Name: _____ ( ) Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS

4047/01

Paper 1 **Solution for student**

Monday 24 August 2020
2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7			
Q4		Q8			

80

This document consists of **15** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1).....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Given that $\dfrac{49^{2x-3}}{7^{3x}} = \dfrac{7^{2x+1}}{343^{x-1}}$, find the value of $\sqrt[3]{343^{x}}$. [4]

$$\frac{49^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{343^{x-1}}$$

$$\frac{\left(7^{2}\right)^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{\left(7^{3}\right)^{x-1}}$$ express as powers of 7

$$\frac{7^{2(2x-3)}}{7^{3x}} = \frac{7^{2x+1}}{7^{3(x-1)}}$$ index laws $\left(a^{m}\right)^{n} = a^{mn}$

$$7^{4x-6-3x} = 7^{2x+1-(3x-3)}$$ $\dfrac{a^{m}}{a^{n}} = a^{m-n}$

$$x - 6 = -x + 4$$

$$2x = 10$$ equate powers

$$x = 5$$

$$343^{\frac{x}{3}} = 7^{x}$$

$$= 16807$$

2 A cuboid has a square base of length $(3+\sqrt{2})$ cm and a height of h cm. The volume of the cuboid is $(19-3\sqrt{2})$ cm^3. **Without using a calculator**, obtain an expression for h in the form $(a+b\sqrt{2})$, where a and b are integers. [4]

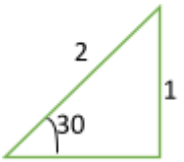
$$h = \frac{19-3\sqrt{2}}{\left(3+\sqrt{2}\right)^2}$$
$$= \frac{19-3\sqrt{2}}{9+6\sqrt{2}+2}$$
$$= \frac{19-3\sqrt{2}}{11+6\sqrt{2}} \times \frac{11-6\sqrt{2}}{11-6\sqrt{2}}$$
$$= \frac{209-114\sqrt{2}-33\sqrt{2}+18(2)}{121-36(2)}$$
$$= \frac{245-147\sqrt{2}}{49}$$
$$= 5-3\sqrt{2}$$

3 (i) **Without using a calculator** , show that $\cot 15° = 2+\sqrt{3}$. [4]

$° = \tan tan\,(45° - 30°)$ or $tan15° = tan(60° - 45°)$

$5° = \frac{\tan tan\,45° - tan30°}{1+tan45°tan30°}$ or $tan15° = \frac{tan60° - tan45°}{1+tan45°tan60°}$

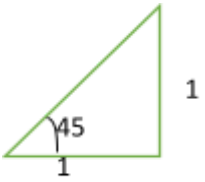
$an15° = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$ or $\tan tan\ 15° = \frac{\sqrt{3}-1}{1+\sqrt{3}}$



$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$5° = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ $\cot cot\ A = \frac{1}{tanA}$

$= \frac{3+2\sqrt{3}+1}{3-1}$



$= \frac{4+2\sqrt{3}}{2}$

$= \frac{2(2+\sqrt{3})}{2} = 2 + \sqrt{3}$

(ii) Hence show that $\text{cosec}^2 15° = 4\cot 15°$. [2]

$$cosec^2 15° = 1 + cot^2 15° = 1 + \left(2 + \sqrt{3}\right)^2$$
$$= 1 + 4 + 4\sqrt{3} + 3$$
$$= 8 + 4\sqrt{3}$$
$$= 4(2 + \sqrt{3})$$
$$= 4cot 15°$$

4 **(i)** Express $\dfrac{2x^3 - 4x^2 - 5x + 3}{(x-3)(x^2-9)}$ in partial fractions. [4]

