# UNIT-I PHYSICAL OPTICS (WAVE OPTICS)

**SYNOPSIS:**

**1. Interference:** Interference can be defined as the superposition of two or more coherent waves with each other such that the intensity modifies maxima and minima which results in a new wave pattern.

**2. Constructive interference:**

The condition for constructive interference is the path difference should be equal to nλ.

i.e., $\Delta = n\lambda$; $n = 0, \pm1, \pm2 \ldots$

**3. Destructive interference:**

The condition for destructive interference is the path difference should be equal to odd multiples of λ/2.

i.e., $\Delta = (2n+1)\,\lambda/2$; $n = 0, \pm1, \pm2 \ldots$

**4. Coherence:**

The two light waves are said to be coherent if they have the same frequency (wavelength) and they are in phase or maintain constant phase relationship.

**5. Conditions for bright fringe in thin film:**

The condition for bright fringe is $2\mu t \cos \cos r = (2n-1)\lambda/2$

**6. Wavelength of a light source**: $\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$,

**7. Minimum intensity position**: $e \sin \sin \theta = \pm m\lambda$ where m = 1, 2, 3 …

**8. Polarization:** Polarization is also called wave polarization. It is an expression of the orientation of the lines of electric flux in an electromagnetic field. Polarization can be constant.

**9. Nichols prism:** A Nicol prism is a type of polarizer, an optical device made from calcite crystal used to produce and analyses plane polarized light.

**10. Double refraction:** In double refraction, the ordinary ray and the extraordinary ray are polarized in planes vibrating at right angles to each other. The refractive indexes for ordinary rays and extraordinary ray can be written as $\mu_0 = \frac{\sin\sin i}{\sin r_1}$ , $\mu_e = \frac{\sin\sin i}{\sin r_2}$

**11. Quarter wave plate:** $t = \frac{\lambda}{4(\mu_0 - \mu_e)}$

This is used to produce circularly and elliptically polarized light.

**12. Half wave plate:** $t = \frac{\lambda}{2(\mu_0 - \mu_e)}$

## PART-A

## SHORT QUESTIONS WITH SOLUTIONS

**1. What is interference?**

Ans: Interference can be defined as the superposition of two or more coherent waves with each other such that the intensity modifies maxima and minima which results in a new wave pattern.

**2. What is constructive interference?**

Ans: If two waves are in phase with the amplitudes $A_1$ and $A_2$ and superpose with each other then the crest of one wave will coincide with the crest of the other wave and the amplitude reaches to a maximum. The resultant wave will have amplitude is $A = A_1+A_2$. The condition for constructive interference is the path difference should be equal to $n\lambda$.

i.e., $\Delta = n\lambda$; $n = 0, \pm1, \pm2 \ldots$

**3. What is destructive interference?**

Ans: If two waves are out of phase with the amplitudes $A_1$ and $A_2$ and superpose with each other then the crest of one wave will coincide with the trough of the other wave and so will tend to cancel out. The resultant wave will have amplitude is $A = |A_1-A_2|$. The condition for destructive interference is the path difference should be equal to odd multiples of $\lambda/2$.

i.e., $\Delta = (2n+1)\ \lambda/2$; $n = 0, \pm1, \pm2 \ldots$

**4. What is coherence?**

Ans: The two light waves are said to be coherent if they have the same frequency (wavelength) and they are in phase or maintain constant phase relationship. Coherence is of two types.

1) Temporal coherence   2) Spatial coherence

**5. What is temporal coherence?**

Ans: If it is possible to predict the phase relation at a point on the wave w.r.t another point on the same wave then the wave has temporal coherence.

**6. What is spatial coherence?**

Ans: If it is possible to predict the phase relation at a point on the wave w.r.t another point on the second wave then the wave has spatial coherence.

**7. Mention any conditions of sustained to get interference**.

Ans: 1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelength and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be large.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow.
8. The sources should be monochromatic.

**8. Why does Newton's ring consist of concentric rings?**

Ans: When a planoconvex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and the glass plate are seen.

**9. What is diffraction?**

Ans: The phenomenon of bending of light round the corners of obstacles and spreading of light waves into the geometrical shadow region of an obstacle placed in the path of light is called diffraction. The phenomenon of diffraction can be divided into two types.

1. Fraunhofer diffraction 2. Fresnel diffraction

**10. Why lenses are necessary for the study of Fraunhofer diffraction?**

Ans: In this diffraction, the source and screen are placed at infinite distances from the obstacle. Due to the above fact, we need lenses to study the diffraction. This diffraction can be studied in any direction. In this case, the incident wave front is plane.

**11. Mention any two differences between interference and diffraction.**

| **Interference of light** | **Diffraction of light** |
|---|---|
| • Interference is due to the interaction of light coming from two different wave fronts originating from the same source. | • Diffraction is due to the interaction of light coming from different parts of the same wave front. |
| • Interference fringes are of the same width. | • Diffraction fringes are not of the same width. |
| • All bright fringes are of the same intensity. | • All bright fringes are not of the same intensity. |
| • All points of minimum intensity are perfectly dark. | • All points of minimum intensity are not perfectly dark. |
| • The spacing between the fringes is uniform. | • The spacing between the fringes is not uniform. |
| • Interference phenomenon does not require an obstacle. | • Diffraction phenomenon requires an obstacle. |
| • Path of the wave stays intact after interference. | • Diffraction changes the path of the incident wave |

**12. What is diffraction grating?**

Ans: An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and act as a slit.

**13. Write the difference between Fresnel and fraunhofer diffraction?**

Ans:

| Fresnel diffraction | Fraunhofer diffraction |
|---|---|
| 1:If the source of light and screen are at finite distance from the obstacle ,then the diffraction called Fresnel diffraction | 1:If the source of light and screen are at infinite distance from obstacle then the diffraction is called fraunhofer diffraction |
| 2:The corresponding rays are not parallel | 2:The corresponding rays are not parallel |
| 3:The wave fronts falling on the obstacle are not plane | 3:The wave fronts falling on the obstacle are plane |

**14. What are engineering applications of interference?**

**Ans: APPLICATIONS**

1. Signal processing
2. Image processing
3. Heat distribution mapping
4. Wave simplification
5. Light simplification (Interference, Diffraction etc.)

**15. What are engineering applications of diffraction?**

Ans: Engineering applications of diffractions are:
1- X-ray diffraction used in crystallography.
2- Diffraction Grating used to transform the light into its spectrum.
3- The most interesting application is the holography (you can check this amazing video.)

**16. What is grating spectrum?**

Ans: **Grating spectrum.**

Grating is optical device used to learn the different wavelengths or colors contained in a beam of light. a diffraction grating is an optical component with a periodic structure, which splits and diffracts light into several beams travelling in different directions.

**17. What is polarization?**

Ans: Polarization is also called wave polarization. It is an expression of the orientation of the lines of electric flux in an electromagnetic field. Polarization can be constant. It affects the propagation of EM fields at infrared, visible, ultraviolet and even X-ray wavelength. It is the the production or acquisition of polarity. It is the partial or complete polar separation of both electrical charges, such as positive and negative in a nuclear, atomic, molecular, or chemical system.

