

MARKING SCHEME
PREBOARD CLASS 12 (MATHS)

SECTION-A		
1	D	1
2	A	1
3	C	1
4	A	1
5	B	1
6	D	1
7	A	1
8	<i>C</i>	1
9	<i>D</i>	1
10	B	1
11	<i>C</i>	1
12	<i>C</i>	1
13	A	1
14	D	1
15	D	1
16	B	1
17	A	1
18	D	1
19	A	1

20	D	1
SECTION-B		
21	We know that, $(\cos \cos x) = x$, if $x \in [0, \pi]$. Now, $\left(\cos \cos \frac{13\pi}{6} \right)$ $= \left[\cos \cos \left(2\pi + \frac{\pi}{6} \right) \right], \left(\because \frac{13\pi}{6} \notin [0, \pi] \right)$ $= \left[\cos \cos \frac{\pi}{6} \right], \left(\because \cos \cos (2\pi + x) = \cos \cos x \right)$ $= \frac{\pi}{6}, \left(\because \frac{\pi}{6} \in [0, \pi] \right)$ OR $(\sqrt{3}) - (-2)$ $= \frac{\pi}{3} - (\pi - 2), [\because (-x) = \pi - x]$ $= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3} \right)$ $= \frac{\pi}{3} - \frac{2\pi}{3}$ $= -\frac{\pi}{3}$	1 1 1
22	$x = a \tan^3 \theta$ Differentiating w.r.t. θ $\frac{dx}{d\theta} = a \times 3 \tan^2 \theta \sec^2 \theta$ Again, $y = a \sec^3 \theta$ Differentiating w.r.t. θ $\frac{dy}{d\theta} = a \times 3 \sec^2 \theta \sec \theta \tan \theta = 3 a \tan \theta \sec^3 \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3 a \tan \theta \sec^3 \theta}{a \times 3 \tan^2 \theta \sec^2 \theta}$ $= \frac{\sec \theta}{\tan \theta}$	0.5 0.5

$$\Rightarrow |\sin \sin \theta| = \frac{1}{\sqrt{14}}$$

$$\Rightarrow \sin \sin \theta = \pm \frac{1}{\sqrt{14}}$$

$$\Rightarrow \theta = \left(\pm \frac{1}{\sqrt{14}} \right)$$

1

25

$$\text{Let, } 2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = k$$

$$\therefore P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}, P(X = x_3) = k, P(X = x_4) = \frac{k}{5}$$

Now,

$$P(X = x_1) + P(X = x_2) + P(X = x_3) + P(X = x_4) = 1$$

$$\Rightarrow \frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$$

$$\Rightarrow \frac{15k+10k+30k+6k}{30} = 1$$

$$\Rightarrow \frac{61k}{30} = 1$$

$$\Rightarrow k = \frac{30}{61}$$

X	x_1	x_2	x_3	x_4
$P(X)$	$\frac{30}{122}$	$\frac{30}{183}$	$\frac{30}{61}$	$\frac{30}{305}$

OR

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

$$B = \{1, 2, 3\}$$

$$A \cap B = \{2\}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{6}$$

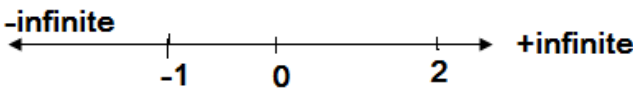
$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\therefore P(A \cap B) \neq P(A) \cdot P(B)$$

