Gizmos

Name: 　　　　Date:

Student Exploration: Compound Interest

Directions: Follow the instructions to go through the simulation. Respond to the questions and prompts in the orange boxes.

Vocabulary: annual percentage yield (APY), compound interest, exponential function, interest, interest rate, principal

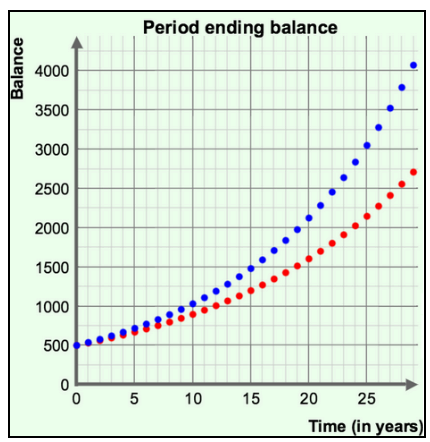
Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

Kim and Kyle are both saving money for their first cars. Their parents said they will add 10% to the amount they save. Kim saves $1600 and Kyle saves $2000.

1. How much will their parents give them? Kim: Kyle:
2. What will they have after adding 10%? Kim: Kyle:

Gizmo Warm-up

Kim and Kyle's parents paid them a little extra on top of the amount they saved. This is essentially an **interest** payment, at an **interest rate**, *r*, of 10%.



Often, though, interest is recurring. When it is paid multiple times, and calculated on the current amount (including previous interest), it is called **compound interest**. In the *Compound Interest* Gizmo, you can explore the effects of different compounding periods, and different interest rates.

1. The ***P*** slider shows the **principal**, or the initial amount of money. Drag the ***P*** slider, and watch the graph.

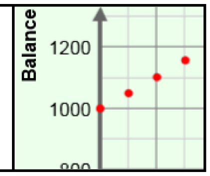
   A. How does the graph change?

   B. Explain why this happens.

2. Drag the ***r*** slider (which shows the interest rate), and watch the graph.

   A. How does the graph change?

   B. Explain why this happens.

Activity A: **Compounding interest**	<u>Get the Gizmo ready</u>: ● On the **CONTROLS** tab, be sure **Annually** is selected and the **END POINTS** tab is chosen. ● Set ***P*** to 1000 and ***r*** to 0.050 (or 5%).	

1. Suzie invests $1000 for 3 years at a rate of 5% compounded annually (once per year). She will earn **compound interest** paid on the principal and the interest from previous periods.

   A. How much interest does Suzie make at the end of year 1?

   B. What is her balance at the end of year 1?

   C. What could you multiply by $1000 to calculate that balance directly?

   Select **Show probe** and drag the probe to *t* = 1 to check your answers.

   D. The year 1 balance is used to calculate the year 2 interest. What do you multiply the year 1 balance by to find the new balance at the end of year 2?

   E. Likewise, the year 3 balance is calculated using the year 2 balance. What multiplier can you use to find the balance after year 3? _____ ___

   F. How many times has $1000 been multiplied by 1.05 by the end of year 3?

   G. In the space to the right, write a formula for the balance after 3 years. Then find the balance. Check in the Gizmo.

   H. Write a formula for the balance (*B*) after *t* years, with interest compounded annually. Use *P* for the principal and *r* for the interest rate.

2. Select **Quarterly** from the dropdown menu. "Quarterly" means interest is paid once per quarter (once 3 months, or 4 times per year). So, if Suzie's account compounds quarterly, instead of annually, the 5% rate gets "chopped" into 4 equal parts.

   A. What rate does that equal? _

   B. So, instead of 1.05, what is the multiplier for each interest payment?

   C. How many times will Suzie receive an interest payment over 3 years?

   D. In the space to the right, write a formula for Suzie's balance after 3 years. Then find the balance. Check in the Gizmo.

3. Select **Monthly**. In this case, Suzie's interest is paid once per month (12 times per year), so the 5% rate gets chopped into 12 equal parts.

   A. What rate does that equal? __

   B. So, instead of 1.05, what is the multiplier for each interest payment?

   C. In the space to the right, write a formula for Suzie's balance after 3
   years. Then find the balance. Check in the Gizmo.

4. Select **Daily**. Now her interest is paid once a day (365 times per year), so the 5% rate gets chopped into 365 equal parts.

   A. What rate does that equal? __

   B. So, instead of 1.05, what is the multiplier for each interest payment?

   C. In the space to the right, write a formula for Suzie's balance after 3
   years. Then find the balance. Check in the Gizmo.

5. Write a formula for the balance (*B*) after *t* years, with interest compounded *n* times per year. Use *P* for the principal and *r* for the interest rate.