**18. Define Nichols prism?**

Ans: A Nicol prism is a type of polarizer, an optical device made from calcite crystal used to produce and analyse plane polarized light. It is made in such a way that it eliminates one of the rays by total internal reflection, i.e. the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism.

**19. What is double refraction?**

Ans: In double refraction, the ordinary ray and the extraordinary ray are polarized in planes vibrating at right angles to each other.

**20. Define quarter wave plate?**

Ans: a crystal plate that changes the phase difference between the two components of polarized light traversing it by one-fourth cycle — compare half-wave plate.

**21. Define half wave plate?**

Ans: a crystal plate that reduces by $^1/_2$ cycles the phase difference between the two components of polarized light traversing it — compares quarter-wave plate.

**22. Write the applications of polarization?**

Ans: **APPLICATIONS OF POLARISATION**

- Polaroid glasses are used to reduce the amount of light that is approachable to eye.
- A polarization is useful in receiving and transmitting wave signals. There has to be an aerial alignment of the polarized waves in the plane in order for them to be able to receive maximum signal.
- Laser is an outcome of polarization of waves.
- 3D movies are possible because of polarization of light.

## UNIT-I PHYSICAL OPTICS (WAVE OPTICS)

## PART-B

## ESSAY QUESTIONS WITH SOLUTIONS

### 1.1. INTERFERENCE – PRINCIPLE OF SUPERPOSITION- INTERFERENCE OF LIGHT-CONDITIONS FOR SUSTAINED INTERFERENCE

**1. Define interference? Write the conditions to get interference.**

Ans: Interference can be defined as the superposition of two or more coherent waves with each other such that the intensity modifies maxima and minima which results in a new wave pattern.

**Conditions for sustained interference**

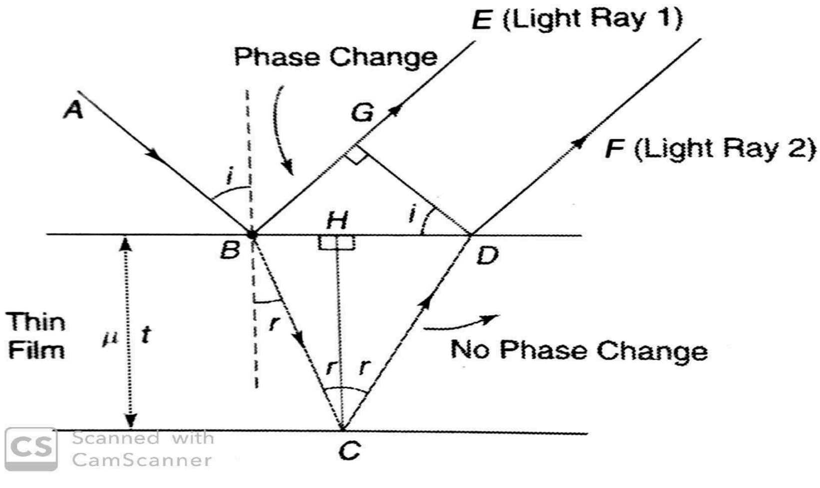
1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelength and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be large.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow.
8. The sources should be monochromatic.

### 1.1.2. INTERFERNCE IN THIN FILMS (REFLECTED LIGHT)

**2. Explain interference in thin film by reflection method.**

Ans: **Interference in thin film by reflection**

When light is incident on a plane parallel thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the lower surface and emerges through the upper surface. These reflected light beams superimpose with each other, producing interference and forming interference patterns.





Consider a transparent plane parallel thin film of thickness ‘t’ with refractive index μ. Let a monochromatic light ray AB be incident at an angle of incidence of ‘i’ on the upper surface of the film. BE and BC are the reflected and transmitted light rays. Let the angle of refraction is ‘r’. The ray BC will be reflected into the film and emerge through the film in the form of the light ray DF. These two light rays superimpose and depending upon path difference between them, they produce interference patterns.

To know the path difference, let us draw the normal DG and BE. From the points D and G onwards, the light rays travel equal distances. By the time the light ray travels from B to G, the transmitted light ray has to travel from B to C and C to D.

The path difference between light rays (1) & (2) is

Path difference = μ(BC+CD) in film – BG in air -----------□ (1)

Consider the ΔBCH, cos $cos\,r = \frac{HC}{BC}$

$$BC = \frac{HC}{\cos cos\,r} = \frac{t}{\cos cos\,r}$$

Similarly, from ΔDCH, CD = $\frac{t}{\cos cos\,r}$

To calculate BG, first BD which is equal to (BH+HD) has to be determined.

From ΔBHC, tan $tan\,r = \frac{BH}{CH} = \frac{BH}{t}$

BH = t tan $tan\,r$

Similarly, HD = t tan $tan\,r$

∴ BD = BH + HD = 2t tan $tan\,r$

From ΔBGD, sin $sin\,i = \frac{BG}{BD}$

BG = BD sin $sin\,i$

= 2t tan $tan\,r$ sin $sin\,i$

From Snell's law, sin $sin\,i$ = μ sin $sin\,r$

∴ BG = 2μt tan $tan\,r$ sin $sin\,r$

∴ From eqn(1), we have

Path difference = $\frac{2\mu t}{\cos cos\,r}$ - 2μt tan $tan\,r$ sin $sin\,r$

= 2μt cos $cos\,r$

At the point B, reflection occurs from the upper surface of the thin film. Light ray(1) undergoes an additional phase change of π or an additional path difference of λ/2.

Total path difference = 2μt cos $cos\,r$ + λ/2

When the path difference is equal to integral multiples of λ then the rays (1) & (2) meet in phase and undergoes constructive interference. The condition for bright fringe is 2μt cos $cos\,r$ = (2n-1)λ/2 where n = 0,1,2,3,………..

When the path difference is equal to half integral multiples of λ then the rays (1) & (2) meet out of phase and undergoes destructive interference. The condition for dark fringe is 2μt cos $cos\,r$ = nλ

Where n = 0, 1, 2, 3…

### 1.1.3. NEWTON'S RINGS- DETERMINATION OF WAVELENGTH

**3. Describe newton's rings theory and determine radius of curvature.**

**Ans:** **<u>Newton's ring</u>**

Newton's rings are one of the best examples for the interference in a non uniform thin film. When a planoconvex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and the glass plate are seen. These circular rings were discovered by Newton and are called Newton's ring.

**<u>Explanation of Newton's ring</u>**

A part of the incident monochromatic light AB is reflected at B in the form of the ray (1) with an additional phase (or path) change. The other part of the light is refracted through BC. Then at C, it is again reflected in the form of the ray (2) with additional phase change of $\pi$ or path change of $\lambda/2$.

As the rings are observed in the reflected light, the path difference between them is $2\mu t \cos cos\, r + \lambda/2$. For air film $\mu=1$ and for normal incidence $r=0$, path difference is $2t+ \lambda/2$.

At the point of contact $t=0$, path difference is $\lambda/2$. Then the incident and reflected lights are out of phase and interface destructively. Hence the central spot is dark.