Hence, A and B are not independent

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1

		1.5
		0.5
SECTION-C		
26	Here, $(\cos y)^x = (\sin x)^y$ Taking log both sides we get $\log (\cos y)^x = \log (\sin x)^y$ Or, $x \log \cos y = y \log \sin x$ Or, $y \log \sin x = x \log \cos y$ Differentiating both sides w.r.t x $y \times \frac{\cos x}{\sin x} + \log \sin x \frac{dy}{dx} = x \times \frac{-\sin y}{\cos y} \frac{dy}{dx} + \log \cos y$ Or, $y \cot x + \log \sin x \frac{dy}{dx} = -x \tan y \frac{dy}{dx} + \log \cos y$ Or, $\frac{dy}{dx} (\log \sin x + x \tan y) = \log \cos y - y \cot x$ Or, $\frac{dy}{dx} = \frac{\log \cos y - y \cot x}{\log \sin x + x \tan y}$	1 1 1
27	Given $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ Differentiating both sides w.r.t. x $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x + 1)(x - 2)$  $f'(x) < 0$ if $x \in (-\infty, -1) \cup (0, 2)$ $\therefore f(x)$ is strictly decreasing in $(-\infty, -1) \cup (0, 2)$ Similarly, $f'(x) > 0$ if $x \in (-1, 0) \cup (2, \infty)$ $\therefore f(x)$ is strictly increasing in $(-1, 0) \cup (2, \infty)$	1 1 1

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9[1 - (1 - \sin 2x)]} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9[1 - (\sin x - \cos x)^2]} dx$$

x	0	$\frac{\pi}{4}$
z	-1	0

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{25 - 9(\sin x - \cos x)^2} dx$$

$$\text{Let, } \sin x - \cos x = z$$

$$\therefore (\cos x + \sin x) dx = dz$$

$$= \int_{-1}^0 \frac{dz}{25 - 9z^2}$$

$$= - \int_{-1}^0 \frac{dz}{9z^2 - 25}$$

$$= - \int_{-1}^0 \frac{dz}{(3z)^2 - 5^2}$$

$$= - \frac{1}{2.5.3} \left[\log \left| \frac{3z-5}{3z+5} \right| \right]_{-1}^0$$

$$= - \frac{1}{30} [\log 1 - \log 4]$$

$$= \frac{1}{30} \log 4$$

OR

$$\int_0^a f(a-x) dx$$

$$\text{Let, } a - x = z$$

x	0	a
z	a	0

$$\therefore dx = - dz$$

1

0.5

1

0.5

$$= - \int_a^0 f(z) dz$$

$$= \int_0^a f(z) dz$$

$$= \int_0^a f(x) dx$$

$$\therefore \int_0^a f(x) dx = \int_0^a f(a - x) dx \text{ (proved)}$$

$$\int_0^1 x^2 (1 - x)^n dx$$

$$= \int_0^1 (1 - x)^2 [1 - (1 - x)]^n dx, \left(\text{Applying } \int_a^b f(x) dx = \int_a^b f(a + b - x) dx \right)$$

$$= \int_0^1 (1 - x)^2 x^n dx$$

$$= \int_0^1 (1 - 2x + x^2) x^n dx$$

$$= \int_0^1 (x^n - 2x^{n+1} + x^{n+2}) dx$$

$$= \left[\frac{x^{n+1}}{n+1} - 2 \cdot \frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{n+3} \right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right]$$

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1

29

$$y = x^2 \dots (i)$$

$$y = |x| \dots (ii)$$

Solving (i) and (ii), we get

$$|x| = x^2$$

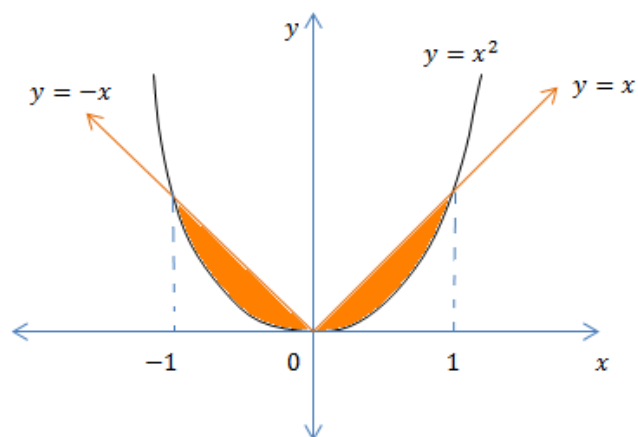
$$\Rightarrow x^2 = x^4$$

$$\Rightarrow x^2 (1 - x^2) = 0$$

$$\Rightarrow x = -1, 0, 1$$

The required area is :

