The abstraction: It is sometimes helpful to approach difficult questions by thinking about the properties that their answers must have.

Relevant collection of objects: Questions for which it helps to think about properties any answer must have, before any answers are actually known.

Examples of objects: Note that some of these questions seem very different from others, and it is only through the abstraction that any similarity can be discovered.

1. I've recently been preparing to run a reading group, and am trying to put together a syllabus. The relevant question is, "What should I put in the syllabus?" To answer this question, I have been trying to determine properties that the syllabus must have for me to be happy with it. Some include:
    a. It should be designed in such a way as to implicitly encourage people to spend the appropriate amount of time and effort on the readings.
    b. The readings should be understandable to participants from a broad range of backgrounds.
    c. The readings and exercises should help people who are interested become better prepared for certain jobs.

    Recognizing these properties allowed me to quickly clarify my vision for what the syllabus should include. They did not describe any single syllabus unambiguously, but this is a question which has multiple acceptable answers.
2. In mathematical induction proofs, a question which arises is, "How can I demonstrate the inductive step?" For any non-trivial proof by induction, the demonstration of the inductive step always has the following property:
    a. It relies on the inductive hypothesis.

    As such, while trying to perform the inductive step, a proof-seeker will always be thinking about how they can utilize the inductive hypothesis.
3. "How do quantum systems change over time?" The definition of the time evolution operator in quantum mechanics was determined based on three physically-motivated properties that it was supposed to have:
    a. Conservation of probability: a quantum system is always guaranteed to be found in one of its possible states, so for any amount of time evolution, the combined probability across all states must remain 1.
    b. Composition: The change in the system that takes place over some time period T must be the same as the change that comes from breaking this

down into a first, smaller time period T_1 and a second smaller time period $T-T_1$.

    i. That is, if U is the time evolution operator, $U(T) = U(T-T_1)U(T_1)$. Equivalently, $U(T_1)U(T_2) = U(T_1+T_2)$ for any T_1 and T_2.

  c. Continuity: When no time passes, a state does not change.

    i. U(0) is the identity operator, which, when applied to a state, returns the exact same state. That is, it does not change anything.

These properties led physicists to define the time evolution operator as $e^{-iHt/\hbar}$. The derivation of the operator's final form is here if you are interested, but the key point is that the answer to the question was found by first looking for properties that the answer needed to have.

4. "Why is there something rather than nothing?" This is a deep question that philosophers have debated for centuries, and in case you haven't already spent some time thinking about it, I will elaborate on why I find it interesting. We live in a universe filled with matter and energy (and their "dark" counterparts) that is governed by physical laws, and all signs seem to indicate that physical laws are the reasons for all interactions in the universe (though of course, we do not understand all physical laws - at least not yet). The question of why there is something rather than nothing is aimed at the one part of this picture that has no explanation: why is there a universe with reasons for things to happen in the first place? This question is an example of the abstraction above because there is a property that any answer must have:

  a. It must itself be a reason. The question "Why?" is equivalent to the question "For what reason?", so we want to answer the question "For what reason is there something rather than nothing?" Clearly, the answer must be a reason.

This is an important insight, because it quickly leads to an infinite regress. Part of the question is "Why are there reasons for things?", so for any reason that attempts to answer it, the same question can be posed again: "Why does that reason exist?" All of this is to say, *any answer to the question "Why is there something rather than nothing?" must have the property of being unsatisfactory*. So the value of the abstraction here is that it allowed me to demonstrate in a fundamental way that the question is unanswerable. Bertrand Russell agreed: "I should say that the universe is just there, and that's all."

Why this abstraction is useful: As evidenced by the examples, there are a wide range of questions for which it is possible and helpful to identify properties of an answer right away, while answering them directly is more difficult. This abstraction provides a general approach which makes many of these questions at least a little bit easier.

How I discovered this abstraction: I noticed that I was seeking necessary properties of the reading group syllabus (Example 1), and it felt like I had utilized similar techniques in other contexts. After thinking for a few minutes I had come up with several examples, so I felt that this was a useful and interesting abstraction.