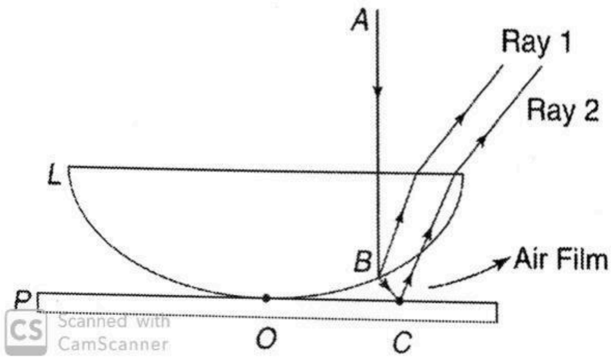
The condition for bright ring is $2t+ \lambda/2 = n\lambda$

$$2t = (2n-1)\lambda/2 \; ; \; n = 1,2,3,\ldots..$$

The condition for dark ring is $2t+ \lambda/2 = (2n+1)\lambda/2$

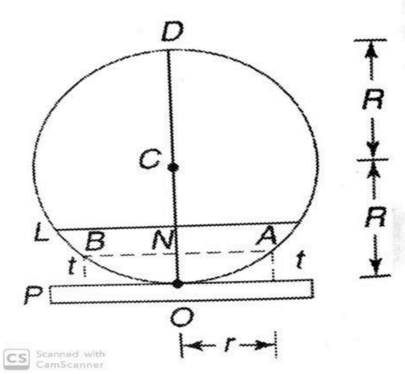
$$2t = n\lambda \; ; \; n = 0,1,2,3,\ldots..$$

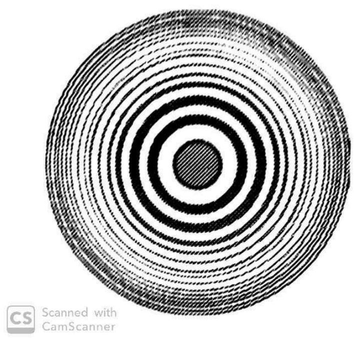
For monochromatic light, the bright and dark ring depends on thickness of the air film. For a Newton's rings system, the focus of points having same thickness lie on a circle having its centre at the point of contact. Thus, we get bright and dark circular rings with the point of contact as the centre.



**Theory of Newton's ring**

To find the diameters of dark and bright rings, let 'L' be a lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness 't'.



From the property of the circle, NA x NB = NO x ND

Substituting the value, r x r = t x (2R-t)

$$r^2 = 2Rt - t^2$$

As t is small, $t^2$ will be negligible

$$r^2 = 2Rt$$

$$t = \frac{r2}{2R}$$

For bright ring, the condition is

$$2t = (2n-1)\lambda/2$$

$$2\left(\frac{r2}{2R}\right) = (2n-1)\lambda/2$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by D/2, the diameter of $n^{th}$ bright ring will by

$$\frac{D^2}{4} = \frac{(2n-1)}{2}\lambda R$$

$$D = \sqrt{2n-1}\sqrt{2\lambda R}$$

$$D \propto \sqrt{2n-1}$$

$$D \propto \sqrt{odd\ natural\ numbers}$$

For dark ring, the condition is

$$2t = n\lambda$$

$$2\frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{natural\ number}$$

With increase in the order (n), the rings get closer and the fringe width decreases and is shown in fig.

**Determination of wavelength of a light source**

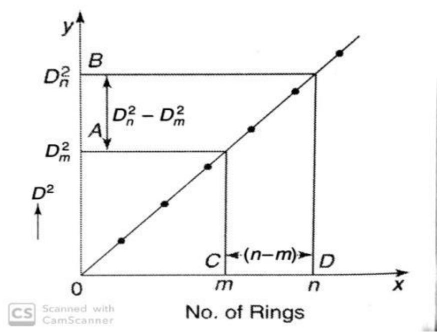
Let R be the radius of curvature of a planoconvex lens, λ be the wavelength of light used. Let $D_m$ and $D_n$ are the diameters of m^th^ and n^th^ dark rings respectively. Then

$$D_m^2 = 4m\lambda R \text{ and } D_n^2 = 4n\lambda R$$

$$D_n^2 - D_m^2 = 4(n-m)\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4\,(n-m)R}, \qquad R = \frac{D_n^2 - D_m^2}{4(n-m)\lambda}$$

Newton's rings are formed with suitable experimental setup. With the help of a travelling microscope, the readings for different orders of dark rings were noted from one edge of the rings to the other edge. The diameters of different orders of the rings can be known. A plot between $D^2$ and the no.of rings gives a straight line as shown in fig.




From the graph,

$$\frac{D_n^2 - D_m^2}{(n-m)} = \frac{AB}{CD}$$

The radius R of the planoconvex lens can be obtained with the help of a spherometer. Using these values in the formula, λ can be calculated.

## 1.1.4. ENGINEERING APPLIACTIONS OF INTERFERNCE

**4. (a) Write the conditions for to get interference.**

**(b) Write the engineering applications of interference**.

Ans: (a) Conditions for sustained interference

1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelength and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be large.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow.
8. The sources should be monochromatic.

**(b) APPLICATIONS**

1. Signal processing
2. Image processing
3. Heat distribution mapping
4. Wave simplification
5. Light simplification (Interference, Diffraction etc.)

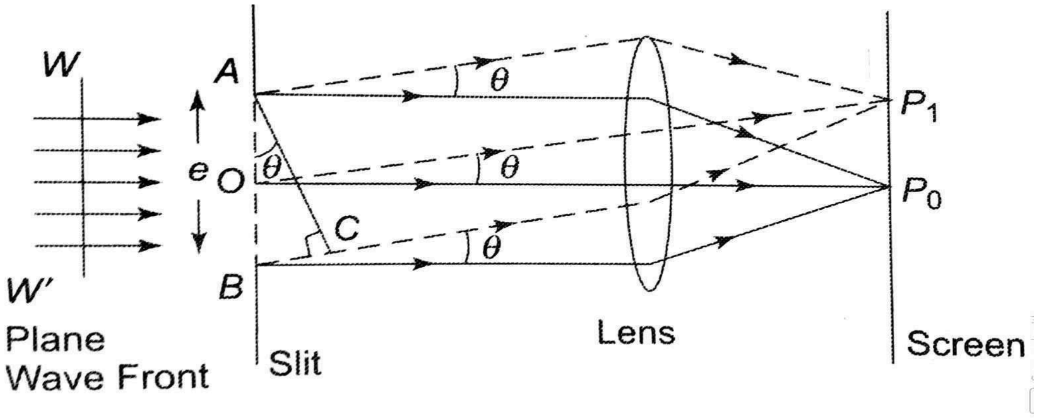
### 1.2. DIFFRACTION – FRAUNHOFER DIFFRACTION-SINGLE AND DOUBLE SLITS

**5. What is diffraction? Explain Fraunhofer diffraction due to single slit.**

**Ans:** The phenomenon of bending of light round the corners of obstacles and spreading of light waves into the geometrical shadow region of an obstacle placed in the path of light is called diffraction.

**<u>Fraunhofer diffraction at single slit</u>**

Consider a slit AB of width ‘e’. Let a plane wave front $WW^1$ of monochromatic light of wavelength $\lambda$ propagating normally towards the slit is incident on it. The diffracted light through the slit is focused by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygens-Fresnel, every point on the wave front in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit, i.e., along the direction $OP_o$ are brought to focus at $P_o$ by the lens. Thus, $P_o$ is a bright central image. The secondary wavelets travelling at an angle $\theta$ with the normal are focused at a point $P_1$ on the screen. Depending on path difference, the point $P_1$ may have maximum or minimum intensities. To find intensity at $P_1$, let us draw the normal AC from A to the light ray at B.