Method 1

$$\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}=\frac{2x^3-4x^2-5x+3}{(x-3)(x-3)(x+3)}\equiv 2+\frac{A}{x+3}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$$

$$\equiv\frac{2(x+3)(x-3)^2+A(x-3)^2+B(x+3)(x-3)+C(x+3)}{(x+3)(x-3)^2}$$

$$2x^3-4x^2-5x+3\equiv 2(x+3)(x-3)^2+A(x-3)^2+B(x+3)(x-3)+C(x+3)$$

when $x=3$, $\quad 54-36-15+3=6C$

$$C=1$$

when $x=-3$, $\quad -54-36+15+3=36A$

$$A=-2$$

when $x=0$, $\quad 3=2(3)(9)-2(9)+B(3)(-3)+1(3)$

$$-36=-9B$$

$$B=4$$

$$\therefore\frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\equiv 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}$$

Method 2

$$\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}=2+\frac{2x^2+13x-51}{(x-3)(x+3)(x-3)}$$

$$\frac{2x^2+13x-51}{(x-3)(x+3)(x-3)}=\frac{A}{x-3}+\frac{B}{x+3}+\frac{C}{(x-3)^2}$$

$$2x^2+13x-51=A(x-3)(x+3)+B(x-3)^2+C(x+3)$$

Let x=3 $\quad 2(3)^2+13(3)-51=C(3+3)$

$$C=1$$

Let x= −3 $\quad 2(-3)^2+13(-3)-51=B(-3-3)^2$

$$B=-2$$

Let $x=0$ $\quad -51=A(-9)-2(9)+3$

$$A=4$$

$$\frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\equiv 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}$$

(ii) Hence find $\int\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}\,\mathrm{d}x$. [2]

$$\int \frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\,dx$$

$$=\int 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}\,dx$$

$$=\int 2-\frac{2}{x+3}+\frac{4}{x-3}+(x-3)^{-2}\,dx$$

$$=2x-2\ln(x+3)+4\ln(x-3)+\frac{(x-3)^{-1}}{-1}+c$$

$$=2x-2\ln(x+3)+4\ln(x-3)-\frac{1}{x-3}+c \quad or\ 2x-\frac{1}{x-3}+\ln\frac{(x-3)^4}{(x+3)^2}$$

5 Liquid is poured, at a constant rate of 15 cm^3/s, into a container. When the depth of liquid in the container is x cm, the volume, $V\ cm^3$, of the liquid in the container is given by

$$V=\frac{1}{4}x(x+16).$$

Find, when $V=20$,

(i) the value of x, [3]

$V=\frac{1}{4}x^2+4x$

(i) When $V=20$, $\frac{1}{4}x^2+4x=20$

$x^2+16x-80=0$

$(x+20)(x-4)=0$

$x=-20(\text{NA})$ or $x=4$

(ii) the rate of change of the depth of liquid at this time. [4]

(ii) $\frac{dV}{dx}=\frac{1}{2}x+4$

When $x=4$, $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$

$=\frac{1}{[\frac{1}{2}(4)+4]}\times 15$ $\qquad \frac{dx}{dv}=\frac{1}{\frac{dv}{dx}}$

$=\frac{15}{6}$

$=2.5$

The rate of change of the depth of liquid at this time is 2.5 cm/s

6 Solve the equation

(i) $2\log_3(x+4)-\log_3(x+2)=2$, [5]

$2\log_3(x+4)-\log_3(x+2)=2$

$\log_3(x+4)^2-\log_3(x+2)=2$ $\quad log_a b^c = clog_a b$

$\log_3\frac{(x+4)^2}{x+2}=2$ $\quad lgP - lgQ = \lg lg\frac{P}{Q}$

$\frac{(x+4)^2}{x+2}=3^2$ $\quad log_a b = x \Longrightarrow a^x = b$

$x^2+8x+16=9(x+2)$

$=9x+18$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2 \quad x=-1$

(ii) $\log_5 y-3\log_y 5=2$. [4]

$\log_5 y-3\log_y 5=2$

$\log_5 y-3\left(\frac{\log_5 5}{\log_5 y}\right)=2$ $\quad log_a b = \frac{log_c b}{log_c a}$

