Introduction:

I just want to start out with a brief introduction to this work, and how I read it. In the last Spaces, I covered Leibniz's historical context. I think going through Leibniz's historical context is important not because his thought depends on his historical context, or is not a timeless teaching, but for basically four reasons:

1.) There is a strong modernist & liberal prejudice in the scholarship on Leibniz
2.) There is a significant historical paradigm shift from Aristotelian science to modern science
3.) Leibniz's use of exoteric writing, or writing for the public according to popular opinion, which does not reflect the fullness of his views
4.) Leibniz's archive did not become publicly available until after World War II! And even today it is still not publicly available, nor are there many English translations.

I compare this to the situation regarding Aristotle's teachings. We did not have adequate translations in the Western world, into Latin, of some of Aristotle's most important works until the 1200's. Leibniz indeed had a more private teaching. And what I argued was that Leibniz, though on the surface appears to be a modernist or liberal, or to take up some modernist positions, upon a closer reading, and indeed upon a very close reading of his more private papers, we are able to discern what his true position is. The way that I would state Leibniz's inner or true position, is that he was an orthodox Aristotlian & Thomist, who sought to give a highly precise logical rigor to

Aristotelian-Thomistic science. This science is best understood by the idea of the interior life, or the unitive way, in Catholic thought. I do think that there are aspects of Leibniz's thought that are incompatible with Catholic thought, but those aspects are less important than his true teaching, just as aspects of Aristotle's thought were considered less important to his true teaching.

In this specific treatise that we are reading through, I hold that Leibniz offers an entirely Aristotelian foundations & philosophy of mathematics, in contrast to Descartes' dualist foundations of mathematics, which separates mathematical inquiry from practice. Leibniz sought to uphold the premodern idea of science, where Virtue, Justice, Beauty, Truth, Goodness, were the primary standard or the criterion of reasoning, and mathematical objects were secondary or derived from these. This is indeed what he proves in this essay, given a closer reading, as I will show.

What this means for us today, is that whereas our modern science almost entirely relies on technical instruments, and it defies our own organic experience of the world, our common sense, and the scientific technocracy requires our implicit or explicit "faith" in its findings, independent of our common sense or the common good, Leibniz's analysis of place, shows us the path to restoration of common sense and the common good, indeed the good life. He allows us to immediately see and touch with our own senses, the real & true science of organic experience, the natural law of reality, which is open and even in a sense relies on, wonder & to faith in God as well as cultivation of virtue and moral actions.

Ultimately what we will see in this essay is that in the fifth paragraph, Leibniz is able to offer an organic logical proof (or definition) for an orthodox understanding of Thomistic-Aristotelian form, in a way that is perfectly understandable to common sense & natural understanding, and it does not in any way close off the understanding of the forms from wonder & faith, but shows the utter harmony of reason & faith, of science & beauty, of philosophy & theology, of logic & poetry.

Comments on "On Analysis Situs" (1679)
First Paragraph:

What is commonly known as *mathematical analysis* is analysis of *magnitude*, not of *situation*, and as such it pertains directly and immediately to arithmetic but is applicable to geometry in only an indirect sense.

He opens with the phrase "what is commonly known" which suggests that in this treatise he is speaking about opinion vs. truth.

What is *commonly known* as mathematical analysis is not mathematical analysis.

Magnitude and arithmetic are what is commonly known as mathematical analysis, but geometry and situation are what is mathematical analysis in reality.

He makes a comparison: as magnitude is to situation, so arithmetic is to geometry, or to put it another way, as magnitude is to arithmetic, so situation is to geometry.

Magnitude is defined in Euclid's *Elements* Book V

And also is dealt with by Aristotle's *Physics* Book IV, 1 - 6

Aristotle's *Physics* Book IV deals with the question of *place*, which is arguably the central question of this treatise - *situs* means *place*

What is commonly known as mathematical analysis pertains directly to arithmetic,

But it is applicable to geometry only in an *indirect* sense

This reiterates the point, that he is making a distinction between what is commonly known versus what is true, as the truth is often only known implicitly or indirectly.

