

Mathematics PBAT

Proofs: “Derivation of Summation”

Assessment: Outstanding

PBAT in Mathematics: Complete the sections below

Introduction: general overview of the course (*Length: approximately a half page*)

Below are some prompts to help you think and write about what you know and understand about “proofs.” Don’t simply answer each question as if it were part of a questionnaire—instead, incorporate your response within your introduction in narrative form.

Possible ideas:

- What are proofs? Do we need proofs? When and how do we use proofs?
- Are proofs important in mathematics? What types of proofs are there? Etc.

Introduction of your specific PBAT topic: What are you going to prove in your paper?

(*Length: it may take less than half a page. It depends on the topic.*)

Provide some brief background information about your topic. State your claim (*what will you prove?*)

Proof (the actual work)

(*Length: It depends on the topic; diagrams and graphs will add pages to the paper.*)

Remember, “more” is not necessarily better; however, make sure to include all relevant information, visuals, tables, diagrams.

Examples:

- if you are doing a Number Theory proof, then write: “let’s assume n is an even number, then we can express $n = 2k$ because ...”
- if you are doing the Pythagorean theorem proof, then state what each variable stands for, etc.
- if you are doing a syllogism, indicate what each letter stands for. Don’t assume anything, even if it appears obvious. State it!

Support each statement you make along the way with evidence.

The writing must be fluent, clear, and easy to read and understand. Refrain from repeating yourself, as it makes the paper confusing. The reader may lose interest in reading as well.

Conclusions

After summarizing your findings, state your final conclusion. Generally, you repeat the introduction of your PBA Task with the difference that now you are confirming it.

Derivation of Summation

Introduction

Joshua N. Cooper once said, “A proof is a mathematical argument that serves to prove something as being ‘true’ or ‘correct.’” Proofs are essential to life and mathematics because they serve to validate one’s statement on a given problem without worry that they are not correct. An example of this in math is the process of checking one’s work that they must conduct when dealing with algebraic calculations. They do this to prove that the answer they found is correct and will work in the given equation. Mathematical proofs however, are not simply equations or numbers but are instead deductive arguments that build on prior knowledge. Proofs are essential to math because it checks our reasoning. If one states that a given function is true for every number, decimal or not, fraction or not, but fails to check it and explain their reasoning behind it, then they have failed to accurately prove their argument and so their statement can be proved wrong. Proofs must be very precise and unequivocal because one cannot leave room for interpretation. If certain vocabulary is not used, the proof falls victim to interpretation and so it can be interpreted in various different ways.

In this PBAT I will be exploring a topic of Number Theory, more specifically, the Derivation of Summation. My given problem was first introduced by Gauss. The story goes that as a child, Gauss was asked to find the sum of the numbers from 1 to 100. The question was

assigned as “busy work” by the teacher, but Gauss found the answer rather quickly by discovering a pattern. Within just a few seconds, Gauss came to his answer of 5050.

The

formula he found to easily add
any set of consecutive integers
was

$$\frac{n(n+1)}{2}$$

, with n representing

the last number in my set of numbers.

Investigation

To find the answer to this question, I began by finding patterns within my set of numbers. I realized that the digits from 1 through 9 were common in the ones column for every set of the next consecutive 10 integers, i.e. from 11 through 20, 21 through 30, 31 through 40, and so on until 91 through 100. Any number within these sets could be seen as the sum of a multiple of 10 plus a number from 1 through 9. For example, $11 = 10 + 1$, $12 = 10 + 2$, $13 = 10 + 3$, $39 = 30 + 9$ and so on. Noticing this pattern, I decided to find the sum of the recurring digits in the ones column: $1+2+3+4+5+6+7+8+9 = 45$. Since I removed these digits from every set of 10 integers, I will only be adding multiples of 10 in addition to the sum of 45.

