# MIDDLETOWN TOWNSHIP PUBLIC SCHOOLS

## Multivariable Calculus - Grade 12
**(Full year Course)**

Written October 2021 - February 2022
Adopted by the Board of Education on May 31, 2022


Written by:

*Kevin Hickey*
*Kara Teehan*

Dan Kushner - Director of Mathematics and Science, K-12

Jessica L. Alfone- Assistant Superintendent for Curriculum and Instruction
Mary Ellen Walker - Superintendent of Schools

**MIDDLETOWN TOWNSHIP BOARD OF EDUCATION**
**EQUAL OPPORTUNITY POLICIES**

The Middletown Township Board of Education affirms its responsibilities to ensure all students in the public schools of this township equal educational opportunity regardless of race, color, creed, religion, sex, ancestry, national origin or social or economic status. Lack of English language skills will not be a deterrent to admission to any program. No otherwise qualified handicapped individuals shall solely by reason of their handicap be denied the benefits of or subjected to discrimination in any activity.

The school system's Affirmative Action Plans for School/Classroom Practices are on file in the Superintendent's office.

**AFFIRMATIVE ACTION GRIEVANCE PROCEDURE**

The Board of Education has established a procedure for staff, students or parents on a student's behalf to follow in filing a complaint dealing with alleged violation, misinterpretation or inequitable application of the policies and practices of the school district relative to provisions of Federal and State anti-discrimination legislation. Details of the grievance procedures are included in the school district's policy manual under Policy #2260.

The Building Principal or designee serves as the first step of this grievance procedure.

The District Affirmative Action Officer is:

Charlene O'Hagan, District Director of Human Resources and Instruction
Middletown Township Board of Education
August T. Miner Administrative Offices
834 Leonardville Road, 2nd Floor
Middletown, New Jersey 07737
(732) 671-3850

The District 504 Compliance Officer is:

Michele Tiedemann, District Director of Special Education
Middletown Township Board of Education
August T. Miner Administrative Offices
834 Leonardville Road, 2nd Floor
Middletown, New Jersey 07737
(732) 671-3850

## DISTRICT PHILOSOPHY OF INSTRUCTION

In order to prepare our students for the ever-increasing demand for a literate, technology-oriented workforce, Middletown Township Public School District embraces an instructional philosophy that is student-centered, inquiry-based, and that differentiates instruction based on student's individual abilities.

Teachers, as facilitators of lifelong learning, challenge students by providing an environment in which the students become active participants engaged in working together on projects and in solving problems that involve or simulate authentic data and events.

Students learn to value a variety of different approaches and are taught to take responsibility for their own meaningful learning as they become more adept at communicating their reasoning and in asking questions to help clarify their thinking and that of their classmates.

## COURSE PHILOSOPHY

All lessons will be taught with an emphasis on four main areas. The first of these will be problem-solving. The primary goal of all mathematics courses is problem-solving. Students will be encouraged to develop their skills so they may be efficient and creative problem solvers. Many exercises will be used to lead students through problem-solving strategies as well as ask for interpretations of the concepts presented. Students will be required to solve many problems in a variety of ways. The solutions will be numerical, graphical and analytical.

Since graphing calculators are readily available, our second main area of emphasis will be to incorporate and integrate technology and we will include Computer Algebra Systems in the instruction. The students will be encouraged to use this technology as a tool for exploration and discovery. It will be an alternate approach to reinforce concepts and serve as an efficient problem-solving tool. The students will also be made aware of the possible misuse of this technology and the need for a total understanding of the concepts presented.

The third important area of emphasis is to continue to help the student with the development of his communication skills. The presentation and assigned problems will incorporate the need for the student to interpret, describe and discuss the finding.

Almost all the early development of calculus was written in the context of real-life problem-solving. The fourth area of emphasis will be the renewed dedication to expose the student to real-life applications of calculus. A major focus of the course will be on applications and will make use of discovery and laboratory projects.

## INTRODUCTION

The goal of the New Jersey Student Learning Standards is to provide consistent standards and prepare students for college and careers, so that when they graduate they will have met benchmarks that will allow them to succeed in college or the 21st century workforce regardless of where in the state they have lived. The curriculum is aligned to the New Jersey Student Learning Standards developed by the New Jersey Department of Education to guide districts as they design curriculum that will support the work of teachers and promote student achievement.

This course is run through Seton Hall University's Project Acceleration and is designed for students who have successfully completed Advanced Placement Calculus BC and is weighted as an AP class as well. It is intended for students who have demonstrated thorough knowledge of

Calculus I and Calculus II. The course expands upon single variable calculus while covering topics in more than one variable including vectors and matrices, parametric curves, partial derivatives, double and triple integrals, and vector calculus in two and three dimensional space. All topics are presented using multiple representations with the use of a graphing calculator. Topics are represented graphically, numerically, algebraically, and verbally.
Prerequisites: Successful completion of AP Calculus BC, teacher recommendation, and a 3 or higher on the AP Calculus BC exam. This will be confirmed over the summer.
Note: This is a Dual Enrollment course and is affiliated with a college/university. Students who enroll in this course are responsible for the tuition as required by each college/university. If applicable, Middletown's teachers have been approved by the respective college/university to teach dual enrollment courses.

| New Jersey Student Learning Standards | 21st Century Themes |
|---|---|
| NJSLS that apply to the course<br>https://www.state.nj.us/education/cccs/ | Summary of NJDOE 21st Century Themes<br>https://www.state.nj.us/education/aps/cccs/career/ |

This course covers vector and multivariable calculus and is the second semester in the MIT freshman calculus sequence, which is used as a model course. Topics include vectors and matrices, parametric curves, partial derivatives, double and triple integrals, and vector calculus in 2- and 3-space.

