Rational Functions | Review B — Key

Teacher explanations · AP Precalculus Topics 1.7–1.11

1. Classify as horizontal, slant, or neither: $f(x) = \frac{3x^2-4x+2}{x^3+2}$

Numerator degree 2; denominator degree 3. Compare degrees and leading coefficients. **Answer: horizontal, y = 0.**

2. Classify as horizontal, slant, or neither: $f(x) = \frac{2x^2+3x-7}{5x^2-3x+2}$

Numerator degree 2; denominator degree 2. Compare degrees and leading coefficients. **Answer: horizontal, y = 2/5.**

3. Classify as horizontal, slant, or neither: $f(x) = \frac{2x^3+5x^2+4x-7}{4x^2-9}$

Numerator degree 3; denominator degree 2. Polynomial division gives quotient x/2 + 5/4 and remainder 17x/2 + 17/4. The remainder over the denominator tends to zero. **Answer: slant, y = x/2 + 5/4.**

4. Classify as horizontal, slant, or neither: $f(x) = \frac{-3x^4+4x^3+9}{x^2-7x-12}$

Numerator degree 4; denominator degree 2. Compare degrees and leading coefficients. **Answer: neither horizontal nor slant.**

5. Classify as horizontal, slant, or neither: $f(x) = \frac{(3x+2)(4x-5)}{(x+4)(x-2)}$

Numerator degree 2; denominator degree 2. Compare degrees and leading coefficients. **Answer: horizontal, y = 12.**

6. Classify as horizontal, slant, or neither: $f(x) = \frac{(x-2)^3(3x+4)^2}{(x-3)^2(x+5)^5}$

Numerator degree 5; denominator degree 7. Compare degrees and leading coefficients. **Answer: horizontal, y = 0.**

7. Classify as horizontal, slant, or neither: $f(x) = \frac{(x-2)^4}{-4x(x-9)^2}$

Numerator degree 4; denominator degree 3. Polynomial division gives quotient $-x/4 - 5/2$ and remainder $123x^2 - 842x + 16$. The remainder over the denominator tends to zero. **Answer: slant, y = −x/4 − 5/2.**

8. Classify as horizontal, slant, or neither: $f(x) = \frac{(x+2)^2}{6x^2(x+3)}$

Numerator degree 2; denominator degree 3. Compare degrees and leading coefficients. **Answer: horizontal, y = 0.**

9. Classify as horizontal, slant, or neither: $f(x) = \frac{8}{x+4}$

Numerator degree 0; denominator degree 1. Compare degrees and leading coefficients. Answer: horizontal, $y = 0$.

10. Solve in interval notation: $\frac{x-4}{x+5} \leq 0$

Factor numerator $x - 4$ and denominator $x + 5$. Critical numbers in order: -5, 4. Test intervals from left to right: +, −, +. Select the required signs; include numerator zeros only for non-strict inequalities and never denominator zeros. Answer: $(-5, 4]$.

11. Solve in interval notation: $\frac{(x+2)^2}{(x-4)(x+6)} > 0$

Factor numerator $(x + 2)^2$ and denominator $(x - 4)(x + 6)$. Critical numbers in order: -6, -2, 4. Test intervals from left to right: +, −, −, +. Select the required signs; include numerator zeros only for non-strict inequalities and never denominator zeros. Answer: $(-\infty, -6) \cup (4, \infty)$.

12. Solve in interval notation: $\frac{-4x(x-8)}{(x+3)^2} \geq 0$

Factor numerator $-4x(x - 8)$ and denominator $(x + 3)^2$. Critical numbers in order: -3, 0, 8. Test intervals from left to right: −, −, +, −. Select the required signs; include numerator zeros only for non-strict inequalities and never denominator zeros. Answer: $[0, 8]$.