The sum of each set will look as such,

Instead of:

$$1+2+3+4+5+6+7+8+9+10 = 55$$

$$11+12+13+14+15+16+17+18+19+20=155$$

$$21+22+23+24+25+26+27+28+29+30=255$$

It looked like:

$$(0+0+0+0+0+0+0+0+0) + \mathbf{10 + 45} = 55$$

$$(10+10+10+10+10+10+10+10+10) + \mathbf{20 + 45} = 155$$

$$(20+20+20+20+20+20+20+20+20) + \mathbf{30 + 45}$$

$$31+32+33+34+35+36+37+38+39+40=355 \quad (30+30+30+30+30+30+30+30+30+30)+\mathbf{40}+\mathbf{45}=355$$

$$41+42+43+44+45+46+47+48+49+50=455 \quad (40+40+40+40+40+40+40+40+40+40)+\mathbf{50}+\mathbf{45}=455$$

As can be seen on the right side of the diagram, I only have multiples of 10, except for the first set/row of 10 integers. I noticed that in addition to 45, each number in the set was the same

multiple of 10, except for the final integer which is 10 greater than the other integers. I tried to create a formula to find the sum of each set using this knowledge, ultimately getting $(9n + (n + 10)) + 45$. The n will represent the multiple of 10 present. Since I have nine of them, I multiply this number 9 times. $(n + 10)$ represents the last multiple of ten that is 10 greater than the recurring multiple of 10. 45 represents the recurring sum present in each set of 10 numbers. I can simplify the formula I found by combining $9n$ and n to make $10n$, and 45 and 10 to make 55. These can be factored from $10n + 55 \rightarrow 5(2n + 11)$.

This final equation works for each individual set of 10 integers, but when combining two or more sets, cannot be used. Using this equation, I found that there is no way to directly relate n from one set of integers to the next set of 10 integers. This means that there is no way for me to transition from a set of multiples of 10 to multiples of 20 to multiples of 30 without performing an overall sum for each individual set. So, there is no way for me continuously increase my n by 10 without manipulating the equation midway through my calculations, unless the formula is a

sum of each different set/row. If I were to set up $\sum_{n=0}^{10} 5(2n + 11)$ a sum, it would look as such

where n indicates a multiple of 10. So $0=0$, $1=10$, $2=20$, $3=30$, $4=40$, and so on until I get to $10=100$.

This equation works for the first 100 numbers but if I do not have sets of 10 integers where the sum of 19 can be isolated, it does not work as well. For instance, if I began from 16 instead of 1, I would have to change the formula being used in the sum so that it reflects a sum that is not 45 because 45 is not consistent in each set being added. Also, if I began with a

negative number, I would have to alter the number that n can represent in the formula as it may not directly represent 10.

After discovering that the formula I found would not work for any number being used, I decided to change my approach to the problem, I would begin the problem similarly to my previous attempt, isolating a specific sum, but instead of 19, being 45, I would isolate the sum of 110, being 55. This would eliminate the final multiple of 10 being greater than the recurring multiple of 10, and leave me with the same multiple of 10 being repeated 10 times. It would look as such

Instead of:

$$1+2+3+4+5+6+7+8+9+10 = 55$$

$$11+12+13+14+15+16+17+18+19+20=155$$

$$21+22+23+24+25+26+27+28+29+30=255$$

It looked like:

$$(0+0+0+0+0+0+0+0+0+0) + 0 + \mathbf{55} = 55$$

$$(10+10+10+10+10+10+10+10+10+10) + \mathbf{55} = 155$$

$$(20+20+20+20+20+20+20+20+20+20) + \mathbf{55} = 255$$

As can be seen, I have the same multiple of 10 being repeated 10 times, except for the first set of 10 numbers. This would give me a multiple of 100 as the sum depending on the multiple of 10 present in the set. So, a set of 10s would result in a sum of 100, a set of 20s would result in a sum of 200, and so on until the final set of 90s that gives me a sum of 900.

After isolating the sum of 110 from every set of 10 numbers, I will have a sum of 55 being repeated 10 times. I would then add this recurring sum to get an overall sum of 550. To find the total sum of all 10 sets (with the sum of 110 isolated) I will simply be finding the sum of the digits from 1 through 9. Since I have 10 sets whose sums are a multiple of 100, with the exception of the first set of 10 numbers, I will be adding 100 through 900. So, if I add 19 and multiply this by 100, I will have the sum of all 10 sets. So, adding 19, I get 45 and multiplying

this by 100 will give me an overall sum of 4500. I will then add 550 to 4500 and get a final answer of 5050.

The formula I found for this approach was $10(10n) + 55$ where n represents the set number. Distributing the 10, my revised formula is $100n + 55$. I categorized my sets into set numbers according to every 10 integers. A table of the set numbers can be seen below.