$$= 2 \left[\int_0^1 y_l dx - \int_0^1 y_p dx \right]$$



1.5

$$= 2 \left[\int_0^1 x dx - \int_0^1 x^2 dx \right]$$

$$= 2 \left[\frac{1}{2} [x^2]_0^1 - \frac{1}{3} [x^3]_0^1 \right]$$

$$= 2 \left[\frac{1}{2} (1 - 0) - \frac{1}{3} (1 - 0) \right]$$

$$= 2 \left[\frac{1}{2} - \frac{1}{3} \right]$$

$$= 2 \cdot \frac{1}{6}$$

$$= \frac{1}{3} \text{sq. units}$$

1

OR

$$y^2 = 16ax \dots (i)$$

$$y = 4mx \dots (ii)$$

Solving (i) and (ii), we get

$$(4mx)^2 = 16ax$$

$$\Rightarrow 16m^2 x^2 - 16ax = 0$$

$$\Rightarrow 16x(m^2 x - a) = 0$$

$$\Rightarrow x = 0, \frac{a}{m^2}$$

The required area is :

$$= \int_0^{\frac{a}{m^2}} y_p dx - \int_0^{\frac{a}{m^2}} y_l dx$$

$$= \int_0^{\frac{a}{m^2}} 4\sqrt{a}\sqrt{x} dx - \int_0^{\frac{a}{m^2}} 4mxdx$$

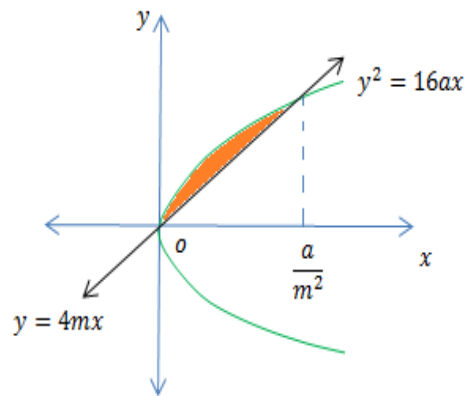
$$= 4\sqrt{a} \cdot \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^{\frac{a}{m^2}} - \frac{4m}{2} [x^2]_0^{\frac{a}{m^2}}$$

$$= \frac{8\sqrt{a}}{3} \cdot \frac{a}{m^2} \cdot \sqrt{\frac{a}{m^2}} - 2m \cdot \frac{a^2}{m^4}$$

$$= \frac{8a^2}{3m^3} - \frac{2a^2}{m^3}$$

$$= \frac{2a^2}{3m^3} \text{sq. units}$$

0.5



1

0.5

0.5

	$\therefore \frac{2a^2}{3m^3} = \frac{a^2}{12}$ $\Rightarrow m^3 = 8$ $\Rightarrow m = 2$	0.5
		0.5
30	$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2 y^2$ $\Rightarrow \frac{dy}{dx} = (1 + x^2) + y^2(1 + x^2)$ $\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$ $\Rightarrow \frac{dy}{1+y^2} = (1 + x^2)dx$ $\Rightarrow \int \frac{dy}{1+y^2} = \int (1 + x^2)dx$ $\Rightarrow y = x + \frac{x^3}{3} + c$ <i>It is given that $y = 0$ when $x = 1$</i> $\therefore 0 = 1 + \frac{1}{3} + c$ $\Rightarrow c = -\frac{4}{3}$ <i>Hence, the particular solⁿ is :</i> $y = x + \frac{x^3}{3} - \frac{4}{3}$ OR $x \frac{dy}{dx} \sin \sin \left(\frac{y}{x} \right) + x - y \sin \sin \left(\frac{y}{x} \right) = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y \sin \sin \left(\frac{y}{x} \right) - x}{x \sin \sin \left(\frac{y}{x} \right)} \dots (i)$ <i>Let, $f(x, y) = \frac{y \sin \sin \left(\frac{y}{x} \right) - x}{x \sin \sin \left(\frac{y}{x} \right)}$</i> $\therefore f(\lambda x, \lambda y) = \frac{\lambda y \sin \sin \left(\frac{\lambda y}{\lambda x} \right) - \lambda x}{\lambda x \sin \sin \left(\frac{\lambda y}{\lambda x} \right)} = \frac{y \sin \sin \left(\frac{y}{x} \right) - x}{x \sin \sin \left(\frac{y}{x} \right)} = f(x, y)$ <i>Hence, f is a homogeneous function of degree 0. Let, $y = vx$</i> <i>Differentiating both sides w. r. t. x</i>	1
		1
		1