$(\log_5 y)^2-3=2\log_5 y$ $\quad (log_5 y)^2 \neq 2(log_5 y)$

$(\log_5 y)^2-2\log_5 y-3=0$

$a^2-2a-3=0$

$(a+1)(a-3)=0$

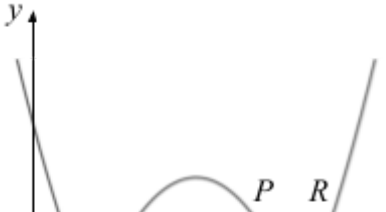
$a=-1 \qquad a=3$

$\log_5 y=-1 \quad \log_5 y=3$

$y=5^{-1} \qquad y=5^3$

$=\frac{1}{5} \qquad =125$

7



The diagram shows the line $y = 24$ and part of the graph of $y = |x^2 - 16x + 39|$.

The graph crosses the line $y = 24$ at the points P and R and meets the x-axis at Q.

(i) Find the coordinates of P, Q and R. [6]

To find P and R, y=24

$|x^2 - 16x + 39| = 24$

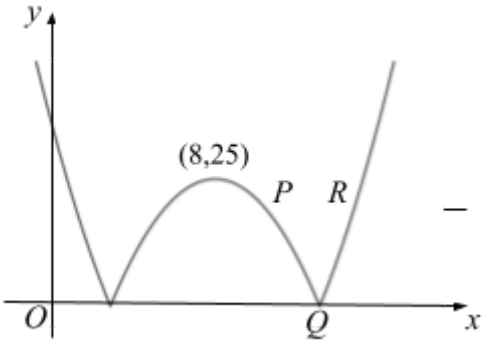
$x^2 - 16x + 39 = \pm 24$

$x^2 - 16x + 39 = -24$ $\quad$ $x^2 - 16x + 39 = 24$

$x^2 - 16x + 63 = 0$ $\quad$ $x^2 - 16x + 15 = 0$

$(x-7)(x-9) = 0$ $\quad$ $(x-1)(x-15) = 0$

$x = 7$, $x = 9$ $\quad$ $x = 1$, $x = 15$



Ans. $P(9,24)$, $R(15,24)$

To find Q , y =0

$x^2 - 16x + 39 = 0$

$(x-3)(x-13) = 0$

$x = 3$, $x = 13$

Ans. $Q(13,0)$

(ii) State the set of values of k for which $|x^2 - 16x + 39| = k$ has 2 solutions. [3]

axis of symmetry $x = \dfrac{3+13}{2}$

$= 8$

$y = |8^2 - 16(8) + 39|$

$= |-25|$

$= 25$

$y = |x^2 - 16x + 8^2 - 8^2 + 39|$

$= |(x-8)^2 - 25|$

Max pt (8,25)

Ans. $k = 0$, $k > 25$

8 It is given that $y_1 = \sin x - 2$ and $y_2 = -3\cos 2x$.

(i) State the amplitude and period, in degrees, of **(a)** y_1, **(b)** y_2. [2]

$= \sin x - 2$ amplitude =1, period=360°

$= -3\cos 2x$ amplitude =3, period =180°

For the interval $0° \le x \le 360°$,

(ii) solve the equation $y_1 = y_2$, [4]

