# ESS Equations by Topic

The skills and techniques students must experience through the course are encompassed within the tools and inquiry process. These tools support the application and development of the inquiry process in the delivery of the environmental systems and societies (ESS) course. Guidance on how the tools can be applied in the learning and teaching of the subject is summarized in each subtopic. *(p. 25 of the ESS Guide)*

## Tool 3: Mathematics

(*p. 28 of the ESS Guide*)

| Skill | Description |
| --- | --- |
| *General mathematical skills* | For manipulating data, students should be able to:<br>• execute basic arithmetic functions: addition, subtraction, multiplication and division<br>• perform calculations involving averages, decimals, fractions, percentages, ratios, approximations, frequencies and reciprocals<br>• calculate measures of central tendency: mean, median and mode<br>• use and interpret standard notation (e.g. $3.6 \times 10^6$)<br>• apply and use the Syst.me International d'Unit.s (International System of Units—SI units) for mass, time, length and their derived units, e.g., speed, area and volume or non-SI metric units<br>• use direct and inverse proportion<br>• plot graphs (with suitable scales and axes) including two variables that show linear and non-linear relationships, independent variable on the x axis, dependent on the y axis<br>• interpret graphs, including the significance of gradients, changes in gradients, intercepts and areas<br>• interpret data presented in various forms: scatter plot, point-to-point line, line of best fit, bar chart, stacked histogram, pie chart, box and whisker plot, kite diagram<br>• evaluate data through statistical tests and quantities (e.g., standard deviation, correlation coefficient, Spearman's rank correlation coefficient, analysis of variance (ANOVA), chi-squared test, t-test)<br>• calculate indices from given formulae, e.g., Simpson's reciprocal index, Lincoln index<br>• calculate natural increase rates and population doubling times from given data. |

# ESS Equations by Topic

| General Equations | |
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| **Percent Change:** Calculate percent change when you want to determine the relative increase or decrease of a value over time, which helps compare changes across different scales or between different ecosystems, populations, or measurements. | **<u>Percent change equation:</u>**<br><br>Percent Change = $\left(\frac{New\ Value - Original\ Value}{Original\ Value}\right) \times 100$ |
| | **<u>Percent change example:</u>**<br>If a population of birds increased from 120 to 150 individuals over a year, what would be the percent change in the population?<br><br>Percent Change = $\left(\frac{150 - 120}{120}\right) \times 100$ = 25 %<br><br>This means the bird population **increased by 25%** over the year. |

| Topic 2: Ecology | |
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| **2.1 Individuals, Populations, Communities, and Ecosystems** | |
| **Knowledge & Understandings:** | **Equations to be able to use and apply:** |
| 2.1.15 Population abundance can be estimated using random sampling, systematic sampling or transect sampling. | **<u>Estimating percentage cover equation:</u>** Percentage cover estimates the proportion of a quadrat occupied by a particular plant or animal species.<br><br>$Percentage\ cover\ = \frac{Area\ of\ species\ within\ quadrat}{Total\ area\ of\ quadrat}\ x\ 100$<br><br>Note: Often, this is done visually by estimating how much of the quadrat's surface is covered by the target species, especially for vegetation.<br><br>**<u>Estimating percentage frequency equation:</u>** Percentage frequency measures how often a species occurs across all sampled quadrats.<br><br>Percentage frequency = $\frac{Number\ of\ quadrat\ squares\ in\ which\ species\ are\ present}{Total\ number\ of\ quadrat\ squares}\ x\ 100$ |
| | **<u>Percentage frequency example:</u>**<br><br>A student was performing quadrat sampling in an urban park and found grass in 89 out of 100 squares in the quadrat. |

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| | Percentage frequency = $\frac{89}{100}\ x\ 100\ =\ 89\%$ |
| 2.1.17 Capture–mark–release–recapture and the Lincoln index can be used to estimate population size for mobile organisms. | **<u>Lincoln's index equation:</u>**<br><br>$Population\ size\ estimate\ =\ \frac{(M \times N)}{R}$<br><br>M = the number of individuals caught and marked initially,<br>N = the total number of individuals recaptured<br>R = the number of marked individuals recaptured |
| | **<u>Lincoln's index example:</u>**<br><br>Scientists wanted to investigate the abundance of leafhoppers in a small grassy meadow. They used sweep nets to catch a large sample of leafhoppers from the meadow. Each insect was marked on its underside with non-toxic waterproof paint and then released back into the meadow. The following day another large sample was caught using sweep nets. Use the figures below to estimate the size of the leafhopper population in this meadow.<br><br>Number caught and marked in first sample (M) = 236<br><br>Number caught in second sample (N) = 244<br><br>Number of marked individuals in the second sample (R) = 71<br><br>Step 1: Write out the equation and substitute in the known values<br>$Population\ size\ estimate\ =\ \frac{(M \times N)}{R}$<br><br>$Population\ size\ estimate\ =\ \frac{(236 \times 244)}{71}$<br><br>Step 2: Calculate the population size estimate (N)<br><br>$Population\ size\ estimate\ =\ \frac{57{,}584}{71}\ =\ 811$<br><br>Estimated population size = 811 individuals |