The result is that many things easily become clear through a consideration of situation, which the algebraic calculus shows only with greater difficulty.

"Many things become clear through a consideration of *situation.*"

Situation is being considered in its own right.

We would often consider situation to be particular, Leibniz is treating situation *as such*, as a general principle.

He is working within the tradition of Aristotle, who shows how the forms were integrated with experience, rather than separate from it.

He is working specifically within the tradition of Book VI of the *Physics*.

"Which the algebraic calculus shows only with greater difficulty" – Leibniz is criticizing his own method of calculus, that he discovered four years earlier!

Why? Because algebraic calculus is Cartesian in its foundations, it deals with the abstract coordinate plane, we will see this unfold over the course of the essay

To reduce geometric problems to algebra, i.e., to reduce problems determined by figures to equation, is often a rather prolonged affair, and further complications and difficulties are necessary to return form the equation to the construction, from algebra back to geometry.

Leibniz is critical of reducing geometric problems to algebra

Because it reduces problems determined by figures to an equation

Which is a rather prolonged affair

And further complications and difficulties are necessary to return form the equation to the construction

He is saying that the algebraic calculus is cumbersome and tedious, and that there is an easier way to accomplish the same thing

Often, too, the constructions produced in this way are not entirely appropriate, unless we are lucky enough to stumble upon unforeseen postulates and assumptions.

This is a very curious sentence.

Postulates and assumptions are the starting point for mathematical inquiries.

He is talking about "stumbling upon" them after the fact.

It is uncanny, and impossible not to notice how he seems to be invoking St. Thomas' a posterior demonstration of the existence of God, proceeding from effects (what is better known to man) to causes (what is better known to God), and that he seems to be looking at the second question from St. Thomas's *Summa*, "Whether sacred doctrine is a science?" St. Thomas says that sacred doctrine is a science because it "*proceeds from the*

light of a higher principle, namely the science of God and the blessed." If we read the passage in full:

"Sacred doctrine is a science. We must bear in mind that there are two kinds of sciences. There are some which proceed from a principle known by the natural light of intelligence, such as arithmetic and geometry and the like. There are some which proceed from principles known by the light of a higher science... just as the musician accepts on authority the principles taught him by the mathematician, so sacred science is established on principles revealed by God." (*Summa P1.Q1.A2)*

St. Thomas in this question invokes geometry & arithmetic as proceeding from higher sciences, and analogizes them to the *sacred science* that proceeds from the light of a higher principle, the science of God, which is exactly what Leibniz is doing in this treatise!

This Descartes himself tacitly admitted in solving a certain problem of Pappus in Book III of his *Geometry*.

I believe what Leibniz is referring to occurs at the very end of Book III of the *Geometry*, where Descartes says the following:

"It should be remarked however, that in many of these problems it may happen that the circle cuts the parabola of the second class so obliquely that it is hard to determine the exact

point of intersection. In such cases this construction is not of practical value. The difficulty could easily be overcome by forming other rules analogous to these, which might be done in a thousand different ways... it is only necessary to follow the same general method to construct all problems, more and more complex, ad infinitum; for in the case of mathematical progression, whenever the first two or three terms are given, it is easy to find the rest. I hope that posterity will judge me kindly, not only as to the things which I have explained, but also as to those which I have intentionally omitted so as to leave to others the pleasure of discovery."

Basically Descartes is saying that his mathematical analysis depends on axioms - 'a general method' - that he himself doesn't know; Descartes is only giving *examples* of his method, through his algebraic constructions, not giving the method itself.

He says that *"it is only necessary to follow the same general method to construct all problems"* and that *"whenever the first two or three terms are given, it is easy to find the rest,"* and yet he is also saying that *"the construction is not of practical value"* and that *"the difficulty can be easily overcome by forming other rules analogous to these, which might be done in a thousand different ways."* and it gets *"more and more complex."*

Leibniz uses the word 'tacitly' to indicate Descartes' suggestion that he *"intentionally omitted"* certain things.

