

1. Chp. 1

- a. The heading of Exercise 1.6 is separated from its body by a page break.
- b. In Definition 1.14, you might want to explain the need for “non-empty” (Chris Dutchyn)
- c. In Question 1.17, why is “let’s allow ourselves ... in the same part” there? (Chris Dutchyn)
- d. Before Definition 1.22, you might want to dispose of the “partial” vs “total” function question ... and tell us that “function” := “total function”. You use it to avoid vacuuous cases later. (Chris Dutchyn)
- e. "Alice’s observation fails to preserve the join operation." Shouldn't this be "The join operation fails to preserve Alice's observation"? Edit: I now see what it's saying, but it was very confusing to me at first. It means that $\phi(x \vee y) \neq \phi(x) \vee \phi(y)$, but "v" hasn't been introduced for Booleans yet, so I never even considered that interpretation. And "preserving order" means something completely different: $x \leq y \Rightarrow \phi(x) \leq \phi(y)$, so I didn't see the relationship. See my question [here](#). (+1 David VandeBunte)
- f. “Finally, we may want to take a disjoint union of two sets, even if they have elements in common. Given two sets X and Y , their disjoint union $X \dot{\cup} Y$ is the set of pairs of the form $(x, 1)$ or $(y, 2)$, where $x \in X$ and $y \in Y$.” – Having very little math background, this doesn't explain what disjoint unions are. This quote from [Wikipedia](#) is short, and found it more useful: “the disjoint union (or discriminated union) of a family of sets is a modified union operation that indexes the elements according to which set they originated in.” (With that said, I love the book, and thank you for putting so much time and effort into it, and for making the topic approachable, even for a lay person. Attila)
- g. Exercise 1.20, “let P be the set of $(-)$ -closed and $(-)$ -connected subsets $\{A_p \mid p \in P\}$ ” — we introduced P as the set of labels; it does not contain subsets A_p : (Alexey Filippov)
- h. In definition 1.22 you could use the contrapositive definition of a injective function (https://en.wikipedia.org/wiki/Injective_function), it could be more natural to interpret as function that preserves distinctness. Alternatively you could add a conclusion at the end of formal notation. (Pawel Sawicz)
- i. Definition 1.22: “A function from S to T is a subset $F \subseteq S \times T$ such that for all $s \in S$ there exists a unique $t \in T$ with $(s, t) \in F$ ” I suspect this definition should have the word “unique” removed; as it stands, it is the definition of an injective function.
- j. In remark 1.35, you use “ \sim ” stacked, but I can’t find any discussion of it before that point ... it shows up in 1.49. Please help. (Chris Dutchyn)
- k. Example 1.37, after table: instead of “... from 4 to 4 ...” there must be “... from 2 to 2 ...” (Henry Chern)
- l. Exercise 1.37: we say there is one path 2 to 3, but there are really infinitely many. (Alexey Preobrazhenskiy)
- m. Example 1.54: The definition of an upper set is wrong. See: <https://davidvandeunte.gitlab.io/executable-notes/notes/ssc-what-is-order.html>
- n. Exercise 1.57: it would be easier if you put “b” left of “e”. (Chris Dutchyn, +1 David VandeBunte)
- o. Solutions for 1.57 and 1.66: the empty set is missing from the upper set preorder (Joachim Hotonnier)
- p. In the solution to exercise 1.57, $\{(b, 1), (b, 2), (c, 1), (c, 2)\}$ is also an upper set (Joeri van Eekelen)
Seconded. I too noticed this was missing. Should the empty set be there as well? (Ian Coulter)
- q. Example 1.58: italics for “opposite preorder” since it’s a term? (Chris Dutchyn)
- r. on p. 20, exercise 1.65 refers to exercise 1.54 and claims we drew the Hasse diagram for $\mathsf{P}(\mathbb{B})$. We drew the Hasse diagram for a *different* 2-element set ($\{1, 2\}$) (Zack Newman)
- s. Exercise 1.66: part 4 refers to Example 1.56 (product preorder), whereas it seems like it should refer to Exercise 1.57 (the one with a diagram). (Ivan Smirnov)
- t. Example 1.66, last line, should refer to 1.57 not 1.56 for the picture of \uparrow ? (Chris Dutchyn)
- u. Exercise 1.66 (Yoneda lemma for preorders): “Show that if $p \leq p'$ in P if and only if $\uparrow(p') \subseteq \uparrow(p)$.” The construction is a little awkward: “show that if ... if and only if”. Maybe omit the first “if”? (Zack Newman)
- v. Question 2: Upper and the Yoneda lemma. $\uparrow \text{Pop} \rightarrow \text{U}(P)$
—— Pop is a tuple (P, \leq) not a set, so is the signature wrong?