$sinx - 2 = -3cos2x$

$3(1 - 2sin^2 x) + sinx - 2 = 0$ $\qquad cos2x = 1 - 2x$

$6sin^2 x - sinx - 1 = 0$

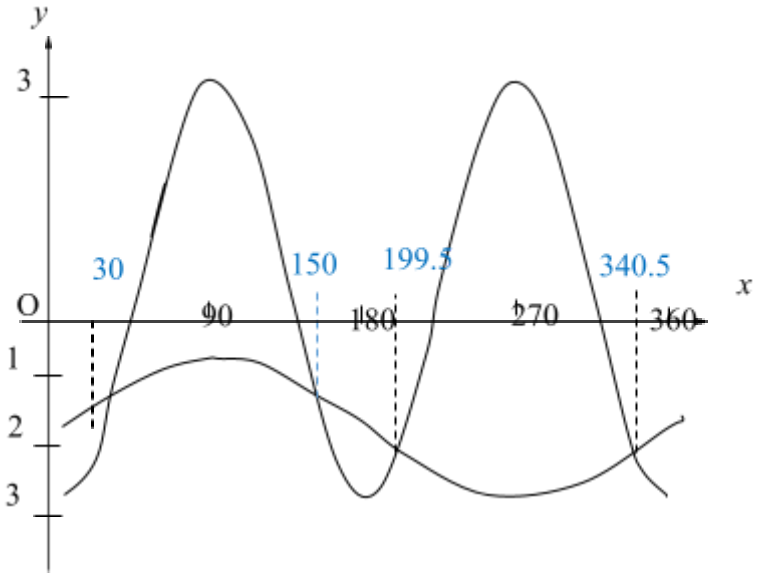
$(3sinx + 1)(2sinx - 1) = 0$

$sinx = -\frac{1}{3}, \qquad x = \frac{1}{2}$

$x = 180° + 19.471°, 360° - 19.471, x = 30°, 150$

$x = 199.5°, 340.5°. \qquad x = 30°, 150°$

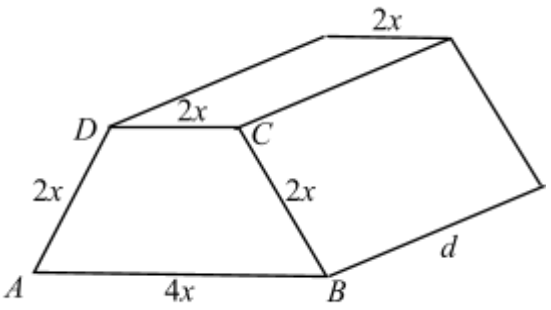
(iii) sketch, on the same diagram, the graphs of y_1 and y_2, [4]



(iv) find the set of values of x for which $y_2 - y_1 > 0$. [2]

$30° < x < 150°,\ 199.5° < x < 340.5°$

9



The diagram shows a prism in which the cross-section is a trapezium $ABCD$.

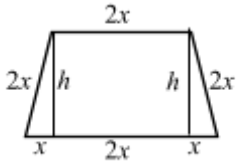
$AB = 4x$ cm, $BC = DC = AD = 2x$ cm. The length of the prism is d cm.

(i) Show that the area of trapezium $ABCD$ is $3\sqrt{3}x^2$ cm². [2]

(i) $h = \sqrt{(2x)^2 - x^2}$

$= \sqrt{3x^2}$

$= \sqrt{3}x$



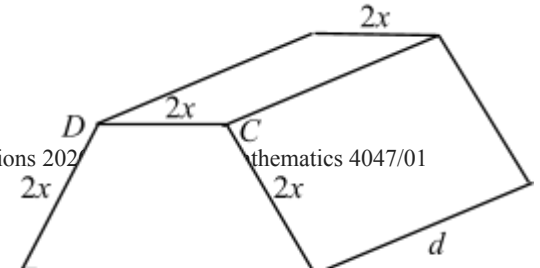
Area of trapezium $ABCD$ $= \frac{1}{2}(\sqrt{3}x)(2x + 4x)$

$= \frac{1}{2}(\sqrt{3}x)(6x)$

$= 3\sqrt{3}x^2$ cm².

The volume of the prism is $3888\sqrt{3}$ cm³.