13. Solve in interval notation: $\frac{x^2+2x-35}{x^2-10x} < 0$

Factor numerator $(x - 5)(x + 7)$ and denominator $x(x - 10)$. Critical numbers in order: -7, 0, 5, 10. Test intervals from left to right: +, −, +, −, +. Select the required signs; include numerator zeros only for non-strict inequalities and never denominator zeros. Answer: $(-7, 0) \cup (5, 10)$.

14. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{4x(x-7)(x+5)}{(x-2)(x+5)}$

Factor: numerator $4x(x - 7)(x + 5)$; denominator $(x - 2)(x + 5)$. Cancel common factors: $(4x^2 - 28x)/(x - 2)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at $x = -5$; vertical asymptotes at $x = 2$.

15. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{(x-4)(x+3)}{(x-4)^2(x-9)}$

Factor: numerator $(x - 4)(x + 3)$; denominator $(x - 9)(x - 4)^2$. Cancel common factors: $(x + 3)/(x^2 - 13x + 36)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at $x =$ none; vertical asymptotes at $x = 4, 9$.

16. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{(x-8)^2(x+2)}{3x(x-8)}$

Factor: numerator $(x - 8)^2(x + 2)$; denominator $3x(x - 8)$. Cancel common factors: $(x^2 - 6x - 16)/(3x)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at x = 8; vertical asymptotes at x = 0.

17. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{x^2-5x-14}{x^2-14x+49}$

Factor: numerator $(x - 7)(x + 2)$; denominator $(x - 7)^2$. Cancel common factors: $(x + 2)/(x - 7)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at x = none; vertical asymptotes at x = 7.

18. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{x^2-16}{x^3+x^2-20x}$

Factor: numerator $(x - 4)(x + 4)$; denominator $x(x - 4)(x + 5)$. Cancel common factors: $(x + 4)/(x^2 + 5x)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at x = 4; vertical asymptotes at x = −5, 0.

19. Identify all x-values of holes and vertical asymptotes: $f(x) = \frac{(x+4)^5(x-6)^3}{(x+4)^3(x-6)^5}$

Factor: numerator $(x - 6)^3(x + 4)^5$; denominator $(x - 6)^5(x + 4)^3$. Cancel common factors: $(x^2 + 8x + 16)/(x^2 - 12x + 36)$. A remaining denominator zero is a vertical asymptote; a fully canceled denominator zero is a hole. Answer: holes at x = −4; vertical asymptotes at x = 6.

20. Write the horizontal or slant asymptote equation: $f(x) = \frac{2x^2+5x-12}{4x^2-3x+5}$

Numerator degree 2; denominator degree 2. Compare degrees and leading coefficients. Answer: horizontal, y = 1/2.

21. Write the horizontal or slant asymptote equation: $f(x) = \frac{5x^3+14}{3x^4-2x^3-10}$

Numerator degree 3; denominator degree 4. Compare degrees and leading coefficients. Answer: horizontal, y = 0.

22. Write the horizontal or slant asymptote equation: $f(x) = \frac{5x^2-4x+6}{x+3}$

Numerator degree 2; denominator degree 1. Polynomial division gives quotient 5x − 19 and remainder 63. The remainder over the denominator tends to zero. Answer: slant, y = 5x − 19.

23. Write the horizontal or slant asymptote equation: $f(x) = \frac{2x^3-3x^2+5x-2}{x^2+2x+4}$

Numerator degree 3; denominator degree 2. Polynomial division gives quotient 2x − 7 and remainder 11x + 26. The remainder over the denominator tends to zero. Answer: slant, y = 2x − 7.