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Hence, $eq^n(i)$ becomes

$$v + x \frac{dv}{dx} = \frac{vx \sin \sin v - x}{x \sin \sin v}$$

$$\Rightarrow vx \frac{dv}{dx} = \frac{vx \sin \sin v - x}{x \sin \sin v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{vx \sin \sin v - x - vx \sin \sin v}{x \sin \sin v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{-x}{x \sin \sin v}$$

$$\Rightarrow x \frac{dv}{dx} = - \frac{1}{\sin \sin v}$$

$$\Rightarrow \sin \sin v \, dv = - \frac{dx}{x}$$

$$\Rightarrow \int \sin \sin v \, dv = - \int \frac{dx}{x}$$

$$\Rightarrow - \cos \cos v = - \log \log |x| + c$$

$$\Rightarrow - \cos \cos \left(\frac{y}{x} \right) = - \log \log |x| + c$$

It is given that $y = \frac{\pi}{2}$ when $x = 1$

$$\therefore - \cos \cos \frac{\pi}{2} = - \log \log 1 + c$$

$$\Rightarrow - 0 = - 0 + c$$

$$\Rightarrow c = 0$$

Hence, the complete solⁿ is :

$$- \cos \cos \left(\frac{y}{x} \right) = - \log \log |x|$$

$$\Rightarrow \cos \cos \left(\frac{y}{x} \right) = \log \log |x|$$

1

0.5

0.5

0.5

		0.5
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31

The region bounded by the points A, B, C and D is the feasible region.

A Cartesian coordinate system with a grid. The x-axis is labeled from -10 to 30 in increments of 5. The y-axis is labeled from 0 to 30 in increments of 10. Three lines are plotted: a green line labeled $x + 3y = 60$ passing through (0, 20) and (60, 0); a blue line labeled $x + y = 10$ passing through (0, 10) and (10, 0); and a red line labeled $y = x$ passing through the origin. The feasible region is the shaded area bounded by these lines and the y-axis, with vertices labeled A(5, 5), B(15, 15), C(0, 20), and D(0, 10).

1.5

Corner Points	Maximize $Z = 3x + 9y$
A(5, 5)	60
B(15, 15)	180
C(0, 20)	180
D(0, 10)	90

Maximum value of Z is 180 and which is at any point on the line segment joining B and C.

1

0.5

SECTION-D

32	<u>For Reflexive:</u> $ab = ba$, for all $a, b \in \mathbb{N}$. $\therefore (a, b) R (a, b)$, for all $a, b \in \mathbb{N}$. Hence, R is reflexive. <u>For Symmetric:</u> Let, $a, b, c, d \in \mathbb{N}$ such that $(a, b) R (c, d)$.	1.5
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$$(a, b) R (c, d)$$

$$\Rightarrow ad = bc$$

$$\Rightarrow da = cb$$

$$\Rightarrow cb = da$$

Hence, $(c, d) R (a, b)$.

$\therefore (a, b) R (c, d) \Rightarrow (c, d) R (a, b)$, for all $a, b, c, d \in N$.

Hence, R is symmetric.

1.5

For Transitive:

Let, $a, b, c, d, e, f \in N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$.

$$(a, b) R (c, d) ; (c, d) R (e, f)$$

$$\Rightarrow ad = bc ; cf = de$$

$$\Rightarrow \frac{c}{d} = \frac{a}{b} ; \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{e}{f}$$

$$\Rightarrow af = be$$

Hence, $(a, b) R (e, f)$.