Set #	Numbers in the set
$n=0$	1,2,3,4,5,6,7,8,9,10
$n=1$	11,12,13,14,15,16,17,18,19,20
$n=2$	21,22,23,24,25,26,27,28,29,30
$n=3$	31,32,33,34,35,36,37,38,39,40
$n=4$	41,42,43,44,45,46,47,48,49,50

So, set 0 would be 110, set 1 would be 1120, set 2 would be 2130. The reason for why the first set is numbered 0 and not 1 is because there is not a multiple of 10 present and so it would not fit the formula. Similar to my previous approach, this formula will only work if I was finding the sum of each individual set of 10 numbers. It does not have one single value for n and so it calculates the sum for an individual set of 10 numbers instead of overall sum. Also, if my initial number is not 1, this formula will not work.

My next approach started with me separating even and odd numbers and creating a formula from this. I decided to separate them because if I split the 100 numbers by even and odd, create an equation to find each of their sums, and combine the two equations, then I would logically have the equation for the sum of all 100 numbers. I began by creating a table for even

in which I would add every consecutive even number (2,4,6,8,10, ... 100) beginning from two and continuing until I could notice a pattern. Similarly, I created a table in which I would add every consecutive odd number (1,3,5,7,9, ... 99) beginning from 1 and continuing until I could notice a pattern. The table on the left represents even numbers and the table on the right represents odd numbers.

# of integers added	Even #	Even addition	Sum
1	2	2	2
2	4	2 + 4	6
3	6	6 + 6	12
4	8	12 + 8	20
n			$n(n + 1)$

# of integers added	Odd #	Odd addition	Sum
1	1	1	1
2	3	1 + 3	4
3	5	4 + 5	9
4	7	9 + 7	16
n			n^2

With regards to the table for even numbers, one can see that the sum for any combination of even numbers is $n(n + 1)$ where n represents half the total number of terms. I found this

because I saw that the numbers of integers being used was always a fraction of the sum. For example, for 2 integers, 2 is onethird of 6, so to find 6, I would multiply 2 by 3 and 3 is equal to

2 + 1 . For 3 integers being used, 3 is onefourth of 12 so I multiply 3 by 4 and 4 is 3 + 1 . For 4 integers being used, 4 is onefifth of 20, so to find 20, I would multiply 4 by 5 and 5 is 4 + 1 .

The pattern here is that the sum is equal to the number of terms, n , multiplied by the number

of terms plus 1, $(n + 1)$, so my formula for finding the sum of all the even numbers is $n(n + 1)$.

Looking at the table for odd numbers now, the formula that I found to calculate the sum for any number of odd numbers is n^2 . I found this because I noticed that the sum was always a result of the number of terms being used multiplied by itself, or to the second power. So, for 2 terms being used, the sum was 4, which is 2 times 2. For 3 terms being used, the sum is 9 which is 3 times 3. For 4 terms being used, the sum is 16 which is 4 times 4. So, using this knowledge, I concluded that for n number of terms, the sum must be n multiplied by itself or n^2 .

Now that I have the formulas for both any number of even and odd numbers, I can combine them to find the sum for all 100 terms. Both $n(n + 1)$ and n^2 combine to become $n^2 + n(n + 1)$, which can be simplified to $n^2 + n^2 + n$. This can be further simplified to get $2n^2 + n$ which can be factored one final time to get my final revised formula of $n(2n + 1)$. In this formula n represents half the number of total terms. Since I split the total number of terms in half to find the formulas for the sum of even and odd numbers, I have to keep the number of terms used in those formulas consistent, which is half the total number of terms. So n in my original problem will be 50. If I plug this number in, I get $50(2(50) + 1) = 50(101) = 5050$. My answer using the formula I found is the same as that which Gauss found so I know that this formula works for the first 100 numbers.

Comparing this formula to Gauss', they both work the same when instead of plugging in

$n = 50$, I plug in $n = \frac{n}{2}$. This still has the same value of $n = 50$ where it is equal to half the

2

number of terms but it is represented differently. $\frac{n}{2}$ into my even and odd
When plugging in 2

$\frac{n}{2} \cdot \frac{n}{2}$
 $\frac{n}{2}$
 $\frac{n^2}{4} + \frac{n^2}{4} + \frac{n}{2}$

equations, it becomes $(\frac{n}{2})^2 + 2(\frac{n}{2} + 1) \cdot \frac{n}{2}$, I get

Distributing the exponent and

Adding like terms, I get $\frac{n^2}{2} + 4$

$+\frac{n}{2}$. Now, I multiply

$\frac{n}{2}$ by 2 to get like terms and add. $\frac{n}{2}$ becomes

$\frac{2n}{4}$ and my sum is now equal to $2n^2 + 2n + 4$. I divide by 2 and get $\frac{n^2 + n}{2}$. Factoring $n^2 + n$ gives me

$n(n + 1)$ and my final formula is $\frac{n(n+1)}{2}$, which is the same formula that Gauss found.