(ii) Express d in terms of x. [1]



(ii) $3\sqrt{3}x^2 d = 3888\sqrt{3}$

$x^2 d = 1296$

$d = \dfrac{1296}{x^2}$

(iii) Show that the total surface area, A cm^2, of the prism is given by

$$A = 6\sqrt{3}x^2 + \frac{12960}{x}.$$ [3]

(iii) $A = 2\left(3\sqrt{3}x^2\right) + \left(2x + 2x + 2x + 4x\right)\left(\dfrac{1296}{x^2}\right)$

$= 6\sqrt{3}x^2 + 10x\left(\dfrac{1296}{x^2}\right)$

$A = 6\sqrt{3}x^2 + \dfrac{12960}{x}$

(iv) Given that x can vary, find the value of x which gives a stationary value of A. [4]

(iv) $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 12\sqrt{3}x - \dfrac{12960}{x^2}$

At $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 0$, $12\sqrt{3}x - \dfrac{12960}{x^2} = 0$

$x^3 = \dfrac{12960}{12\sqrt{3}}$

$x \approx 8.543$

$x = 8.54$ (to 3 s.f.)

(v) Explain why this value of x gives the smallest possible surface area of the prism. [1]

(v) $\dfrac{d^2A}{dx^2} = 12\sqrt{3} + \dfrac{25920}{x^3}$

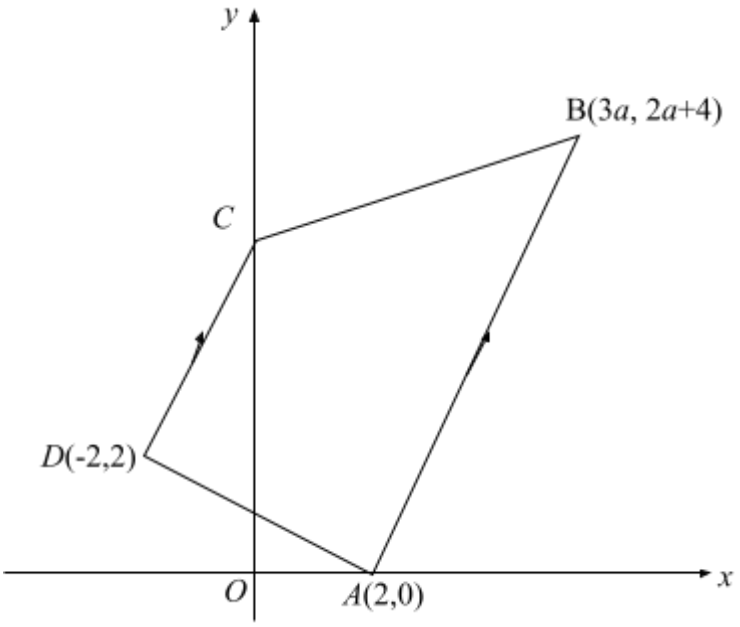
When $x \approx 8.543$, (or $x^3 = 360\sqrt{3}$)

$\dfrac{d^2A}{dx^2} \approx 62.3538$ (or $36\sqrt{3}$)

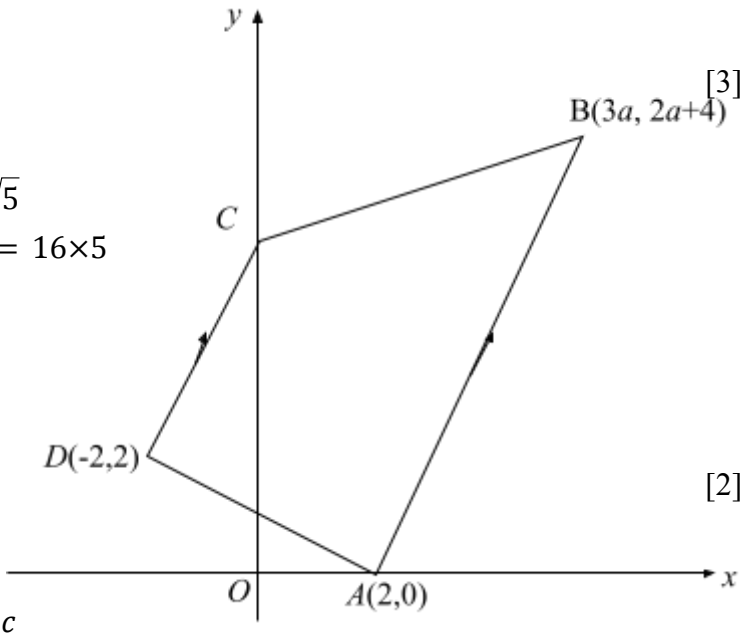
$\dfrac{d^2A}{dx^2} > 0$

Therefore this value of x gives the smallest possible surface area of the prism.