The path difference between the wavelets from A & B in the direction θ is given by

$$\text{Path difference} = BC = AB \sin sin\,\theta \;= e\sin sin\,\theta$$

$$\text{Corresponding phase difference} = \frac{2\pi}{\lambda} \text{ x path difference}$$

$$= \frac{2\pi}{\lambda} e\sin sin\,\theta$$

Let the width of the slit be divided into n equal parts and the amplitude of the wave from each part is ‘a’. The phase difference between any two successive waves from these parts would be

$$\frac{1}{n}\,[\text{total phase}] = \frac{1}{n}\,[\frac{2\pi}{\lambda} e\sin sin\,\theta\,] = d$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin sin\, d/2}{\sin sin\, d/2}$$

$$= a\,\frac{\sin sin\,(\pi e \sin sin\,\theta\,/\lambda)}{\sin sin\,(\pi e \sin sin\,\theta\,/n\lambda)}$$

$$= a\,\frac{\sin sin\,\alpha}{\sin sin\,\alpha/n} \quad \text{where } \alpha = \pi e \sin sin\,\theta\,/\lambda$$

$$= a\,\frac{\sin sin\,\alpha}{\alpha/n} \quad < \text{since } \alpha/n \text{ is very small} >$$

$$= na\,\frac{\sin sin\,\alpha}{\alpha}$$

$$= A\frac{\sin sin\,\alpha}{\alpha}$$

$$\text{Intensity} = I = R^2 = A^2\,(\frac{\sin sin\,\alpha}{\alpha})^2 \text{ --------------□(1)}$$

**Principal maximum**

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\;[\,\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \;\ldots\ldots\,]$$

$$= A\,[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\ldots]$$

I will be maximum, when the value of R is maximum. For maximum value of R, the negative terms must vanish i.e., $\propto\ = 0$

$$\pi e \sin sin\,\theta\,/\lambda = 0$$

$$\sin sin\,\theta\ = 0$$

$$\theta = 0$$

Then R = A

$$\therefore I_{max} = R^2 = A^2$$

The condition $\theta = 0$ means that the maximum intensity is formed at $P_o$ and is known as principal maximum.

**Minimum intensity position**

I will be minimum, when $\sin sin\,\alpha\ = 0$

$$\alpha = \pm\pi,\ \ \pm 2\pi,\ \ \pm 3\pi,\ \ \pm 4\pi, \ldots\ldots\ldots$$

$$\alpha = \pm\ m\pi$$

$$\pi e \sin sin\,\theta\,/\lambda = \pm\ m\pi$$

$$e \sin sin\,\theta\ = \pm m\lambda \text{ where } m = 1,2,3,\ldots\ldots\ldots$$

Thus, we obtain the points of minimum intensity on either side of the principal maximum. For m = 0, $\sin sin\,\theta\ = 0$, which corresponds to principal maximum.

**Secondary maxima**

In between these minima, we get secondary maxima. The positions can be obtained by differentiating the expression of I wrt α and equating to zero. We get

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\ [\ A^2 (\frac{\sin sin\,\alpha}{\alpha})^2\ ] = 0$$

$$A^2\, 2\, \frac{\sin sin\,\alpha}{\alpha}\, \frac{\alpha \cos cos\,\alpha - \sin sin\,\alpha}{\alpha^2} = 0$$

Either $\sin sin\,\alpha\ = 0$ or $\alpha \cos cos\,\alpha - \sin sin\,\alpha\ = 0$

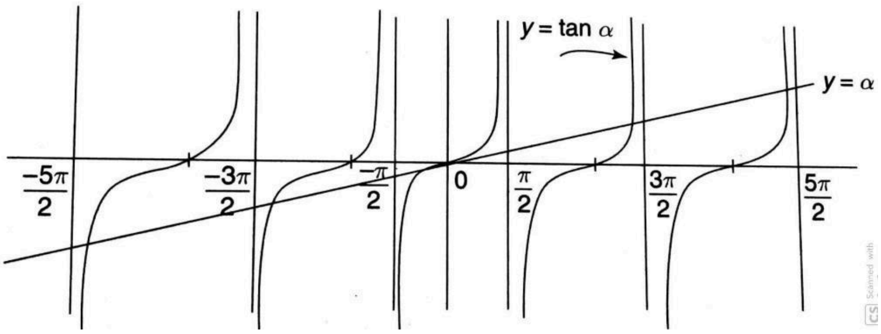
$\sin sin\,\alpha\ = 0$ gives position of minima

Hence the position of secondary maxima are given by

$$\alpha \cos cos\,\alpha - \sin sin\,\alpha\ = 0$$

$$\alpha = \tan tan\,\alpha$$

The values of α satisfying the above eqn are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan tan\, \alpha$ on the same graph. The points of intersection of the two curves gives the values of α which satisfy the above equation. The plots of $y = \alpha$ and $y = \tan tan\, \alpha$ are shown in fig.




The points of intersection are

$$\alpha = 0,\ \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots$$

Using the above values in eqn (1), we get the intensities in various maxima.

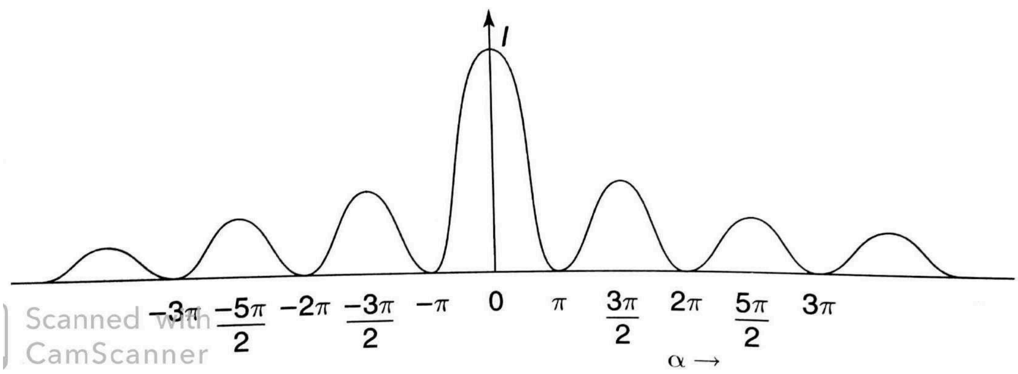
$\alpha = 0,\ I_o = A^2$ ( principal maximum)

$\alpha = \frac{3\pi}{2},\ I_1 = A^2\left[\frac{\sin sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2 \cong \frac{A^2}{22}$ ( 1st secondary maximum)