- w. Exercise 1.66: This question is extremely hard for outsiders to follow (David VandeBunte). See: <https://math.stackexchange.com/questions/3307889>
<https://davidvandeunte.gitlab.io/executable-notes/notes/ssc-what-is-order.html>
- x. 1.69: replace “non-identity” with “non-isomorphism”, otherwise I’ll use two three-element sets with the obvious (trivial) isomorphism $\{a, b, c\}$ and $\{1, 2, 3\}$... I know, I’m evil... (Chris Dutchyn)
- y. Beginning of Section 1.3.2, through the paragraph after Definition 1.93: When reading this for the first time, it was a bit confusing that the function indicating a generative effect was sometimes called f , and other times called ϕ (Daniel Irving Bernstein)
- z. Proposition 1.78. In the last sentence of the proof, instead of ‘ f ’, there must be f_{\downarrow} (Henry Chern)
- aa. Exercise 1.79. Instead of “Let P and Q be preorders” there must be “Let P and Q be sets with a preorder relation” (Henry Chern)
- bb. Discussion after exercise 1.66, “to know an element is the same as knowing its upper set — that is, knowing its web of relationships with the other elements of the preorder” — it is not quite clear that we introduced “its upper set” for $\uparrow p$. (Alexey Filippov)
- cc. Exercise 1.79 — If the reader is not familiar with the ‘ f^4 ’ notation from elsewhere, they (like me) might not remember from 1.26 what it is and get confused (like me). This might make the exercise look more difficult than it actually is. A quick remark about preimages would help here. (Lisa Bylinina)
- dd. In Example 1.97 (Oct 12 version, p.27), x is first used as an element in \mathbb{Z} and then as an element in \mathbb{R} ; similarly, y is first used as an element in \mathbb{R} and then as an element in \mathbb{Q} . The change in notation is a bit reader unfriendly as we are still in the same example. (Julio Song)
- ee. Page 263 the explanation for exercise 1.98 says “largest” when it means “smallest”.
- ff. Exercise 1.101 solution to part 2 could use the adjoint functor theorem, rather than just brute calculation.
- gg. Exercise 1.101: Since the given function preserves meets, it should have a left adjoint by the adjoint functor theorem (Theorem 1.115). See also: <https://davidvandeunte.gitlab.io/executable-notes/notes/ssc-galois-connections.html>
- hh. In Exercise 1.104, is there a reason to reuse labels 11, 12, 13, etc. for the partitioned T ? I feel like it makes it a little harder to reason about what’s happening, with no benefit. (Valentin Robert)
- ii. Proposition 1.107, minor note: it might make sense to have a colon after “equivalent”. (Ivan Smirnov)
- jj. Proposition 1.111, minor wording: “Suppose ... any subset” \rightarrow “Suppose that ... is any subset” or “Let ... be any subset”. Also, “Right adjoints preserve meets” may not be the best label for the whole proposition since it also handles the left adjoints as well. (Ivan Smirnov)
- kk. The second row ($p=1, q=2$) for the solutions for Exercise 1.114 appear to be wrong. It is true that both $f(p) \leq q$ and $p \leq f(q)$. (Nicholas Fazzio)
- ll. In Example 1.117, the names of the sets change from A and B to X and Y and then back.
- mm. Example 1.117: This is a bit confusing. It could be clarified that there are two adjoint pairs here, (1) $f_{\downarrow}!$ and $f^{\wedge*}$, and (2) $f^{\wedge*}$ and f_{\downarrow}^* . (+1 David VandeBunte)
- nn. Just before 1.135, did you want to change the font of “ Cl ” as the function from $\text{Rel}(S) \rightarrow \text{Pos}(S)$? You use sans-serif in part 3 of exercise 1.135. (Chris Dutchyn)

2. Chp. 2

- a. Definition of symmetric monoidal preorder should possibly be (strict) symmetric monoidal preorder?