# ESS Equations by Topic

| 2.2 Energy and biomass in ecosystems | |
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| **Knowledge & Understandings:** | **Equations to be able to use and apply:** |
| 2.2.13 Gross productivity (GP) is the total gain in biomass by an organism. Net productivity (NP) is the amount remaining after losses due to cellular respiration. | **<u>Gross productivity (GP) equation:</u>** GP is the total gain in biomass by an organism or community in a given area or time period. It includes all the energy captured by organisms.<br><br>GP = Total energy captured<br><br>**<u>Net productivity (NP) equation:</u>** NP is the amount of energy or biomass remaining after accounting for energy lost through respiration.<br><br>NP = GP - Respiration (R) |
| | **<u>Gross productivity example:</u>**<br><br>Imagine a pond ecosystem where the energy captured by aquatic plants is equal to 1,000 kJ per day.<br><br>GP = 1,000 kJ/ day<br><br>**<u>Net productivity example:</u>**<br><br>Imagine the same pond ecosystem, where gross productivity is 1,000 kj/ day and respiration (R) is 300 kJ/ day.<br><br>NP = 1,000 - 300 = 700 kJ/day<br><br>This means the **NP is 700 kJ/day of energy**, which is available for growth and reproduction in the pond ecosystem. |
| 2.2.14 The number of trophic levels in ecosystems is limited due to energy losses.<br><br><u>Application of skills:</u> Work out the efficiency of transfer between trophic levels. | **<u>Efficiency of energy transfer between trophic levels equation:</u>** This measures the percentage of energy or biomass transferred from one trophic level to the next (e.g. from plants to herbivores).<br><br>$Efficiency\ (\%) = \frac{Energy\ at\ higher\ trophic\ level}{Energy\ at\ lower\ trophic\ level}\ x\ 100$ |
| | **<u>Efficiency of energy transfer between trophic levels example:</u>**<br><br>Imagine an ecosystem where the energy from the plants is 10,000 kJ $m^{-2}$ $yr^{-1}$ and the energy from the herbivores is 1,000 kJ $m^{-2}$ $yr^{-1}$. Calculate the efficiency of energy transfer between the trophic levels. |

# ESS Equations by Topic

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$$Efficiency\ (\%) = \frac{1{,}000}{10{,}000}\ x\ 100 = 10\%$$

This means that 10% of the energy is transferred from plants to herbivores.</td></tr>
<tr><td rowspan="2">2.2.17 Ecological pyramids are used to represent relative numbers, biomass or energy of trophic levels in an ecosystem.<br><br><u>Application of skills:</u> Create pyramids of numbers, biomass and energy from given data.<br><br>Follow experimental procedures on how to find biomass and energy from biological samples (plant material only).</td><td>

**<u>Biomass Pyramid Equation:</u>** Ecological pyramids are graphical representations of trophic level data. They can represent:

- Numbers: the count of organisms
- Biomass: the mass of living material
- Energy: the energy stored in that biomass

When measuring biomass in an ecosystem from plant material, use:

$$Biomass\ (g/\ m\text{^}2) = \frac{Dry\ Mass\ of\ Sample\ (g)}{Sample\ Area\ (m\text{^}2)}$$
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**<u>Biomass pyramid worked example:</u>**

**Step 1:** Sample collection

- Collect plant material from a 1 $m^2$ quadrat.
- Dry the sample until constant mass is reached (to remove water).
- Measure dry mass: 200 g.