$\alpha = \frac{5\pi}{2},\ I_2 = A^2\left[\frac{\sin sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2 \cong \frac{A^2}{62}$ ( 2nd secondary maximum)

and so on

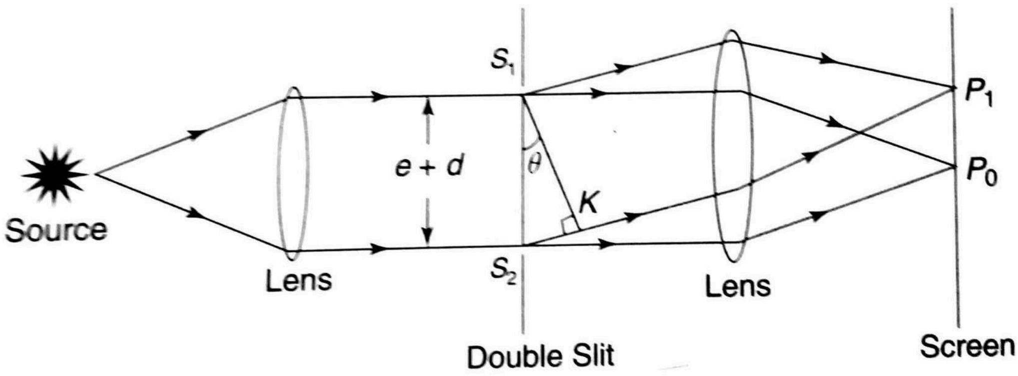
**Intensity distribution**




**6. Explain Fraunhofer diffraction due to double slit.**

**Ans: Fraunhofer diffraction at a double slits**

Let $S_1$ & $S_2$ be double slits of equal widths 'e' and separated by a distance d. the distance between the middle points of the two slits is (e+d). A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focused by a lens on the screen placed in the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference, the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superimposed. When a plane wavefront is incident normally on two slits, the secondary wavelets from the slits travel uniformly in all directions. The wavelets travelling in the direction of incident light come to a focus at $P_0$ while the wavelets travelling in a direction making an angle θ, come to focus at $P_1$.




From the study of diffraction due to single slit, the resultant amplitude = $A\frac{\sin sin\,\alpha}{\alpha}$ where $\alpha = \frac{\pi e \sin sin\,\theta}{\lambda}$. Since we use double slit, from each slit we get a wavelet of amplitude $A\frac{\sin sin\,\alpha}{\alpha}$ in a direction θ. These two wavelets interfere and meet at a point $P_1$ on the screen. To calculate the path difference between the wavelets, let us draw a normal $S_1K$ to the wavelets through $S_2$.

$$\text{Path diff} = S_2K$$

$$= (e+d)\sin sin\,\theta$$

$$\text{Phase diff} = \frac{2\pi}{\lambda}(e+d)\sin sin\,\theta = \delta$$

To find the resultant amplitude at $P_1$ we use vector addition method is which the two sides of a triangle are represented by the amplitudes through $S_1$ & $S_2$. The third side gives the resultant amplitude.

$$(OH)^2 = (OG)^2 + (GH)^2 + 2(OG)(GH)\cos cos\,\delta$$

$$R^2 = \left(A\frac{\sin sin\,\alpha}{\alpha}\right)^2 + \left(A\frac{\sin sin\,\alpha}{\alpha}\right)^2 + 2\left(A\frac{\sin sin\,\alpha}{\alpha}\right)\left(A\frac{\sin sin\,\alpha}{\alpha}\right)\cos cos\,\delta$$

$$= 2A^2\left(\frac{\sin sin\,\alpha}{\alpha}\right)^2[1 + \cos cos\,\delta\,]$$

$$= 2\left(A\frac{\sin sin\,\alpha}{\alpha}\right)^2(1 + 2cos^2\delta/2 - 1)$$

$$= 4A^2\left(\frac{\sin sin\,\alpha}{\alpha}\right)^{2cos^2\left(\frac{\pi(e+d)\sin sin\,\theta}{\lambda}\right)}$$

$$\text{Let } \beta = \frac{\pi(e+d)\sin sin\,\theta}{\lambda}$$

$$R^2 = 4A^2\left(\frac{\sin sin\,\alpha}{\alpha}\right)^{2cos^2\beta}$$

The resultant intensity $I = R^2 = 4A^2\left(\frac{\sin sin\,\alpha}{\alpha}\right)^{2cos^2\beta}$

From the above expression, it is clear that the resultant intensity is the product of two factors, i.e.,

1) $A^2(\frac{\sin\sin\alpha}{\alpha})^2$ which represents the diffraction pattern due to a single slit.

2) $cos^2\beta$ which gives the interference pattern due to wavelets from double slits. The resultant intensity is due to both diffraction and interference effects.

**Diffraction effect**

The diffraction term $A^2(\frac{\sin\sin\alpha}{\alpha})^2$ gives the principal maximum at the centre of the screen with alternate minima and secondary maxima of decreasing intensity. We get principal maxima for $\theta = 0$. We get minima for $\sin sin\,\alpha = 0$

$$\alpha = \pm\, m\pi$$

$$\pi e \sin sin\,\theta\,/\lambda = \pm\, m\pi$$

$$e \sin sin\,\theta = \pm m\lambda \text{ where } m = 1,2,3,\ldots\ldots$$

The position of secondary maxima occurs for

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots$$

**Interference effect**

The interference term $cos^2\beta$ gives the equidistant bright and dark fringes. The maxima will occur for $cos^2\beta = 1$

$$\beta = \pm\, n\pi$$

$$\pi(e + d) \sin sin\,\theta\,/\lambda = \pm\, n\pi$$

$$(e + d) \sin sin\,\theta = \pm n\lambda \text{ where } n = 0,1,2,3,\ldots\ldots$$

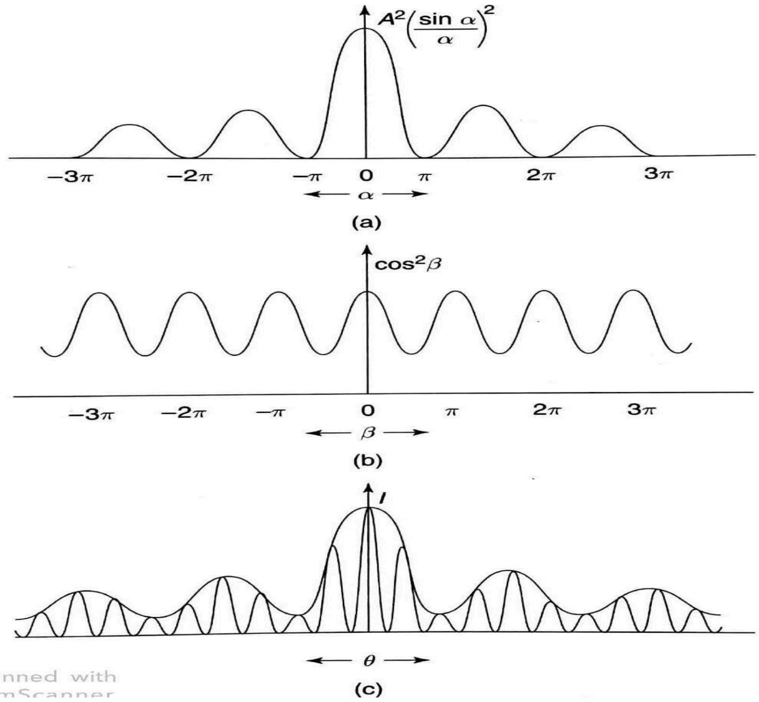
The minima will occur for $cos^2\beta = 0$

$$\beta = \pm(2n+1)\pi/2 \text{ where } n = 0,1,2,\ldots.$$

$$(e + d) \sin sin\,\theta = \pm(2n+1)\lambda/2$$

**Intensity distribution**

When both effects are combined then we get the resultant intensity variation. From fig, it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference effect.