In other words, with this sentence, Leibniz is saying that he is looking for unforeseen axioms, the unknown postulates that begin mathematical reasoning.

In other words, he is looking for the true foundations of mathematical analysis, not the philosophical foundations as Descartes has laid them out.

It brings us back to the prior sentence, about unforeseen postulates and assumptions, and one wonders whether the postulate and assumption that he is looking for, is God!

In fact algebra, whether using numbers or symbols, adds, subtracts, multiplies, divides, extracts roots, all of which are arithmetical.

Going back to the first sentence, he shows the analogy of magnitude with arithmetic

Cartesian analysis reduces magnitudes to equations, which is a prolonged affair

We have to consider what he means by arithmetical, and what he means by *magnitude*

Magnitude is a fundamental notion, it has something do with the indivisible or continuous

Again, there is magnitude as treated by Aristotle, and magnitude as treated by Euclid

We will settle on saying that by magnitude he means something like indivisible shape, though it may not be entirely accurate

The comparison of magnitude to arithmetic is something like the comparison of the continuous to the discrete

For *logistics* itself, or the science of magnitude and proportion in general, deals only with general or indeterminate number and with the species of operations performed on it, since *magnitude* is in fact measured by the number of determinate parts, yet this number may vary for the same fixed thing, depending upon which measure or unit is assumed.

His use of the term "logistics" here, in distinction to his earlier use of "algebraic calculus" is notable, because he is subtly differentiating his own mathematical analysis from Descartes mathematical analysis

Particularly as he defines logistics as the "science of magnitude and proportion in general"

Again it is important to understand what he means by magnitude. The Fifth book of Euclid's *Elements*, begins with a definition of magnitude. Euclid's definition is redundant, he uses the term magnitude to define itself:

- ***Definition 1***

- *A magnitude is a part of a magnitude, the less of the greater, when it measures the greater.*
- ***Definition 2***
- *The greater is a multiple of the less when it is measured by the less.*
- ***Definition 3***
- *A ratio is a sort of relation in respect of size between two magnitudes of the same kind.*
- ***Definition 4***
- *Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another.*

And again, we should look at how Aristotle treats the problem of magnitude & place in Book VI of the *Physics* (Hippocrates G. Apostle translation):

"(d) Further, of which things would one posit a place to be a cause? For it can be no cause in any of the four senses of "cause," whether as matter of things (for no thing consists of it), or as form or formula of things or as end or as a mover of things. (e) Again, if a place is itself a being, where will it be? Thus, Zeno's difficulty must be met with an argument; for if every thing is in a place, clearly a place too is in a place, and this goes on to infinity. If, then, the place of each body is what primarily contains it, it would be a boundary; so it would seem that the place of each body is its form or shape, by which the magnitude or the matter of the magnitude is bounded, for this is the boundary of each. If we view the problem in this manner, then, a place would be the form of a body; but insofar as a place is regarded as the

interval of the magnitude, it would be the matter of a body (for this is distinct from the magnitude), and this is what is contained and limited by the form, as by a surface or a boundary.

He concludes that place is neither matter nor form,

"Again, if a place were matter or a form, how would a thing travel to its own place? Well then, if a place is none of the three, that is, neither the form nor the matter nor yet a sort of interval always present with but distinct from the thing which is displaced, it must be what remains of the four alternatives, namely, the containing body's boundary which is in contact with what is contained (and by "the contained body" I mean the one that is movable with respect to locomotion). A place is thought to be an object of importance and difficult to grasp, both because it appears to be present with the matter and the shape [of a thing] and because the displacement of a body in locomotion occurs in a container which is at rest; for within [the inner boundary of the container] there appears to exist an interval distinct from the magnitudes in motion. A place, then, is this, namely, the primary motionless boundary of that which contains. And it is in view of this that the Center of the heaven and the last [inner surface] of the rotating parts.... stays in the same condition.