Now I can test this formula to see if it works when the number of terms to be added cannot be evenly split. I tested this formula to see if it worked using a set of numbers that began from one but did not split evenly. My set began at 1 and ended at 17. This gave me total number of 17 terms that split into 8 even and 9 odd. Seeing as I do not have one value for n , I decided to use the average number of even and odd numbers. I divided 17 by 2 and got 8.5 as my value for n . Plugging this value into my equation $n(2n + 1)$, I get $8.5(2(8.5) + 1) = 8.5(18) = 153$. Doing the manual addition, the total sum is also 153. Now that I know the equation works for a set of numbers that begins at 1 but does not evenly split, I will test it with a set of numbers that does not begin at 1. The set of numbers I used to test this began at 3 and ended at 17, giving me a total of 15 terms. This would result in 7 even terms and 8 odd terms, and since I do not have the same amount of even and odd terms, this means that my n will not be the same. Again, I decided to take the average number of even and odd numbers. So I divided 15 by 2 and got 7.5 as my value for n . Plugging this into my equation, I get $7.5(2(7.5) + 1) = 7.5(16) = 120$. Performing the manual addition resulted in a sum of 150, proving that this equation does not work when given a set of numbers that does not begin at 1. Seeing that the revised formula did not work, I decided to test out if I would get 150 if I separated both formulas to find the sum for even numbers and odd numbers. I plugged in 8 into the equation for odd numbers getting $8^2 = 64$ for the sum of my

odd numbers. For even numbers, I plugged in 7 and got $7(7 + 1) = 7(8) = 56$. Adding 56 and 64 results in 120, proving that the equations still do not work. One reason that the equation does not

work is that taking the average implies that the first even number is 2 and the first odd number is

1. This will give me the incorrect sum for even numbers and odd numbers which will ultimately give me an incorrect total sum.

My final approach began with me deciding to group the numbers by first and last integer, continuously increasing the first number by one and decreasing the final number by one. This would result in the sum of each pair being the same, which I could use to find the final sum of all 100 numbers. I decided to use this approach after I noticed that from the first 10 number, I could form 5 pairs whose sums would all equal 11. So, if I multiply the common sum by the number of pairs in a set, I would find the total sum for the set. This would give me an equation where if I had the total number of pairs in a set and the sum of each pair, I could find the total sum. I began by using the first 10 numbers to test if it worked. It looked as such where

Instead of: $1+2+3+4+5+6+7+8+9+10=55$

It looked like:

$1+10=11$	$2+9=11$	$3+8=11$	$4+7=11$	$5+6=11$
-----------	----------	----------	----------	----------

As can be seen, each pair of two numbers has a sum of 11 which is common between all 5 pairs, giving me a total sum of 55. Doing this showed me that my total number of integers was being split in half due to the pairing. After seeing that this worked for the first ten numbers, I decided to test it out for the first twenty numbers. The pairs for the first 20 numbers looked as such,

$1+20=21$	$2+19=21$	$3+18=21$	$4+17=21$	$5+16=21$
$6+15=21$	$7+14=21$	$8+13=21$	$9+12=21$	$10+11=21$

Each pair has a common sum of 21 and I have ten pairs, so 21 multiplied by 10 gives me a total sum of 210. After testing this method out for the first ten and the first twenty numbers, I decided to test it with all one hundred numbers. I began by adding 1 and 100, giving me a sum of 101.

Using the method for the first ten and twenty numbers, this would logically give me the sum for the first 100 numbers. All 50 pairs can be seen below.

1+100=101	2+99=101	3+98=101	4+97=101	5+96=101
6+95=101	7+94=101	8+93=101	9+92=101	10+91=101
11+90=101	12+89=101	13+88=101	14+87=101	15+86=101
16+85=101	17+84=101	18+83=101	19+82=101	20+81=101
21+80=101	22+79=101	23+78=101	24+77=101	25+76=101
26+75=101	27+74=101	28+73=101	29+72=101	30+71=101
31+70=101	32+69=101	33+68=101	34+67=101	35+66=101
36+65=101	37+64=101	38+63=101	39+62=101	40+61=101
41+60=101	42+59=101	43+58=101	44+57=101	45+56=101
46+55=101	47+54=101	48+53=101	49+52=101	50+51=101

As can be seen from the table, each pair has the recurring sum of 101. Since I have a total of 50 pairs and each shares the common sum of 101, if I multiply the recurring sum by the total number of pairs, I would find the sum for all the numbers, so $101 \cdot 50 = 5050$.