## 1.2. DIFFRACTION GRATING- GRATING SPECTRUM

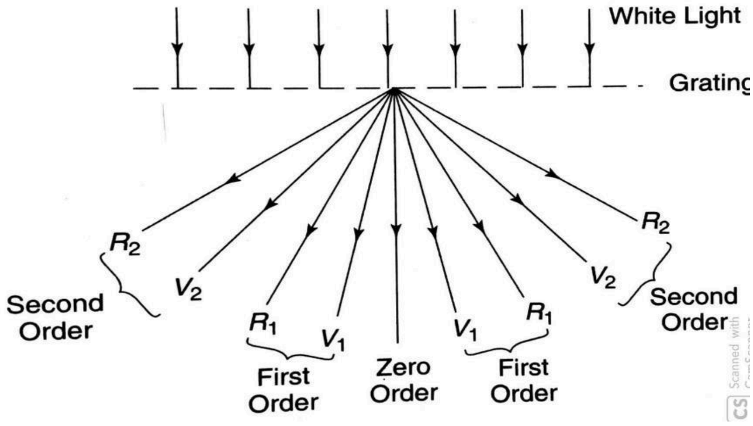
**7. Define grating spectrum? Discuss diffraction grating.**

**Ans:** **<u>Diffraction grating spectrum</u>**

When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as diffraction spectrum.

The condition to form the principal maxima in a grating is given by

$\theta = \pm m\lambda$




**Diffraction grating**

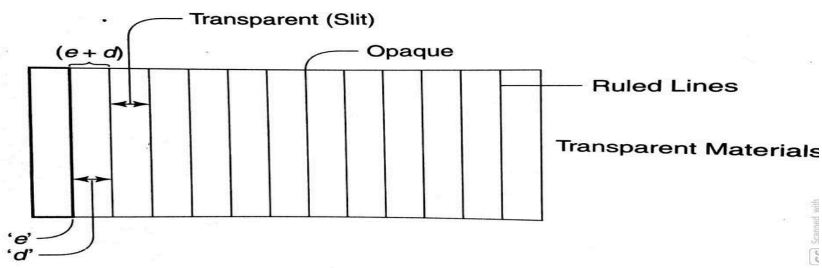
An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and act as a slit.

Let 'e' be the width of the line and 'd' be the width of the slit. Then (e+d) is known as grating element. If 'N' is the number of lines per inch on the grating then

$$N\,(e+d) = 1'' = 2.54 \text{ cm}$$

$$(e+d) = \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to wavelengths of light.




**1.2.2. DETERMINATION OF WAVELENGTH-ENGINEERING APPLICATIONS OF DIFFRACTION**

**8. Write engineering applications of diffraction.**

**Ans:**

Engineering applications of diffractions are:

1- X-ray diffraction used in crystallography.

2- Diffraction Grating used to transform the light into its spectrum.

3- The most interesting application is the holography (you can check this amazing video.

4- Diffraction is used in recording of 3D image using the process holography.

5- Diffraction is used in the study of DNA.

6- Ordinary CD's and DVD's are everyday examples of diffraction used to demonstrate the effect by reflecting sunlight.

7-The diffraction phenomenon is effectively used in explaining the structure of crystals

**9. Write the determination of wavelength of light?**

**Ans:** Principle:

When light is incident on a grating, it diffracts into its constituent wavelengths. Hence, we get a spectrum consisting of different series of spectral lines. When a plane diffraction grating is illuminated normally by a monochromatic beam of light of wavelength λ, the principal maximum are formed in the direction given by

$(a + b) \sin\theta = n \lambda$

Where ( a+b) is the grating element, n is the order of maxima and λ is the wavelength of the light. The grating element is measured directly from the number of rulings per inch written on the grating by manufacturer. If there are N lines per inch.

$$N(a+b) = \text{inch} = 2.54 \text{ cm}$$

$$(a+b) = 2.54/N$$

$$(2.54/N) \sin\theta = n\lambda$$

$$\lambda = \frac{2.54 sin\theta}{nN} = \frac{2.54\ sin\theta}{n\ x\ 15000}$$

Knowing the angle of diffraction (θ), the order of spectrum (n), and number of lines (N) wavelength of light can be calculated.

## 1.3.1. POLARIZATION – POLARIZATION BY DOUBLE REFRACTION

**10. Define polarizations? Discuss polarization by double refraction.**

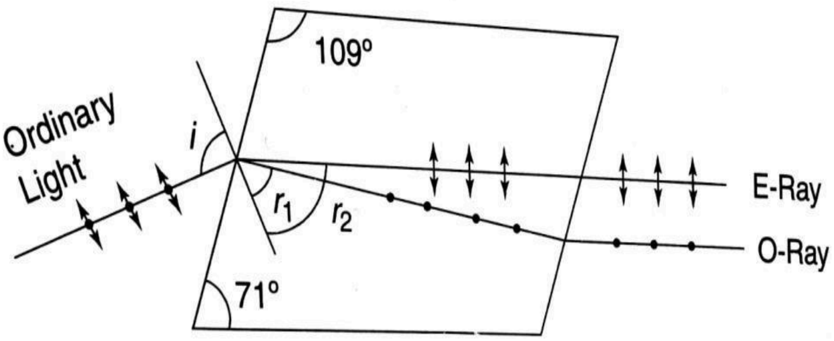
**Ans: <u>Polarization:</u>**

The experiments on interference and diffraction have shown that light is a form of wave motion. But these effects, do not give an idea about the type of wave motion. The phenomenon of polarization has helped to establish that light waves are transverse in nature.

**<u>Polarization by double refraction</u>**

In 1869, Bartholinus discovered that when a beam of unpolarized light incident on the uniscopic crystals light quartz, calcite. Then, the beam will split up into two refracted beams. This is known as double refraction.

Let us consider, a beam of unpolarized light incident on the calcite crystal at an angle of incidence 'i' as shown in fig.




Inside the crystal the ray breakup into O-ray and E- ray. The O-ray travels along BD and makes an angle of refraction $r_1$. The E-ray travels along BC and makes an angle of refraction $r_2$. The refractive indexes for ordinary rays and extraordinary ray can be written as

$$\mu_0 = \frac{\sin sin\, i}{sin r_1} \quad , \quad \mu_e = \frac{\sin sin\, i}{sin r_2}$$

Here, In the case of calcite $\mu_0 < \mu_e$ because $r_1 < r_2$. Here, the velocity of light for ordinary ray inside the crystal will be less than the extraordinary ray. It is observed that $\mu_e$ varies.

Therefore, ordinary ray travels with the same speed in all directions while the extraordinary ray has different speeds in difference directions.