**Step 2:** Calculate biomass

$$\text{Biomass} = \frac{200\ \text{g}}{1\ \text{m}^2} = 200\ \text{g/m}^2$$

**Step 3:** Compare trophic levels

- Producers (plants): 200 $g/m^2$
- Primary consumers (herbivores): 20 $g/m^2$
- Secondary consumers (carnivores): 2 $g/m^2$

**Step 4:** Draw the pyramid

- Producers at the base (200 $g/m^2$)
- Primary consumers above them (20 $g/m^2$)
- Secondary consumers at the top (2 $g/m^2$)
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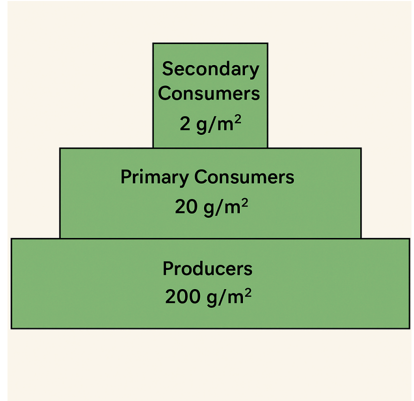
| |  |
| *HL ONLY:*<br><br>2.2.25 Net primary productivity is the basis for food chains because it is the quantity of carbon compounds sustainably available to primary consumers.<br><br><u>Application of skills:</u> Use laboratory and field techniques for measuring primary and secondary productivity and work out GP and NP from data. | **<u>Gross primary productivity (GPP) equation:</u>** GPP is the rate at which plants (primary producers) store chemical energy or biomass through photosynthesis. It represents the total energy captured by plants per unit area and per unit time. GPP is expressed in kJ $km^{-2}$ $yr^{-1}$.<br><br>$$GPP = \frac{Total\ energy\ captured\ by\ photosynthesis}{Area\ x\ Time}$$<br><br>Note: For aquatic ecosystems, GPP can be expressed per unit volume instead (e.g., kJ $m^{-3}$ $yr^{-1}$).<br><br>**<u>Net primary productivity (NPP) equation:</u>** NPP is the rate at which energy is stored in plant biomass after accounting for plant respiratory losses (R). It represents the energy available to higher trophic levels, supporting food chains and webs.<br><br>$$NPP = GPP - R$$<br><br>• GPP = Gross Primary Productivity<br>• R = Respiration by plants<br><br>Notes:<br>• NPP represents the energy that is actually stored as plant biomass and can be passed on to herbivores and decomposers.<br>• In aquatic ecosystems, NPP would often be expressed per unit volume (e.g. kJ $m^{-3}$ $yr^{-1}$).<br><br>**<u>Gross secondary productivity (GSP) equation:</u>** GSP is the total energy or biomass assimilated by consumers (e.g. herbivores or carnivores). It is calculated by subtracting the mass of faecal loss from the mass of food eaten.<br><br>$$GSP = Food\ eaten - Faecal\ loss$$<br><br>**<u>Net secondary productivity (NSP) equation:</u>** NSP is the |

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| | energy or biomass remaining after subtracting respiratory losses from the GSP. It represents the energy available for growth and reproduction of the consumer.<br><br>$$NSP = GSP - R$$ |
| | **<u>Gross primary productivity example:</u>**<br><br>Imagine a grassland ecosystem where over one year, the total energy captured by photosynthesis is 1,200,000 kJ and the area studied is 1 km$^2$. Calculate the GPP.<br><br>$$\mathrm{GPP} = \frac{1,200,000\ \mathrm{kJ}}{1\ \mathrm{km}^2 \times 1\ \mathrm{year}} = 1,200,000\ \mathrm{kJ\ km^{-2}\ yr^{-1}}$$<br><br>The **GPP** in this grassland is **1,200,000 kJ per km$^2$ per year**.<br><br>**<u>Net primary productivity (NPP) example:</u>**<br><br>Imagine a forest ecosystem where the Gross Primary Productivity (GPP) is 2,000,000 kJ m$^{-2}$ yr$^{-1}$ and Plant Respiration (R) = 1,800,000 kJ m$^{-2}$ yr$^{-1}$. Calculate the NPP.<br><br>$$\mathrm{NPP} = 2,000,000 - 1,800,000 = 200,000\ \mathrm{kJ\ m^{-2}\ yr^{-1}}$$<br><br>So, the **NPP** is **200,000 kJ per m$^2$ per year.**<br><br>**<u>Gross secondary productivity (GSP) example:</u>**<br><br>Imagine a group of herbivores that have eaten 5,000 g m$^{-2}$ yr$^{-1}$ of food with 1,500 g m$^{-2}$ yr$^{-1}$ of faecal loss. Calculate the GSP.<br><br>$$GSP = 5,000\ g\ m^{-2}\ yr^{-1} - 1,500\ g\ m^{-2}\ yr^{-1}$$<br><br>So, the **GSP** is **3,500 g per m$^2$ per year**.<br><br>**<u>Net secondary productivity (NSP) example:</u>**<br><br>Continuing with the example above, the GSP = 3,500 g m$^{-2}$ yr$^{-1}$ and the respiration loss is 2,000 g m$^{-2}$ yr$^{-1}$. Calculate the NSP.<br><br>$$NSP = 3,500\ g\ m^{-2}\ yr^{-1} - 2,000\ g\ m^{-2}\ yr^{-1}$$ |