24. Write the horizontal or slant asymptote equation: $f(x) = \frac{3x(x-5)(x+4)}{(x-8)^3}$

Numerator degree 3; denominator degree 3. Compare degrees and leading coefficients. **Answer: horizontal, y = 3.**

25. Write the horizontal or slant asymptote equation: $f(x) = \frac{-4x^2(x+5)^2}{(x+2)^2(x-2)^3}$

Numerator degree 4; denominator degree 5. Compare degrees and leading coefficients. **Answer: horizontal, y = 0.**

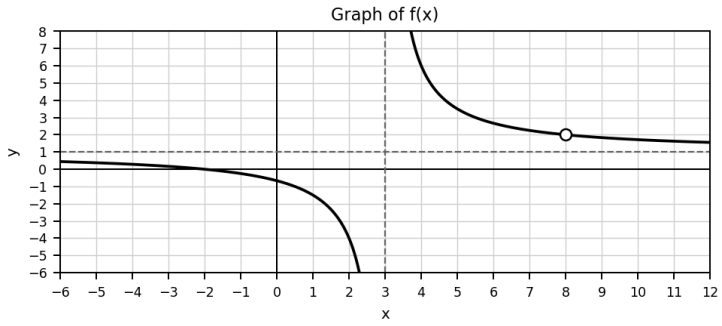
26. Which expression matches the graph?

A. $\frac{(x+2)(x-3)}{(x-3)(x-8)}$

B. $\frac{(x+2)(x-8)}{(x-3)(x-8)}$

C. $\frac{(x-2)(x+8)}{(x+3)(x+8)}$

D. $\frac{(x+2)(x-8)}{(x-2)(x-3)}$



The graph has zero −2, vertical asymptote 3, and hole 8. Only B cancels x−8 while keeping x−3 in the denominator. **Answer: B.**

27. Which expression has a hole at x = 4 and vertical asymptote at x = −3?

A. $\frac{(x-4)(x-3)}{(x-4)(x+3)}$

B. $\frac{(x-3)(x+3)}{(x-4)(x+3)}$

C. $\frac{(x-4)(x+3)}{(x-4)(x-3)}$

D. $\frac{(x-4)(x+3)}{(x-3)(x+3)}$

Cancel x−4 in A; x+3 remains below. Other options have the wrong hole or asymptote. **Answer: A.**

28. Which statement is correct for $g(x) = \frac{(x-3)^3(x+2)^2}{(x-3)^2(x+2)^4}$

A. VA $x = 3$; HA $y = 1$.

B. VA $x = -2$; HA $y = 2$.

C. VA $x = 3$; HA $y = -1$.

D. VA $x = -2$; HA $y = 0$.

Cancel common factors to get $(x-3)/(x+2)^2$. Denominator zero -2 remains; numerator degree 1 < denominator degree 2. **Answer: D.**

29. Which horizontal asymptote does $h(x)$ have? $\frac{(x-3)^5 - x^5}{4x^4}$

A. $y = 1$ B. $y = 5$ C. $y = -3$ D. $y = \frac{-15}{4}$

E. None

Expand the highest terms: $(x-3)^5 = x^5 - 15x^4 + \ldots$. Subtract x^5 to leave degree 4 with leading coefficient -15. Divide by leading coefficient 4. **Answer: D, $y = -15/4$.**

30. Which values are zeros of $r(x) = \frac{x^2+x-12}{x^3-7x^2+12x}$

A. $x = 3$ only

B. $x = -4$ only

C. $x = 0$ and 4 only

D. $x = 0$, 3, and 4

Factor numerator $(x+4)(x-3)$ and denominator $x(x-3)(x-4)$. $x=3$ is excluded; -4 makes only numerator zero. **Answer: B.**

Video support by problem type

End behavior and horizontal asymptotes (A 1–8; B 1–9, 20–25, 28–29)

Watch the worked example, then apply the same method to the listed questions.

Vertical asymptotes and one-sided behavior (A 9–22; B 14–19, 26–28)

Watch the worked example, then apply the same method to the listed questions.

Holes and removable discontinuities (A 11–12, 17, 20–22; B 14–19, 26–30)

Watch the worked example, then apply the same method to the listed questions.

Slant asymptotes and division (B 1–9, 20–25)

Watch the worked example, then apply the same method to the listed questions.

Rational inequalities (A 23–26; B 10–13)

Watch the worked example, then apply the same method to the listed questions.

Writing rational functions from features (B 26–30)

Watch the worked example, then apply the same method to the listed questions.