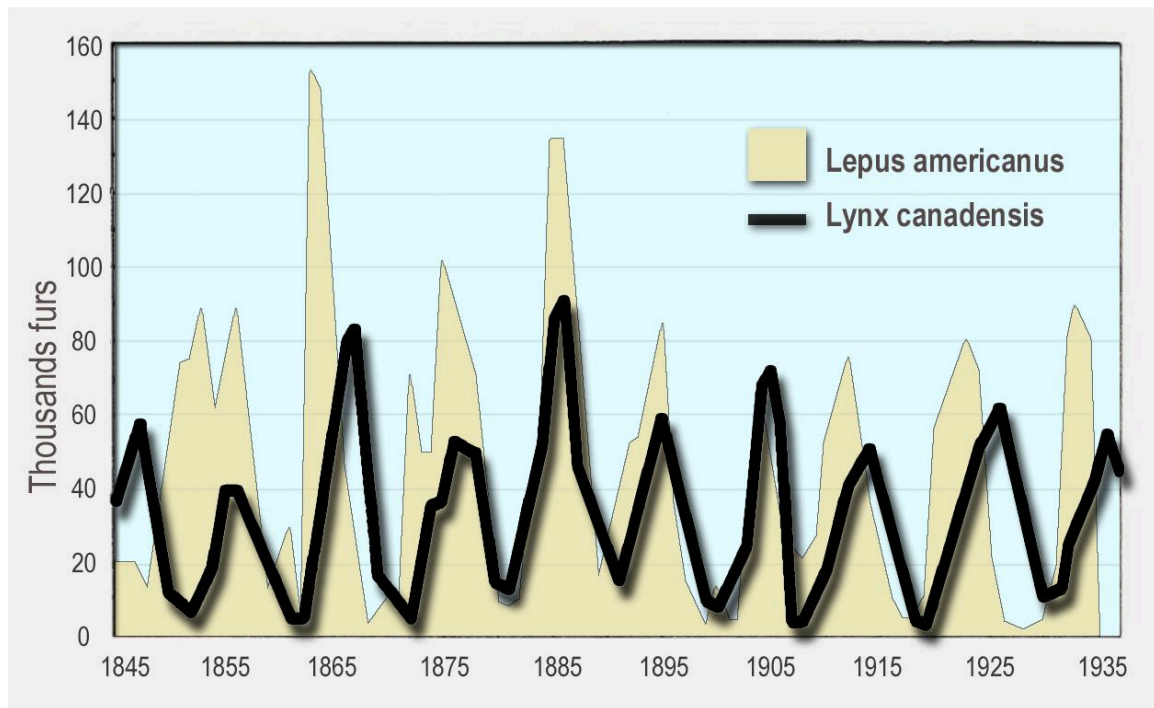
Key Points:

- **Oscillatory Dynamics:** The equations capture a real feature of predator/prey dynamics, which is that the populations of predators and prey tend to oscillate over time. When the

prey population increases, the predator population also increases because there is more food. However, as predators become more abundant, they reduce the prey population, which eventually causes a decline in the predator population due to lack of food.

- ****Stable Cycles:**** In a simplified model, these oscillations can continue indefinitely, leading to stable cycles of predator and prey populations.

Real data on Lynxes and Hares



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