B =

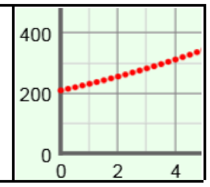
6. Set ***P*** to 500, ***r*** to 0.100 (10%), and select **Annually** from the dropdown menu. Select the **ALL TIME** tab above the graph. This graph shows the balance at all times.

   A. Why do you think the graph has flat steps, instead of a smooth curve?

   B. Be sure **Show probe** is selected. Drag the probe to the right. What happens at the "breaks" in the graph? .

   C. What is the balance after the first year? Why?

   D. What is the balance after the second year? Why?

Activity B: **Continuously compounding interest**	<u>Get the Gizmo ready</u>: ● Set ***P*** to 300 and ***r*** to 0.025 (2.5%). ● Select the **END POINTS** tab above the graph.	

1. Interest can be compounded at various intervals – annually, quarterly, monthly, and daily. In the Gizmo, select **Annually**, **Quarterly**, **Monthly**, and then **Daily** from the dropdown menu.

   A. What happens to the graphs as you compound more and more often?

   B. Imagine compounding interest more and more frequently – every hour, every minute, every second, etc. What if you could take that to its extreme and compound interest continuously? What do you think would happen? Would you get super-rich? Explain.

   Select **Continuously** from the dropdown menu to check your answer.

2. Select **Show additional function**. With **Continuously** still selected for the top function, choose **Monthly** for the bottom function. Set ***P*** to 600, and set both ***r*** sliders to 0.08 (8%).

   A. When interest is compounded at regular intervals (yearly, quarterly, monthly, or daily) recall that the balance (B) is given by $B = P(1 + \frac{r}{n})^{nt}$. ($P$ = principal, r = interest rate, n = number of times interest is compounded per year, and t = number of years.)

   In the space to the right, find the balance for an account with interest compounded monthly, with $P = 600$, $r = 0.08$, and $t = 10$ years. Then check in the Gizmo.

   B. As the number of compounding periods gets bigger and bigger (as n goes to infinity), the $(1 + \frac{r}{n})^{n}$ part of $B = P(1 + \frac{r}{n})^{nt}$ approaches the number e, or about 2.718… . This means that the formula for continuously compounded interest is $B = P \cdot e^{rt}$.

   In the space to the right, find the balance for an account with interest compounded continuously, with $P = 600$, $r = 0.08$, and $t = 10$ years. Then check in the Gizmo.

3. Check that ***P*** is 600, both ***r*** sliders are 0.08, and **Continuously** and **Monthly** are selected. Choose the **END POINTS** tab, and click the **–** button once to zoom out on the graph.

   A. Select **Show probe**. Use the probe to fill in the balances in the table below.

	5 years	10 years	15 years	20 years	25 years
Monthly					
Continuously					

   B. How do the balances after 25 years compare?

   C. How does the rate of increase early (steepness for low values of *t*) compare to late?

4. Set ***P*** to 600, and both ***r*** sliders to 0.08. Change one of the functions to compounding **Daily**.

   A. Use the probe to fill in the balances in the table below.

	5 years	10 years	15 years	20 years	25 years
Daily					
Continuously					

   B. How do the balances after 25 years compare?

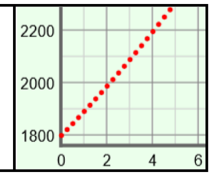
   C. How does the rate of increase early (steepness for low values of *t*) compare to late? That is a common trait of many **exponential functions**, like these.

5. Set one function to **Annually** and the other to **Continuously**.

   A. Switch back and forth between the **END POINTS** and **ALL TIME** tabs. Which graph is identical on both tabs? .

   B. Compare the **END POINTS** and **ALL TIME** graphs for other compounding periods. What is true about the continuously compounded graph? Why does this make sense?

<table>
<tr>
<td>Activity C:

Practice with compound interest</td>
<td><u>Get the Gizmo ready</u>:
Be sure the CONTROLS tab is selected.
Select the ALL TIME tab above the graph.</td>
<td>
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1. Jake deposits $300 in an account that pays 5% interest compounded quarterly.

   A. Write a formula for the balance of the account after *t* years. Then use the formula to find the balance of the account after the first quarter (after 3 months). Show your work to the right. Check your answer in the Gizmo.

   B. Use the formula you wrote above to fill in the table below. Check in the Gizmo.

Number of quarters	1 quarter	2 quarters	3 quarters	4 quarters
Time in years (*t*)				
Balance				

   C. When do you think the balance will be at least $400? Why? Check in the Gizmo.

2. Jake deposits another $300 in an account that pays 5% continuously compounded interest.

   A. When do you think the balance will be at least $400? Why? Check in the Gizmo.

   B. Write a formula for the balance of this account after *t* years.

   C. Use the formula you wrote above to fill in the table below. Check in the Gizmo.

Number of years	1 year	2 years	3 years	4 years
Balance				