- b. In section 2.2.2, “We say that a wiring diagram is valid if the monoidal product of the elements on the left is less than the monoidal product of those on the right” — I think this should say that the monoidal product of the elements of *each box* have to be less than (or equal to) the monoidal product of the elements on its right. (This seemed to really confuse my colleagues, who interpreted it in a consistent but weird way in which a diagram indicating something like $2 \leq 10 \leq 4$ is considered valid.)
- c. Related to the above, in section 2.2.2 you say what it means to draw a wire above a wire, but you don’t explicitly say in that section what it means to draw a box above another box, or a box above a wire — I think it would be helpful to say explicitly that drawing a box above a box means both inequalities are true.
- d. 2.2.2, definition of a valid wiring diagram: should “less than” be “less than or equal to”? (Ivan Smirnov)
- e. Section 2.2.4 ... the Booleans: only two different structures: can’t I build some out of NAND and NOR too? (Chris Dutchyn)
- f. Example 2.32: use “ $\forall n$ for all n ” so you’re using the same letter/variable on the right hand side of \vdash (you previously said “ $m \mid n$...”) (Chris Dutchyn) (+1 David VandeBunte)
- g. 2.2.5, Definition 2.41: why drop “map” as in “monoidal monotone map” ... now monotone is an adjective (as in “a monotone map”) and a noun (as in “a monoidal monotone”) and the reader needs to take undue care. Alternatively, go back and change all the other “a monotone map” uses to just “a monotone”. (Chris Dutchyn)
- h. Example 2.42: The floor function is $\mathbf{R} \rightarrow \mathbf{Z}$, rather than $\mathbf{R} \rightarrow \mathbf{N}$ (consider floor of π). Perhaps it might make sense to rephrase the whole example in terms of \mathbf{Z} and \mathbf{R} ? That seems to work better. (Alexey Filippov) (+1 David VandeBunte)
- i. Exercises 2.43 and 2.44 — don’t they presuppose that ∞ is a member of the set of real numbers, a real number as well? Seems like it’s a silent

- assumption here that one might want to make explicit, saying we're dealing with an affinely extended real number system. Also, what would happen to these structures if we had the regular set of real numbers, without ∞ ? It feels like there's a need for a short clarifying discussion here. (Lisa Bylinina)
- j. This is with regard to a possible mistake in the solution to Exercise 2.45. I believe that the uniqueness part of this claim is false. Namely, wouldn't $f(n) = 2^n$ be another monoidal monotone between these two monoidal posets? Or perhaps I'm unclear on something in the definition. (Daniel Irving Bernstein) (+1 David VandeBunte)
- k. I had exactly the same remark about 2.45: any m^n seems to work — since it's not just me I hope you give an explanation here. I would also add that many exercises are of the short form “Why? Why not?” and it doesn't seem like a good idea to end an answer with another “Why?” Unless this is an exercise in level shifting and you plan to add “Answers to Exercises in Answers to Exercises”. (Tomasz)
- l. Me three ... 2.45 seems to be incorrect (ChrisDutchyn)
- m. The footnote after exercise 2.52 (about Hausdorff distance) seems to be split across pages 60 and 61, leading to a seemingly unnumbered and irrelevant footnote on page 61. (Luke Worth) (+1 David VandeBunte)
- n. Exercise 2.62: In part 2 of this exercise it's claimed that you can get an M-Category where the hom object from x to y is computed as follows: for each path p from x to y , take the intersection of the sets labelling the edges in p . Then, take the union of these sets. However, if we imagine a graph where there is no path from a vertex A to itself, wouldn't this violate condition (a) in the definition of V-categories?