### 1.3.2. NICOL'S PRISM- HALF WAVE AND QUARTER WAVE PLATES

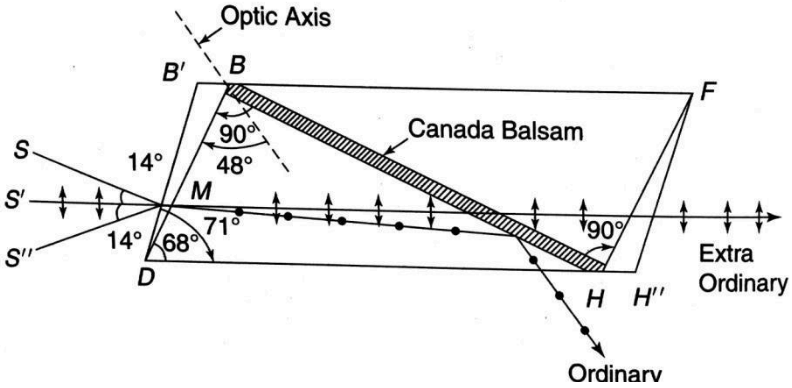
**11. Explain the construction and working of nicol's prism?**

**Ans: Principle:**

Nicol prism is an optical device which is used to produce and analyze the plane polarized light. This is invented by William nicol in 1828. This is made by double refracting calcite crystal. Nicol prism eliminated the ordinary beam by utilizing the phenomenon of total internal reflection at thick film of canadabalsam, separating the two pieces of calcite. This device is known as nicol prism.

**Construction:**

It is constructed from the calcite crystal PQRS having length 3 times of its width. Its end faces are PQ and RS are cut, such that the angles in the principle section become $68^0$ and $112^0$. In the place of $71^0$ and $109^0$. Here, the crystal is cut diagonally into two parts. The surfaces flat these parts are polished does the polished surfaces are connected together in special cement known as canadabalsam.




**Working:**

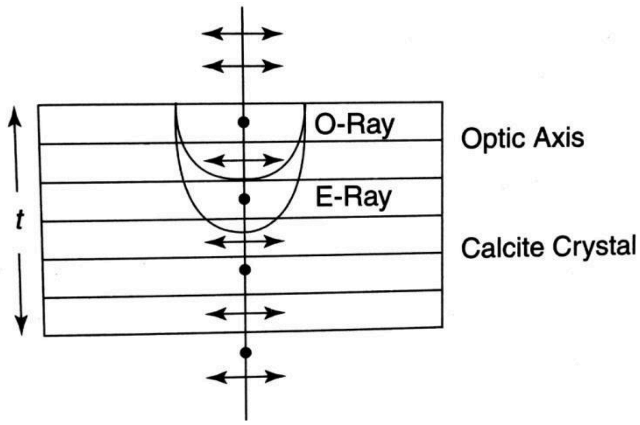
When a beam of unpolarised light is incident on the face PQ, it splits into two refracted rays named as ordinary ray (O- ray) and extraordinary ray (E-ray). These two rays are plane polarized rays whose vibrations are at right angles (transverse waves) to each other. The refractive rays of canadabalsam cement are 1.55 which lies refractive indexes of O-ray and E-ray.

It is clear the canadabalsam layer act as rarer medium for the O-ray and denser medium for E-ray. When O-ray of light travels in the calcite crystal and enters into canadabalsam layer, it passes from denser medium to rarer medium. Here, the angle of incidence is greater than the critical angle. Now, the incident ray is totally internally reflected from the crystal and only E-ray is transmitted through the prism. Therefore, fully polarized ray is generated with the help of Nicol prism.

**12. Discuss the quarter wave plate?**

**Ans: Quarter wave plate:**

A quarter wave plates is a device of double refracting uniaxial when plane polarized light is incident on calasite crystal the light splits up into O-ray and E-ray. Here, the velocity of light for E-ray is more than O-ray. A crystal plane which introduces a phase difference $\frac{\pi}{2}$ and path difference $\frac{\pi}{4}$ between O-ray and E-ray is called as 'Quarter ray plate'.




Let $\mu_0$ is the refractive index of O-ray, $\mu_e$ is the refractive index of E-ray, it is thickness of the crystal. Then the path difference

$$\Delta = \mu_0 t - \mu_e t$$

$\Delta = t(\mu_0 - \mu_e)$-------------------1

In quarter wave plate, the path difference between O-ray and E-ray.

$\Delta = \frac{\lambda}{4}$ ------------------------------2

Now,

Comparing equation 1 and 2

$$t(\mu_0 - \mu_e) = \frac{\lambda}{4}$$

$$\therefore \lambda = 4t(\mu_0 - \mu_e)$$

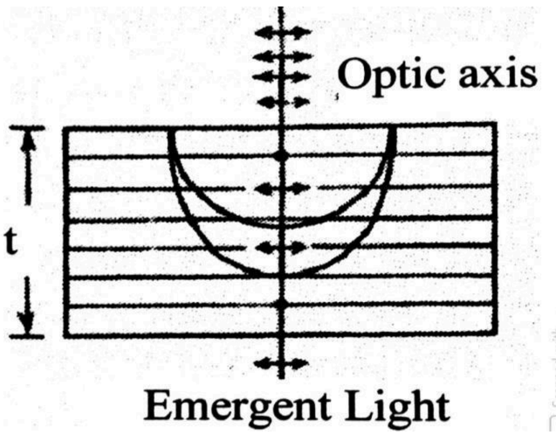
$$t = \frac{\lambda}{4(\mu 0 - \mu e)}$$

This is used to produce circularly and elliptically polarized light.

**13. Discuss the Half wave plate?**

**Ans: Half wave plate:**

A crystal plane which introduces a phase difference π path difference $\frac{\lambda}{2}$ between O-ray and E-ray is called 'half wave plate'.




Let $\mu_0$ is the refractive index of O-ray, $\mu_e$ is the refractive index of E-ray, it is thickness of the crystal. Then the path difference,

$\Delta = \mu_0 t - \mu_e t$

$\Delta = t(\mu_0 - \mu_e)$-------------------1

In half wave plate, the path difference between O-ray and E-ray.

$\Delta = \frac{\lambda}{2}$ ------------------------------2

Now,

Comparing equation 1 and 2

$t\,(\mu_0 - \mu_e) = \frac{\lambda}{2}$

$\therefore \lambda = 2t\,(\mu_0 - \mu_e)$

$t = \frac{\lambda}{2(\mu 0 - \mu e)}$

## 1.3.3. ENGINEERING APPLICATIONS

**14. Write the engineering applications of polarization?**

**Ans: Engineering applications of polarization:**

1. This is used in polarized sunglass.
2. This is used in photographic.
3. This is used in 3D movies.
4. This is used to impure the colour contrast.
5. This is used in liquid crystal display screens (LCD).
6. This is used for chemical analysis.
7. A polarization is useful in receiving and transmitting wave signals. There has to be an aerial alignment of the polarized waves in the plane in order for them to be able to receive maximum signal

**SOLVED PROBLEMS**

**1. *T*wo coherent sources of intensity 10 *w/m*$^2$ and 25 *w/m*$^2$ interfere to from fringes. Find the ratio of maximum intensity to minimum intensity.**

Solutions: We know the intensity ($I$) = $a^2$ [square of amplitude]

$$\text{on this bases } \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{10}{25}$$

$$\text{or} \quad \frac{a_1}{a_2} = \frac{3.1623}{5}$$

$$\text{i.e } a_1 = \left[\frac{3.1623}{5}\right] a_2 = 0.6324 a_2$$

Ratio of $I_{max}$ to $I_{min}$ is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left[0.6324\, a_2 + a_2\right]^2}{\left[06324\, a_2 - a_2\right]^2}$$

$$= \frac{\left(1.6324\, a_2\right)^2}{\left[-0.3675 a_2\right]^2} = \frac{2.6647}{0.1351} = 19.724$$

***2.*** **In a double slit experiment a light of *λ* = 5460 *Å* is exposed to slits which are 0.1 *mm* a part. The screen is placed 2 *m* away from the slits. What is the angular position of the 10*th* maximam and 1*st* minimum?**

Solutions: The given data are

Wave length of light ($\lambda$) = 5460 Å = $5460 \times 10^{-10}$ m

Separation between slits ($2d$) = 0.1 mm = $1 \times 10^{-4}$ m

Distance of screen ($D$) = 2 m

Angular position of 10th maximum ($\varnothing_{max\ 10}$) = ?