Basically it is safe to say that magnitudes are something like continuous & indivisible, but nevertheless measurable according to a given criterion; this is more or less what he is saying in this sentence

The science of magnitude and proportion in general would refer to the "general method" that Descartes speaks about *tacitly*, but does not give explicitly

The science of magnitude and proportion in general, deals only with general or indeterminate number and with the species of operation performed on it

It deals with *variables* and the specific operations by which they are transposed or rearranged

Since magnitude is in fact measured by the number of determinate parts, yet this number may vary for the same fixed thing, depending on which measure or unit is assumed

In other words, the criterion of measure matters, the beginning postulates and assumptions matter

They determine how the magnitude is measured into its determinate parts, though the magnitude itself may be continuous

Again this emphasis that the beginning postulates or criterion are significant, is suggestive of the idea that God is the beginning postulate or criterion

It is not surprising, therefore, that the science of magnitude in general is a kind of arithmetic, since it deals with indeterminate numbers.

It is notable that he says “the science of magnitude in general” whereas before he had said “the science of magnitude *and proportion* in general”

This indicates, again, that he is distinguishing his teaching from Descartes and form the algebraic calculus.

The point that “it is not surprising” is humorous in relation to his earlier point “unless we are lucky enough to stumble upon unforeseen postulates”

The science of magnitude in general does include *proportion*, and so it is not surprising that it is a kind of arithmetic, and it deals with indeterminate numbers because in the

Cartesian analysis the criterion or the measure or the foundational philosophy of mathematics is unstated.

Second Paragraph:

The ancients had another kind of analysis, different from algebra, which was concerned rather with considering situation.

The ancients had another kind of analytic geometry

Algebra was from the Arabs

The ancient analytic geometry was concerned with considering situation

It deals with *data* and with *positions* of unknown entities and their *loci*.

There are three main considerations here:

- Data means the *given* points
- *Positions* of unknown entities are unknown points
- The *loci* is a range where the relation is preserved between the given and the positions of unknown points

For example, if I have a given point A and a given point B

Then I establish an unknown point Y that is *congruent* to B

Then the resulting *loci* is a sphere

The *loci* is like a clean sweep across all unknown points Y that preserve the same relation A:Y as A:B

This is the trend of Euclid's book *De datis*, on which there exists a commentary by Marinus.

Marinus succeeded Proclus as the head of the neoplatonic school

There are ten definitions of *given* or *data* given by Marinus

- Agnoston - not knowable
- Algon - irrational
- Aporon - not available
- Atakton - ambiguous
- Ek duo onomaton - with two names
- Gnorimon - known
- Onomaton - one word definition
- Porimon - available
- Rheton - rational
- Tetagmenon - orderly

The conclusion reached by Marinus in this essay is that the proper definition of *data* or the given is as follows:

"*The only people, then, that seem to reach the notion of the given are those who declare it to be both gnorimon and porimon."* That is - both known and available.

And earlier, he states the precise relationship between *gnorimon* and *porimon*:

"The similarity between gnorimon (known) and porimon (available) is easy for anyone to distinguish, but the difference is hard to grasp. For by nature they are close to one another, so that they seem to be coextensive. But even here a certain inherent different will be seen if we observe them accurately. For it is evident and gnorimon (known) that there is [precisely] one tangent to a spiral from a singl point, but for all that the problem is not now porimon (available) as well, since it has not yet been grasped. And so everything that is porimon is also gnorimon. Therefore the gnorimon extends more widely than the porimon."

Apollonius particularly, but others as well dealt with plane, solid, and linear loci; more recent thinkers have reconstructed the doctrine of plane and solid loci from his propositions as preserved by Pappus, but in such a way as to show merely the truth rather than the source of the ancient doctrine.

Yet this kind of analysis does not reduce the matter to a calculation, nor is it carried through to the first principles and elements of situation, as its necessary for a perfect analysis.