10 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC. The coordinates of the points A, B and D are (2, 0), (3a, 2a+4) and (−2, 2) respectively, where a is a positive integer. The length of AB is $4\sqrt{5}$ units.



(i) Show that $a = 2$. [3]

$$\sqrt{(3a-2)^2 + (2a+4)^2} = 4\sqrt{5}$$

$$9a^2 - 12a + 4 + 4a^2 + 16a + 16 = 16\times5$$

$$13a^2 + 4a - 60 = 0$$

$$(a + 30)(a - 2) = 0$$

$$a = -\frac{30}{13}\ (NA),\ a = 2$$

(ii) Find the coordinates of C. [2]

ɔe (0,y)

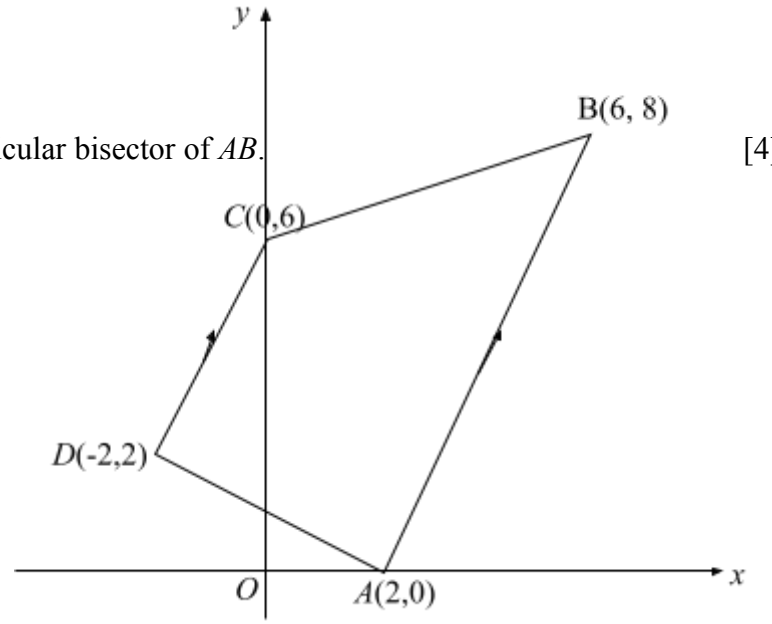
$y = 2x + c$

$= \frac{8-0}{6-2}$ OR $2 = 2(-2) + c$

$= 2$ OR $c = 6$

$y = 2(0) + 6$

(iii) Find the equation of the perpendicular bisector of AB. [4]



, B (6,8)

ɔoint of AB=$(\frac{6+2}{2}, \frac{0+8}{2})$

$= (4, 4)$

$-\frac{1}{2}$ $m_1 \times m_2 = -1$

on of the perpendicular bisector of AB

$4 = -\frac{1}{2}(4) + c$

$c = 6$

$y = -\frac{1}{2}x + 6$

(iv) Hence, or otherwise, determine if C lies on the perpendicular bisector of AB. [1]

let x=0 $\quad y = -\frac{1}{2}(0) + 6$

y=6 or

let y=6. $\quad 6 = -\frac{1}{2}x + 6$

$x = 0$

6) lies on the perpendicular bisector of AB

(v) Find the area of the trapezium $ABCD$. [2]

$ABCD = \frac{1}{2}|2\ 6\ 0\ 0\ 8\ 6 \quad -\ 2\ 2\ 2\ 0|$

$= \frac{1}{2}(16 + 36 + 12 - 4)$

$= \frac{1}{2}(60)$

$= 30$ sq units