$\therefore (a, b) R (c, d) ; (c, d) R (e, f) \Rightarrow (a, b) R (e, f)$, for all $a, b, c, d, e, f \in N$.

Hence, R is transitive.

Hence, R is an equivalence relation on $N \times N$.

2

OR

For one-one:

Let, $x_1, x_2 \in N$ (Domain) such that $f(x_1) = f(x_2)$

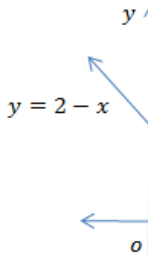
$$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$

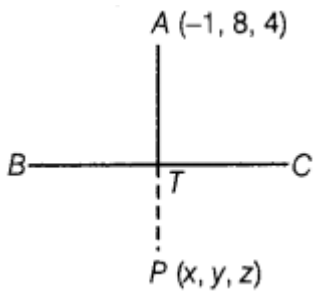
$$\Rightarrow x_1^2 - x_2^2 + x_1 - x_2 = 0$$

$$\Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$

	$\Rightarrow (x_1 - x_2) = 0 \left(\because x_1, x_2 \in N \Rightarrow x_1 + x_2 + 1 \neq 0 \right)$ $\Rightarrow x_1 = x_2$ $\therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \text{ for all } x_1, x_2 \in N \text{ (Domain).}$ Hence, f is one-one. <u>For onto:</u> Let us take 1 from N (Co – domain) such that $f(x) = 1$ $\Rightarrow x^2 + x + 1 = 1$ $\Rightarrow x^2 + x = 0$ $\Rightarrow x(x + 1) = 0$ $\Rightarrow x = 0, -1$ But, $0, -1 \notin N$ (Domain) Hence, for 1 in co-domain, we do not get its pre image in domain. $\therefore f$ is not onto.	2.5 2.5
33	$A = [1 \ 1 \ 1 \ 0 \ 1 \ 3 \ 1 \ -2 \ 1]$ $ A = 1(1 + 6) - 1(0 - 3) + 1(0 - 1) = 7 + 3 - 1 = 9 (\neq 0)$ $\therefore A^{-1}$ exists. $adj A = [1 \ 3 \ -2 \ 1 \ \ -0 \ 3 \ 1 \ 1 \ \ 0 \ 1 \ 1 \ -2 \ \ -1 \ 1 \ -2 \ 1 \ \ 1 \ 1 \ 1 \ 1 \ \ -1 \ 1 \ 1 \ 1 \ \ -1 \ 1 \ 1 \ 1]$ $\therefore A^{-1} = \frac{adj A}{ A } = \frac{1}{9} [7 \ -3 \ 2 \ 3 \ 0 \ -3 \ -1 \ 3 \ 1]$ $x + y + z = 6, y + 3z = 7, x - 2y + z = 0$ The above system of equations can be written as: $AX = B$, where, $A = [1 \ 1 \ 1 \ 0 \ 1 \ 3 \ 1 \ -2 \ 1], X = [x \ y \ z], B = [6 \ 7 \ 0]$ $\therefore X = A^{-1}B = \frac{1}{9} [7 \ -3 \ 2 \ 3 \ 0 \ -3 \ -1 \ 3 \ 1] [6 \ 7 \ 0] = \frac{1}{9} [42 - 21 + 0 \ 18 + 0 + 0 \ 0 + 0 + 0]$ $\therefore x = \frac{7}{3}, y = 2, z = \frac{5}{3}$	1 2

		1
		1
34	$\int_1^4 [x - 1 + x - 2 + x - 3] dx = \int_1^4 x - 1 dx + \int_1^4 x - 2 dx + \int_1^4 x - 3 dx$ Now, $\int_1^4 x - 1 dx$ $= \int_1^4 (x - 1) dx$ $= \left[\frac{x^2}{2} - x \right]_1^4$ $= \left[(8 - 4) - \left(\frac{1}{2} - 1 \right) \right] = 4 + \frac{1}{2} = 4.5$ Now, $\int_1^4 x - 2 dx$ $= \int_1^2 (2 - x) dx + \int_2^4 (x - 2) dx$ $= \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^4$ $= \left[(4 - 2) - \left(2 - \frac{1}{2} \right) \right] + [(8 - 8) - (2 - 4)]$ $= 2 - 1.5 + 2$ $= 2.5$ Now,	 1.5