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| | So, the **NSP** is **1,500 g per m$^2$ per year**. |
| *HL ONLY:*<br><br>2.2.28 Ecological efficiency is the percentage of energy received by one trophic level that is passed on to the next level. | **Ecological efficiency equation:** Ecological efficiency measures the percentage of energy transferred from one trophic level to the next in an ecosystem. It quantifies how efficiently energy is passed along the food chain.<br><br>$\text{Ecological Efficiency (\%)} = \left(\frac{\text{Energy at higher trophic level}}{\text{Energy at lower trophic level}}\right) \times 100$ |
| | **Ecological efficiency example:**<br><br>Imagine a grassland ecosystem where the energy produced by the producers is 10,000 kJ/m$^2$/year and the energy of the herbivores is 1,500 kJ/m$^2$/year. Calculate the ecological efficiency equation.<br><br>$\text{Ecological Efficiency} = \left(\frac{1,500}{10,000}\right) \times 100 = 15\%$<br><br>So 15% of the energy at the plant level is transferred to herbivores. |

## Topic 3: Biodiversity & Conservation

### 3.1 Biodiversity and evolution

| **Knowledge & Understandings:** | **Equations to be able to use and apply:** |
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| 3.1.8 Simpson's reciprocal index is used to provide a quantitative measure of species diversity, allowing different ecosystems to be compared and for change in a specific ecosystem over time to be monitored.<br><br>Application of skills: Collect data in order to work out Simpson's reciprocal index for diversity. | **Simpson's reciprocal index equation:**<br><br>Calculate diversity (D) if provided with data and the formula in which *N* is the total number of individuals in the population and *n* is the number of individuals of a single species.<br><br>$D = \frac{N\,(N-1)}{\Sigma\, n\,(n-1)}$ |
| | **Simpson's reciprocal index example:**<br><br>Suppose you have a community with:<br><br>• Species A: 5 individuals<br>• Species B: 8 individuals<br>• Species C: 7 individuals |

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$$D = \frac{N\,(N-1)}{\Sigma\, n\,(n-1)}$$

Step 1: Calculate N:

$$N = 5 + 8 + 7 = 20$$

Step 2: Calculate N(N-1):

$$N\,(N - 1) = 20\, x\, 19 = 380$$

Step 3: Calculate n (n-1) for each species:

- Species A:
  $$n\,(n - 1) = 5\,(4) = 20$$
- Species B:
  $$n\,(n - 1) = 8\,(7) = 56$$
- Species C:
  $$n\,(n - 1) = 7\,(6) = 42$$

Step 4: Sum n (n-1) values:

$$20 + 56 + 42 = 118$$

Step 5: Calculate D:

$$D = \frac{380}{118} = 3.22$$

So, the Simpson's Reciprocal Index for this community is **3.22.**

The value of D will be higher where there is greater richness (number of species) and evenness (similar abundance), with 1 being the lowest possible value.

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## Topic 8: Human Populations & Urban Systems

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**8.1 Human populations**

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**Knowledge & Understandings:**

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**Equations to be able to use and apply:**

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8.1.1 Births and immigration are inputs to a human population.

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**Crude birth rate equation:** Crude birth rate (CBR) is the number of live births per 1,000 people in a population per year. It is a quantitative measure of a population input.

$$CBR = \frac{total\ number\ of\ live\ births}{total\ population}\, x\, 1000$$

**Immigration rate equation:** Immigration rate is the number of immigrants per 1,000 population per year. It is a quantitative measure of a population input.