Noticing that my total number of pairs was equal to half my total number of terms and that the recurring sum was equal to the initial number plus the final number, I decided to create a formula.

My formula was $\frac{x}{2}(x + 1)$

where x represents the final number in the set, which is also

equal to the total number of terms. I divide x by 2 to find the total number of pairs and inside the parenthesis, I add 1 to x to find the recurring sum, which is equal to the the sum of the final and initial integer. So, if given a set of numbers begins at 1 and ends at 67, 67 would represent my value for x and I would simply plug it into the equation: $\frac{67}{2}(67 + 1) = 33.5(68) = 2,278$.

Therefore, the sum for the first 67 terms when using the formula is 2,278 and when manually adding is also 2,278. Since I got the same answer using the formula as I would when manually adding I know that this formula works for any set of numbers beginning from one.

As I did for my previous attempts, I checked if the formula would work for the sum of consecutive integers if the set began with numbers other than one. After testing the formula out however, I found that the formula did not work anymore. The amount of pairs that I found were equal to half the amount of terms, but the recurring sum was not accurate seeing as the set did not begin with one. For example, if my set began at 4 and ended at 17, the recurring sum using the formula would be 18 (17+1). After finding this out, I discovered that n is This sum however, is not accurate and the actual recurring sum is 21 when I add the initial and final number, (4+17), (5+16); etc..

This made me change my formula slightly, where instead of adding one inside of the parenthesis, I would add the *initial* and the *final* terms for the set. These would be represented by x_i , and the final term would be x_f . To find the total number of terms, t , in the set I subtract the initial term, x_i , from the final term, x_f , and add one. I add one because the equation $t = (x_f - x_i)$ only counts the number of terms from the initial to the final, failing to take into account the

initial term, so I add one to include the initial term in the total number of terms. It would look as

such, $t = (x_f - x_i) + 1$. My revised formula would be $\sum_{n=x_i}^{x_f} (t + x_i)$.

To get to my final formula from my original formula, I had to substitute a lot of the values. With the $\frac{x}{2}(x + 1)$ formula, I had to substitute the x being divided by two ($\frac{x}{2}$) to represent the total number of terms. I knew that this would not be equal to the final number so I had to give assign a different variable to it, being t . After finding this, I had to find how to represent the final number plus the initial number within the parenthesis $(x + 1)$. I decided to keep the same

variable of x but instead the final integer would be x_f and the initial term would be represented by

be x_i . This would give me my $\sum_{n=x_i}^{t+x_i}$ where I divide the total number of

terms by 2 to find the total pairs and add the final and initial term to find the recurring sum.

Now, if I test this formula with a set of numbers starting at 4 and ending at 19, I would

begin by finding the total and 19 as x_f and solve for my number of terms. I would plug in 4 as x_i

t value: $t = (19 - 4) + 1 = 15 + 1 = 16$. Now that I know my value for t , I can move onto the final

formula. I plug in 16 as my t and 19 as x_f and solve for the sum. value, plug in 4 as x_i

$\frac{16}{2}(19 + 4) = 8(23) = 184$. My sum for a set of numbers starting at 4 and ending at 19 using my formula is 184 and using manual addition, it is also 184. Now, I have my final formula to find the sum of any set of numbers regardless of it begins from one or not.

Comparing Gauss' formula to my own, there are some noticeable differences and similarities. When finding the sum between the final and initial number, $(x_f + x_i)$ is the same as $(n + 1)$ when n is considered the final term and 1 is the initial term. Also, $\frac{n}{2}$ is the same as $\frac{n+1}{2}$ when n is even.

is considered the total number of terms in the set and not the final number. Also, because both equations were found working in pairs, both equations feature the total number of terms being divided by 2.

My equation differs from Gauss' because his equation does not require you to find the total number of terms. Since n represents the total number of terms, there is no need to find them. Also, since he only dealt with the first 100 numbers, his initial term was 1. Since not every set begins with 1, I substituted 1 with x_i to represent the initial term.

