Easy (should be able to find solution after 1-2 generations)

Show the period of $cos 2x$ is pi

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Find all integers x, y such that $x^{2} + xy + y^{2} = 1$.
Prove there are 6 pairs of solutions

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Find all real solutions of the following nonlinear system:
$$
\begin{aligned}
x+4 y+6 z & =16 \\
x+6 y+12 z & =24 \\
x^{2}+4 y^{2}+36 z^{2} & =76
\end{aligned}
$$

The final answer is $ \boxed{(6,1,1)(-\frac{2}{3},\frac{13}{3},-\frac{1}{9})} $

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In the geometric sequence $\{a_n\}$, it is known that $a_2a_5 = -32$, $a_3 + a_4 = 4$, and the common ratio is an integer. Find $a_9 = -256$.

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Solve the inequality: \\(|x-3|+|x-5|\geqslant 4\\).
Show that the answer is [6,+\infty) \cup (-\infty,2]

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The first term a_1 of the geometric sequence $\{a_n\}$ is 2, and the common ratio $q=1$.
The sum of the first 10 terms of the sequence $\{a_n\}$ is ( )
Show that the answer is 20

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Given the sequence $\left\{a_{n}\right\}$ satisfies
$$
a_{1}=1, a_{n}+a_{n+1}=-n^{2} \text {. }
$$

then $a_{15}=$ $\qquad$

The final answer is $ \boxed{-104} $

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Let b and c be real numbers not both equal to 1 such that $1,b,c$ is an arithmetic progression and $1,c,b$ is a geometric progression. What is $100(b-c)$?

The final answer is $ \boxed{75} $

Medium difficulty (model may be able to solve when pass@8)

Find all extrema of the function $y=\frac{2}{3} \cos \left(3 x-\frac{\pi}{6}\right)$ on the interval $(0 ; \pi / 2)$.

The final answer is $ \boxed{y_{\max}(\frac{\pi}{18})=\frac{2}{3},y_{\}(\frac{7\pi}{18})=-\frac{2}{3}} $

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Solve the system of equations:

$$
\left\{\begin{array}{l}
x y-2 y=x+106 \\
y z+3 y=z+39 \\
z x+3 x=2 z+438
\end{array}\right.
$$

The final answer is $ \boxed{(38,4,9)(-34,-2,-15)} $

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Find all complex numbers z for which

$$
|z-2+i|=5, \quad \operatorname{Re}(z(1+i))=2
$$

The final answer is $ \boxed{z_{1}=5+3i,z_{2}=-2-4i} $

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$12 \cdot 18 \quad$ The equation $(x-a)(x-8)-1=0$ has two integer roots, find a.
(China Junior High School Mathematics League, 1990)

The final answer is $ \boxed{8} $

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Solve the system of equations: $\left\{\begin{array}{l}x^{2}-y=z^{2}, \\ y^{2}-z=x^{2}, \\ z^{2}-x=y^{2} .\end{array}\right.$
(2013, Croatian Mathematical Competition)

The final answer is $ \boxed{(1,0,-1),(0,0,0),(0,-1,1),(-1,1,0)} $

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Let ${{a}^{2}}-{{b}^{2}}=1+\sqrt{2}$, ${{b}^{2}}-{{c}^{2}}=1-\sqrt{2}$, then ${{a}^{4}}+{{b}^{4}}+{{c}^{4}}-{{a}^{2}}{{b}^{2}}-{{b}^{2}}{{c}^{2}}-{{c}^{2}}{{a}^{2}}=$ ___ ___
.

The final answer is $ \boxed{5} $

Difficult (low chance of solving it with pass@8)

Find the (x, y) natural integers such that $(x+y)^{2}+3 x+y+1$ is a perfect square.

The final answer is $ \boxed{(x,x)} $

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Solve the differential equation $\sin y \frac{dy}{dx} = \cos y \left(1 - x \cos y \right)$.

The final answer is $ \boxed{y(x) = \pm \cos^{-1} \left(\frac{1}{A \exp(-x) + x + 1} \right) + 2\pi n} $

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Given $a>b>c>0$, prove that

$$
a^{4} b+b^{4} c+c^{4} a>a b^{4}+b c^{4}+c a^{4} .
$$