- o. After Exercise 2.78. There is no edge (only the path) from A to C , so the matrix for X in equation (1.77) must be: $\begin{matrix} & A & B & C \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{matrix} 0 \\ \infty \\ \infty \end{matrix} & \begin{matrix} 2 \\ 0 \\ \infty \end{matrix} & \begin{matrix} \infty \\ 3 \\ 0 \end{matrix} \end{matrix}$ (then you need to correct the matrix $X \times Y$) (Henry Chern)
- p. Exercise 2.84: Additional explanation for coming up with an idea of what the hom element might be: if we assume Bool is monoidal closed, we can then conclude how the hom element has to look: We have $a \& b \leq c$ iff $a \leq b \multimap c$, in particular for $a = \text{true}$ we see: $b \leq c$ iff $\text{true} \leq b \multimap c$, i.e. **$b \leq c$ iff $b \multimap c$** . So we choose $(b \multimap c) := (b \leq c)$ (which is the same as $b \Rightarrow c$). (Hero Wanders)
- q. Remark 2.89: Replace $X(x,x)$ with $V(x,x)$ (Hero Wanders)
- r. There is double repetition of “the” occurs several times in the text. (Konstantin Nisht)
- s. 2.96: <https://math.stackexchange.com/questions/3846876/joins-and-meets-in-preorder> (David VandeBunte)
- t. Remark 2.97. The correspondence with the Hausdorff distance here is: $\sup \rightarrow \text{meet}$, $\inf \rightarrow \text{join}$, but in Example 2.91 the join is the supremum in the actual order of Cost, as explained in the footnote. Is the confusion (again) due to reverse orders? That is, the join is the standard infimum on $[0, \infty]$, and that's what we want to recover? (Tomasz)
- u. Proof of Proposition 2.98 uses the adjoint functor theorem and states that $(v \multimap)$ preserves joins. However, $(v \multimap)$ is a right adjoint. Shouldn't we use the left adjoint, $(- \multimap v)$, instead? (Zhengqun Koo)
- v. In the proof of proposition 2.98, “We need to show that if Eq. (2.88) holds then $- \multimap v : V \rightarrow V$ has a right adjoint.... (Louise Nielsen)
- w. 2.104: <https://math.stackexchange.com/q/4213631/245548>
- x. Ch. 2.6, pg 76 — a typo, “Enrichment is a fundamental notion in category theory, and **we will we** return to it in Chapter 4,..” (Alexey Filippov)
3. Chp. 3
- a. There seems to be inconsistency of notation throughout the whole book when referring to objects of a category. Sometimes it's referred as “ $c \in \text{Ob}(C)$ ” (definition 3.6), sometimes as “ $c \in C$ ” (page 247., when specifying that a predicate is a sheaf morphism) edit: Although now I see this is explicitly mentioned on page 93 when defining a category. (Bruno Gavranović)
- b. Example 3.5: For $m+1 \leq i \leq m+n$ instead of setting $(f+g)(i) = m' + g(i)$ you should probably set $(f+g)(i) = m' + g(i-m)$ or alternatively (to avoid using subtraction) define it by $m' + g(j)$ with $1 \leq j \leq n$ being the unique natural number satisfying $j + m = i$; Reason: g is not defined for $n < i$ (Hero Wanders)
- c. Example 3.42: It must send morphisms in \mathcal{A} to morphisms in \mathbb{N} . The second \mathbb{N} should be the same as the first, because the first \mathbb{N} represents the pre-order (\mathbb{N}, \leq) , but the \mathbb{N} only represents a set, i.e. it has no morphisms (Harmon Nine). (+1 David VandeBunte)
- d. On page 281, the solution to part 2 of exercise 3.48 doesn't sound right: $D(f)$ is not an inclusion because it sends a set of people to a set of the gifts they give themselves.
- e. Remark 3.59: This claims that $F;G$ is a natural isomorphism. If this is the case, where is the natural transformation (with an inverse) on which this natural isomorphism is based? Is it between F and G , is this the composition of F and G ($F;G$), and if it is the latter, what are the two functors that it is between? (Harmon Nine) (+1 David VandeBunte)
- f. The paragraph after Exercise 3.76 seemed to be confusing: an instance on **1** is the same thing as a set (not Category **Set**). So let's let's identify **1-Inst** with **Set**(the category). Are we confusing set with **Set** here? (Hao Deng)
- g. Exercise 3.78 solution: change name Emory to Emmy
4. Chp. 4
- a. Immediately preceding Def 4.2: Recall that if $X = (X, \leq)$ is a preorder, then its opposite $X^{\text{op}} = (X, \geq)$ has $x \geq y$ iff $y \leq x$. It is not immediately clear what is intended. I see now that the definition of op implies that if $x \leq y$ in X then $y \leq x$ in X^{op} , but the notation for the opposite preorder makes use of \geq rather than \leq which makes sense iff $x \geq y$ is equivalent to $y \leq x$.