Proof by Induction

Now that I have found my final equation for any set of numbers, I will prove that the original $\frac{n(n+1)}{2}$ equation works for every number. My method of proving this equation will be

proof by induction. I will begin by proving that the equation works for the base case, or $n = 1$.

I will then make the assumption that the equation works for any number k , or $n = k$. If I can prove that the formula also works for the next term or $n = k + 1$, then by induction the formula must work for any term afterwards. Proving that the equation works for $k + 1$ will create a "domino effect" where for any number k , I will have proven that it works for the any term afterwards. Assuming that the formula works for $k = 29$, if I can prove that the formula also works for $n = k + 1$ (30), I know that it will work for any number after that. Assuming that the formula works for $k = 1, 789$, if I prove that the equation works for $n = k + 1$ (1, 790), I know that it will also work for any number after that.

The first step is to prove that the equation works when I plug in 1 for n .

1

$$\frac{n}{2} = \frac{1}{2}$$

$$\sum_{n=1}^1 = \frac{1(1+1)}{2} = \frac{1(2)}{2} = 0.5(2) = 1$$

Therefore the sum for $n = 1$ is one, and since my only number is 1, the sum must be 1.

Next, I assume that the formula works for any number k , or for $n = k$

$$\sum_{n=1}^k = \frac{k(k+1)}{2}$$

Now I prove that the equation works for the next term, or $n = k + 1$

$$\sum_{n=1}^{k+1} = 1 + 2 + 3 + 4 + \dots + k + (k + 1)$$

Assuming that the sum for any number up to and including k is $\frac{k(k+1)}{2}$, I can substitute

everything up to $k + 1$ with its sum.

$$\sum_{n=1}^{k+1} = \frac{k(k+1)}{2} + (k + 1)$$

So, now I want to find a common denominator so that I can properly add my two numbers.

$$\sum_{n=1}^{k+1} = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k(k+1) + 2(k+1)}{2}$$

Now I add both numbers.

$$\sum_{n=1}^{k+1} = \frac{k(k+1) + 2(k+1)}{2}$$

$$k \left(\frac{k+1}{2} \right) = \frac{k(k+1)}{2}$$

Seeing that $(k + 1)$ is common to both terms, I can factor this out. It will look as such

$$\sum_{n=1}^{k+1} \frac{(k+1)k}{2}$$

This can be rewritten one final time where instead of $(k + 2)$, I have $((k + 1) + 1)$. My final sum will look as such

$$\sum_{n=1}^{k+1} \frac{(k+1)(k+2)}{2} = \frac{k}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

This final sum proves that the equation works for $n = k + 1$. It proves that the original equation of $\frac{n}{2}(n + 1)$ applies when $n = k + 1$. If I were to substitute every value for n in my original equation with $k + 1$, I would get the same sum as I did for my final sum above. Proving that the equation works for the base case, or 1, and assuming that it works for any number k has proven that it also works when $n = k + 1$. Now that I know that the equation works for any number k , through induction, I also know that it works for the following term. So if $n = 74$, I know that it will have to work for $n = 75$. Now that I know it works for $n = 75$, it must work for $n = 76$ and so on.

Conclusion

In conclusion, there are many different ways one can approach a problem and many different ways one can solve it. With this problem for example, I approached the problem in many different ways, some that gave me a fitting formula when finding the sum of the first 100 consecutive numbers. However, when I tested the formula using a set of numbers other than the first 100, it would not work. This would make me try out a different approach and create different formulas until I finally found my final formula. Numbers are so interconnected that we

can find many different strategies, patterns, and relationships when given a problem. With this problem for example, the same formula did not work for every case. With the $n(2n + 1)$, I found that it worked with any set of numbers beginning from one but not with a set of numbers other

than one. However, with the $\sum_{i=1}^t (x_i + x_i)$ formula, I found that it worked for both sets of numbers

that begin from one as well as sets of numbers that do not begin with one. This process of finding the answer to a problem and attempting to validate it shows why proofs are important. They serve to validate one's argument and provide logical reasoning for the answer one finds to a given problem. It is important to be accurate when making claims because if one is not accurate with both their reasoning and the work they provide, their proof can be proven false. My experience exploring the topic of number theory is one of constant trial and error. When I thought that I had found a formula, I had to test if it worked for any set of numbers and when I found that it didn't, I had to move on to a different approach. When writing the paper, I had to be as clear and specific with my explanation as possible. I found that the way in which I explained certain things was not as clear as I thought and when people read it, they were confused. This made me have to change my explanation and include additional details that I assumed people knew.