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| | $$Immigration\ Rate\ = \frac{total\ number\ of\ immigrants}{total\ population} x\ 1000$$ |
| | **<u>Crude birth rate example:</u>**<br><br>A country has 25 000 live births in a year, and the total population is 500 000. Calculate the crude birth rate.<br><br>$$CBR\ = \frac{25,000}{500,000} x\ 1000$$<br><br>CBR = 50 births per 1 000 individuals<br><br>**<u>Immigration rate example:</u>**<br><br>Suppose a country has 15,000 immigrants in one year and the country's total population is 3,000,000 people.<br><br>$$Immigration\ Rate\ = \frac{15,000}{3,000,000} x\ 1000$$<br><br>This means the country has an **immigration rate of 5 immigrants per 1,000 people per year.** |
| 8.1.2 Deaths and emigration are outputs from a human population. | **<u>Crude death rate equation:</u>** Crude death rate is the number of deaths per 1,000 people in a population per year. It is a quantitative measure of a population output.<br><br>$$CDR\ = \frac{total\ number\ of\ deaths}{total\ population} x\ 1000$$<br><br>**<u>Emigration rate equation:</u>** Emigration rate is the number of emigrants per 1,000 population per year. It is a quantitative measure of a population output.<br><br>$$Emigration\ Rate\ = \frac{total\ number\ of\ emigrants}{total\ population} x\ 1000$$ |
| | **<u>Crude death rate example:</u>**<br><br>In a given year, a country recorded 15,000 deaths, and the total population is 750,000. Calculate the crude death rate.<br><br>$$CDR\ = \frac{15,000}{750,000} x\ 1000$$<br><br>CDR = 20 deaths per 1,000 individuals<br><br>**<u>Emigration rate example:</u>**<br><br>Suppose a country has 12,000 emigrants in one year and 4,000,000 people as its total population. |

# ESS Equations by Topic

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| | $$Emigration\ Rate\ = \frac{12{,}000}{4{,}000{,}000} x\ 1000$$<br><br>The emigration rate is **3 emigrants per 1,000 people per year.** |
| 8.1.3 Population dynamics can be quantified and analysed by calculating total fertility rate, life expectancy, doubling time and natural increase.<br><br><u>Application of skills:</u> Work out natural increase rates and doubling times from given data. | **<u>Total fertility rate equation:</u>** Total fertility rate is the average number of births per woman of childbearing age.<br><br>TFR is calculated by summing the age-specific fertility rates (ASFR) and multiplying the result by five.<br><br>$$TFR\ =\ \Sigma\ ASFR\ x\ 5$$<br><br>**<u>LIfe expectancy equation:</u>** Life expectancy is the average number of years that a person can be expected to live, usually from birth, if demographic factors remain unchanged.<br><br>You will not need to calculate life expectancy from an equation, but rather interpret data from life expectancy tables.<br><br>**<u>Doubling time (DT) equation:</u>** Doubling time is the number of years it would take a population to double its size at its current growth rate; it can be calculated by using the rule of 70. To do this, divide 70 by the growth rate (as a percentage).<br><br>$$DT\ = \frac{70}{growth\ rate\ \%}$$<br><br>**<u>Natural increase rate (NIR) equation:</u>** Natural increase is birth rate minus death rate, expressed as a number per 1,000 or as a percentage (the birth rate minus the death rate is divided by 10).<br>$$NIR\ = \frac{(CBR - CDR)}{10}$$ |
| | **<u>Total fertility rate example:</u>**<br><br>A country has the following fertility rates per 1 000 women in each age group:<br><br>• 15-19 years: 20 births per 1 000 women<br>• 20-24 years: 85 births per 1 000 women<br>• 25-29 years: 100 births per 1 000 women<br>• 30-34 years: 80 births per 1 000 women<br>• 35-39 years: 40 births per 1 000 women<br>• 40-44 years: 10 births per 1 000 women<br>• 45-49 years: 2 births per 1 000 women |

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| | Calculate the total fertility rate.<br><br>$$TFR = \Sigma\, ASFR\ x\ 5$$<br>$$TFR = (20 + 85 + 100 + 80 + 40 + 10)\ x\ 5$$<br>$$TFR = 1,685\ births\ per\ 1,000\ women$$<br>$$TFR = 1.685\ children\ per\ woman$$<br><br>This means that, on average, a woman in this country is expected to have approximately 1.69 children over her lifetime based on current fertility rates.<br><br>**Doubling time example:**<br><br>A population has a growth rate of 2% per year Calculate the doubling time.<br>$$DT = \frac{70}{2} = 35\ years$$<br><br>**Natural increase rate example:**<br><br>A country has a CBR of 25 births per 1 000 individuals and a CDR of 10 deaths per 1 000 individuals. Calculate the natural increase rate.<br>$$NIR = \frac{(25 - 10)}{10} = 1.5\%$$ |