- b. Solution to Exercise 4.9: The author almost meant to refer to Definition 2.69 rather than Definition 2.41 (which is specific to monotone maps) (David VandeBunte).
- c. Definition 4.21: It isn't immediately obvious (to me) that this construction indeed produces a profunctor. Maybe it's worth including a proof, or an exercise asking for a proof? (Daniel Irving Bernstein)
- d. Type: The definition of 4.21 stipulates X, Y, Z as categories, and then uses the same equation as 4.20 (which used P, Q, R); specifically, it says to take the join over all $q \in Q$, which should read over all $q \in Y$ (or better yet, match the letters to the categories or vice versa). (+1 David VandeBunte)
- e. Lemma 4.27: The arrow between P and Q indicating the profunctor ϕ is missing its vertical line (Daniel Irving Bernstein)
- f. Type: The arrows giving the profunctors in lemma 4.31 are missing the vertical bar.
- g. Rough Definition 4.45: it would be nice to see “well behaved” precisely defined in a remark after this (Daniel Irving Bernstein)
- h. Example 4.49, fourth bullet point. It seems like this should say $(f \times g) \circ (f' \times g') = (f \circ f') \times (g \circ g')$. See a longer explanation under “**Commutative diagram for snake equations**” in <https://davidvandeBunte.gitlab.io/executable-notes/notes/ssc/4-5.html>.
5. Chp. 5
- a. Page 193, the diagram in example 6.37 is printed in black while text references blue and red arrows. Juan Manuel Gimeno. (David VandeBunte: likely refers to Example 6.42 on pg. 194, but this issue seems to have been fixed)
6. Chp. 6
- a. Theorem 6.77: “...and whose morphisms are *equivalence* classes...” should be “...and whose morphism are *isomorphism* classes...”, because we do not have any equivalence relation. Also, neatlab refers to isomorphism classes in the page about decorated cospans. (Konstantin Nisht)
- b. Definition 6.97, section iii: Probably function composition sign (circle) in the right side of the equation should be replaced with semicolon sign for composition of morphisms, because we are still using categorical terms. (Konstantin Nisht)
7. Chp. 7
- a. On page 227, section 7.2.1, in the “Cartesian closed” paragraph, the sentence starting with “So now you've transformed a two-player game...” is very hard to read, and possibly nonsensical around the “valued in...” part. (Valentin Robert)
- b. Example 7.54: Quotes behave strange in the last paragraph. (Konstantin Nisht)
- c. (file not found :)) Example 7.54: **But if it is not in H, the mathematics requires us to ask more questions: is its source in H? is its target in G? both? neither?** we are talking about the subobject classifier of graphs; the morphism $(0, V; 0)$ surely corresponds to the target being in H , the subgraph, rather than G ? (AKoziell)
- d. Example 7.58: Is it really true that $V \rightarrow U = U$? We can make a union of that answer with the interval $(4; +\infty)$. (Konstantin Nisht)
- e. 7.61: <https://math.stackexchange.com/q/3912771/245548>
- f. Just above Eq. 7.63 the set of S predicates is denoted as “ $\setminus \Omega^E$ ”. Was this meant to be “ $\setminus \Omega^A S$ ”, since E is never mentioned again? (Bruno Gavranović); In the first paragraph of 7.4.5 they denote it correctly. (Rostislav Svoboda)
8. General

- a. How about linking the number of an exercise to its solution and vice versa in the PDF file? (Marcin Ciura)
- b. How about using three digits after the period in the numbering? For instance, Exercise 1.1 would become Exercise 1.001. This way, searching the PDF file will be easier. (Marcin Ciura)

Check for:

- More than 13 posts: <https://math.stackexchange.com/search?q=%22seven+sketches%22>
 - I'm not sure it would be worth checking on 13 posts; the 14th could be ignored (unless there are e.g. 33 later).
- <https://math.stackexchange.com/search?q=%22Applied+Category+Theory%22>