Paper-Folding Proofs of the Pythagorean Theorem

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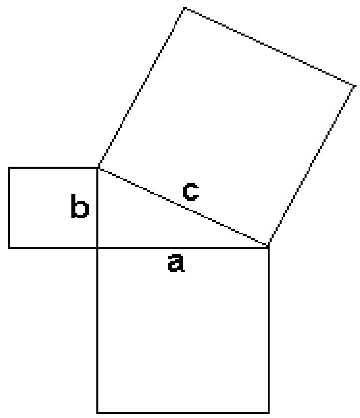
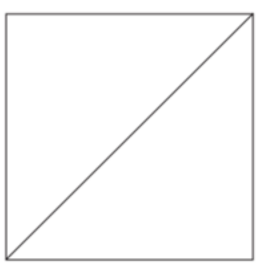
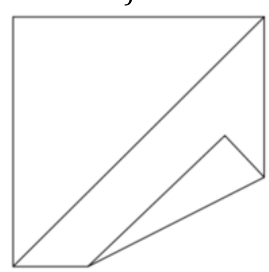
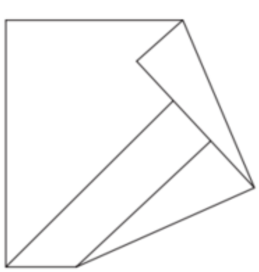
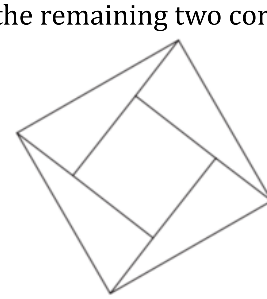
The Pythagorean Theorem tells us that the sum of the squares of the legs of a right triangle equals the square of the hypotenuse, or $a^2 + b^2 = c^2$, where a and b are the lengths of the legs, and c is the length of the hypotenuse.

Geometrically this can be interpreted as a statement about areas: the sum of the areas of a square of length a and a square of length b equals the area of a square of length c.



In this activity, we will prove this statement in **three** different ways using paper-folding. All three of the proofs require the following folds:

1. Fold the square in half along the diagonal from bottom left corner to top right to make a crease, then unfold. 	2. Fold the bottom left corner up so that the bottom edge is parallel to the diagonal fold. (Try to be as accurate as possible here!) 
3. Fold the top right corner in so that it lines up with the side of the corner that was folded down in the previous step. (This edge should be *perpendicular* to the diagonal.) 	4. Repeat with the remaining two corners. 

Proof Method 1: **The Unfolded Square**

Unfold the corners so that your paper lies flat again. You should see four right triangles, with a square in the middle. Label the long leg of each triangle "*a*", the short leg "*b*", and the hypotenuse "*c*".

1. Compute the area of the large square directly.	2. Compute the area of the large square by looking at the smaller interior shapes.
3. Compare the two areas that you calculated above - what do you see??	

Proof Method 2: **The Folded Square**

Refold the corners of your paper once again, so that now all of the triangles lie in the *interior* of the inner square. Then repeat the steps from proof 1.

1. Compute the area of the larger square directly.	2. Compute the area of the larger square by looking at the smaller interior shapes.
3. Compare the two areas that you calculated above - what do you see??	

Proof Method 3: **Conservation of Area**

Keep the paper with the corners folded down. This proof uses the idea that the area of a shape remains unchanged even if it is cut into pieces, and rearranged in a different configuration.

Cut off two of the triangles (cutting through both layers) and rearrange the pieces, then compare the area of the original shape, and the new shape you created.

Tips: (1) It is pretty hard to "undo" cutting off a corner of your paper, so I would suggest using one person's paper for your first attempt, and saving the other papers in the group in case repeat attempts are necessary! (2) Remember what we are trying to prove: "The sum of the smaller ***squares*** *equals the larger* ***square.****"*

1. Compute the total area *before* rearranging.	2. Compute the total area *after* rearranging.
3. Compare the two areas that you calculated above - what do you see??	