	$\int_1^4 x - 3 dx$ $= \int_1^3 (3 - x) dx + \int_3^4 (x - 3) dx$ $= \left[3x - \frac{x^2}{2} \right]_1^3 + \left[\frac{x^2}{2} - 3x \right]_3^4$ $= \left[\left(9 - \frac{9}{2} \right) - \left(3 - \frac{1}{2} \right) \right] + \left[(8 - 12) - \left(\frac{9}{2} - 9 \right) \right]$ $= 4.5 - 2.5 - 4 + 4.5 = 2.5$ $\therefore \int_1^4 [x - 1 + x - 2 + x - 3] dx$ $= 4.5 + 2.5 + 2.5$ $= 9.5$	1.5
		1.5
		0.5
35	The equation of a line joining the points B (0, -1, 3) and C (2, -3, -1) is $\vec{r} = (0\hat{i} - \hat{j} + 3\hat{k}) + \lambda[(2\hat{i} - 3\hat{j} - 1\hat{k}) - (0\hat{i} - \hat{j} + 3\hat{k})]$ $\Rightarrow \vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 2\hat{j} - 4\hat{k})$ $\Rightarrow \vec{r} = (2\lambda)\hat{i} + (-2\lambda - 1)\hat{j} + (-4\lambda + 3)\hat{k}, \lambda \in \mathbb{R}$ So any point on line BC is to the form (2λ, -2λ - 1, -4λ + 3) Let foot of the perpendicular drawn from point A to the line BC be T(2λ, -2λ - 1, -4λ + 3).  Now, DR's of line AT is (2λ + 1, -2λ - 1 - 8, -4λ + 3 - 4) or (2λ + 1, 2λ - 9, -4λ - 1).	1
		1

	Since, AT is perpendicular to BC. $\therefore 2 \times (2\lambda + 1) + (-2) \times (-2\lambda - 9) + (-4) \times (-4\lambda - 1) = 0$ $[\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$ $\Rightarrow 4\lambda + 2 + 4\lambda + 18 + 16\lambda + 4 = 0$ $\Rightarrow 24\lambda + 24 = 0, \lambda = -1$ $\therefore$ Coordinates of foot of perpendicular is $T(2 \times (-1)), -2 \times (-1) - 1, -4 \times (-1) + 3 \text{ or } T(-2, 1, 7)$ Let P(x, y, z) be the image of a point A with respect to the line BC. So, point T is the mid-point of AP. $\therefore$ Coordinates of T = Coordinates of mid-point of AP $\Rightarrow (-2, 1, 7) = [(x-1)/2, (y+8)/2, (z+4)/2]$ On equating the corresponding coordinates, we get $-2 = (x-1)/2, 1 = (y+8)/2 \text{ and } 7 = (z+4)/2$ $\Rightarrow x = -3, y = -6 \text{ and } z = 10$	1 2
SECTION-E		
36	Let A, B and C be the events that calculation is done by Jayant, Sonia and Olivia respectively. Let D be the event that there is an error in the calculation. Then, $P(A) = \frac{50}{100}$, $P(B) = \frac{20}{100}$, $P(C) = \frac{30}{100}$ and $P(D/A) = 0.06$, $P(D/B) = 0.04$, $P(D/C) = 0.03$ (i) The probability that Sonia processed the calculation and committed a mistake $= \frac{P(D/B) P(B)}{P(D/A) P(A) + P(D/B) P(B) + P(D/C) P(C)}$ $= \frac{0.04 \times \frac{20}{100}}{0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}}$ $= \frac{8}{47}$ (ii) The total probability of committing a mistake in processing the calculation $= P(D/A) P(A) + P(D/B) P(B) + P(D/C) P(C)$ $= 0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}$ $= 0.047$	1 1