Angular position of 1st minimum ($\varnothing_{min\ 1}$) = ?

Condition for maximum intensity is $x_n = \dfrac{n\lambda D}{2d}$

$$\text{Distance of 10th maximum, } X_{max\ 10} = \frac{10 \times 5460 \times 10^{-10} \times 2}{1 \times 10^{-4}}$$

$$= 0.1092 \text{ m}$$

$$\therefore \tan\theta_{\max 10}\ \frac{0.1092}{2} = 0.0546 \text{ radians}$$

$$= 3°7'37''$$

Condition for minimum intensity is $x_n \dfrac{(2n-1)\lambda D}{4d}$

Distance of 1st minimum, $x_{\min 1} = \dfrac{\lambda D}{4d} = \dfrac{\lambda D}{2\times 2d} = \dfrac{5460\times 10^{-10}\times 2}{2\times 10^{-4}}$

$$= 5460\times 10^{-6}\text{ m}$$

Angular position of 1st minimum is $(\tan\phi_{\min 1}) = \dfrac{x_{\min 1}}{D}$

$$= \frac{5460\times 10^{-6}}{2} = 0.00273 \text{ radians} = 0.156° = 0°9'23''$$

**4. In a Newton's rings experiment the diameter of the 15th ring was found to be 0.59 cm and that of the 5th ring is 0.336 cm. If the radius of curvature of the lens is 100 cm, find the wave length of the light.**

**Solutions:** The given data are

Diameter of Newton's 15th ring ($D_{15}$) = 0.59 cm = $0.59\times 10^{-2}$ m

Diameter of Newton's 5th ring ($D_5$) = 0.336 cm = $0.336\times 10^{-2}$ m

Radius of curvature of lens ($R$) = 100 cm = 1 m

Wave length of light ($\lambda$) = ?

$$\lambda = \frac{D^2_{n+m} - D^2_n}{4mR} = \frac{(0.59\times 10^{-2})^2 - (0.336\times 10^{-2})^2}{4\times 10\times 1}$$

$$= \frac{0.3481\times 10^{-4} - 0.112896\times 10^{-4}}{40} = \frac{0.235204\times 10^{-4}}{40}$$

$$= 0.00588\times 10^{-4}\text{ m} = 5880\times 10^{-10}\text{ m} = 5880\text{ Å}$$

**5. Newton's rings are observed in the reflected light of wave length 5900 Å. The diameter of 10th dark ring is 0.5 cm. Find the radius of curvature of the lens used.**

**Sol:** The given data are

Wave length of light ($\lambda$) = 5900 Å= $5900\times 10^{-10}$ m

Diameter of 10th Newton's dark ring ($D_{10}$) = 0.5 cm = $0.5\times 10^{-2}$ m

Radius of curvature of lens ($R$) = ?

Formula is $D^2_n = 4n\lambda R$

$$\text{or } R = \frac{D_n^2}{4n\lambda} = \frac{(0.5\times10^{-2})^2}{4\times10\times5900\times10^{-10}}$$
$$= \frac{0.25\times10^{-4}}{236\times10^{-7}} = 1.059\text{ m}$$

**6. Calculate the thickness of air film at the 10th dark ring in a Newton's rings system, viewed normally by a reflected light of wave length 500 nm. The diameter of the 10th dark ring is 2 mm.**

**Sol:** Given data are

Wave length of light ($\lambda$) = 500 *nm* = $500 \times 10^{-9}$ *m*

Number of the dark ring viewed ($n$) = 10

Diameter of 10th dark ring ($D_{10}$) = 2 *mm* = $2 \times 10^{-3}$ m

Radius of 10th dark ring $(r_{10}) = \dfrac{D_{10}}{2} = 1\times10^{-3}$ m

Thickness of air film ($t$) = ?

Condition for dark ring is

$$D_n = \sqrt{4n\lambda R}$$

$$\text{or } D_n^2 = 4n\lambda R$$

$$\text{or } R = \frac{D_n^2}{4n\lambda} = \frac{(2\times10^{-3})^2}{4\times10\times500\times10^{-9}} = \frac{4\times10^{-6}}{40\times500\times10^{-9}} = 0.2\text{ m}$$

$$\text{Also } t = \frac{r_n^2}{2R} = \frac{(10^{-3})^2}{2\times0.2} = 2.5\times10^{-6}\text{ m} = 2.5\ \mu\text{m}$$

**7. Two slits separated by a distance of 0.2 mm are illuminated by a monochromatic light of wave length 550nm. Calculate the fringe width on a screen at a distance of 1 m from the slits.**

**Sol:** The given data are

Separation between the slits ($2d$) = 0.2 mm = $0.2 \times 10^{-3}$ m

Wave length of light ($\lambda$) = 550 nm = $550 \times 10^{-9}$ m

Distance of the screen ($D$) = 1 m

Fringe width ($\beta$) = ?

$$\beta = \frac{\lambda D}{2d} = \frac{500\times10^{-9}\times1}{0.2\times10^{-3}} = \frac{500\times10^{-6}}{0.2} = 2750\times10^{-6}\text{ m}$$
$$= 2.75\times10^{-3}\,\mathbf{m} = 2.75\text{ mm}$$

**8. Calculate the thickness of a quarter wave plate for a monochromatic light having a wavelength of 600nm, if the refractive indices of ordinary and extraordinary rays in the medium are 1.5442 and 1.533 respectively.**

**Solutions:** wavelength of the monochromatic light, $\lambda$= 600nm = 600 x $10^{-9}$m

$\mu_0$=1.5442

$\mu_e$ = 1.5533

We know that , $t = \frac{\lambda}{4(\mu_0 - \mu_e)}$

$= \frac{600\ x10^{-9}}{4(1.5533-1.5442)}$

= 16.84µm

= 0.0165mm

**9. A plane polarized light is incident on a crystal cut parallel to the axis. Find the least thickness for which the ordinary and extraordinary rays in the medium are 1.5533 and 1.5442. Wavelength of incident light is 500nm.**

**Solutions:** wavelength of the monochromatic light, $\lambda$= 500nm = 500 x $10^{-9}$m

$\mu_e$=1.5442

$\mu_o$ = 1.5533

We know that , $t = \frac{\lambda}{2(\mu_0 - \mu_e)}$

$= \frac{500\ x10^{-9}}{2(1.5533-1.5442)}$

= 27.47nm