As its name suggests, multivariable calculus is the extension of calculus to more than one variable. That is, in single variable calculus you study functions of a single independent variable. In multivariable calculus we study functions of two or more independent variables. These functions are interesting in their own right, but they are also essential for describing the physical world.

Many things depend on more than one independent variable. Here are just a few:

1. In thermodynamics pressure depends on volume and temperature.
2. In electricity and magnetism, the magnetic and electric fields are functions of the three space variables $(x,y,z)$ and one time variable $t$.
3. In economics, functions can depend on a large number of independent variables, e.g., a manufacturer's cost might depend on the prices of 27 different commodities.
4. In modeling fluid or heat flow the velocity field depends on position and time.

Single variable calculus is a highly geometric subject and multivariable calculus is the same, maybe even more so. In your calculus class you studied the graphs of functions $y=f(x)$ and learned to relate derivatives and integrals to these graphs. In this course we will also study graphs and relate them to derivatives and integrals. One key difference is that more variables means more geometric dimensions. This makes visualization of graphs both harder and more rewarding and useful.

By the end of the course you will know how to differentiate and integrate functions of several variables. In single variable calculus the Fundamental Theorem of Calculus relates derivatives to integrals. We will see something similar in multivariable calculus and the capstone to the course will be the three theorems (Green's, Stokes' and Gauss') that do this.

After completing this course, students should have developed a clear understanding of the fundamental concepts of multivariable calculus and a range of skills allowing them to work effectively with the concepts.

The basic concepts are:

1. Derivatives as rates of change, computed as a limit of ratios
2. Integrals as a 'sum,' computed as a limit of Riemann sums

The skills include:

1. Fluency with vector operations, including vector proofs and the ability to translate back and forth among the various ways to describe geometric properties, namely, in pictures, in words, in vector notation, and in coordinate notation.
2. Fluency with matrix algebra, including the ability to put systems of linear equation in matrix format and solve them using matrix multiplication and the matrix inverse.
3. An understanding of a parametric curve as a trajectory described by a position vector; the ability to find parametric equations of a curve and to compute its velocity and acceleration vectors.
4. A comprehensive understanding of the gradient, including its relationship to level curves (or surfaces), directional derivatives, and linear approximation.
5. The ability to compute derivatives using the chain rule or total differentials.
6. The ability to set up and solve optimization problems involving several variables, with or without constraints.
7. An understanding of line integrals for work and flux, surface integrals for flux, general surface integrals and volume integrals. Also, an understanding of the physical interpretation of these integrals.
8. The ability to set up and compute multiple integrals in rectangular, polar, cylindrical and spherical coordinates.
9. The ability to change variables in multiple integrals.
10. An understanding of the major theorems (Green's, Stokes', Gauss') of the course and of some physical applications of these theorems.

Curriculum units for all grade levels follow the same format and include the following:
• The NJSLS that are aligned with the unit, including 21st Century Themes
• Enduring Understandings
• Essential Questions
• Student Learning Outcomes
• Suggested Resources/Materials

The recommended activities section on the following pages includes activities for the purpose of differentiating instruction to meet the needs of special education students, English language learners, students at risk of school failure, and gifted students. The following are suggested modifications for teachers to use in each unit as appropriate:

- Structure lessons around questions that are authentic, relate to students' interests, social/family background and knowledge of their community.
- Provide students with choices including multiple options for how they can represent their understandings (e.g., conversations via digital tools such as SKYPE, experts from the community helping with a project, journal articles, biographies, multisensory techniques --auditory/visual aids: pictures, illustrations, graphs, charts, data tables, multimedia, modeling; etc.).
- Provide multiple grouping opportunities for students to share their ideas and to encourage work among various backgrounds and cultures (e.g. multiple representation and multimodal experiences).
- Engage students with a variety of Science and Engineering practices to provide students with multiple entry points and multiple ways to demonstrate their understandings. (Science specific)
- Use project-based learning to help students engage with content in an authentic way.
- Structure learning around explaining or solving a social or community-based issue.
- Provide ELL students with multiple literacy strategies.
- Collaborate with after-school programs or clubs to extend learning opportunities.

The district's expectation is for **ALL** teachers planning instruction for students with IEP's to thoroughly read and implement modifications and accommodations accordingly and consult with co-teacher.

## ADOPTED TEXTBOOK/PROGRAM and RESOURCES

**Calculus Early Transcendentals,** Ninth Edition, © 2021 Stewart, Clegg, Watson

Resources include an online homework system with ebook access for each student, a set of classroom textbooks for each classroom per teacher. Teachers will have access to a Google Team Drive which includes MIT resources and supplementary assessments, projects, lessons, district quarterlies, activities, and technology links.

# Recommended Time Frame and Sequence

(including Quarterly Assessments)

**1st Marking Period:**

Unit 1 - Parametric Equations and Polar Coordinates (9 blocks)
Unit 2 - Vectors and the Geometry of Space (13 blocks)

**2nd Marking Period:**

Unit 3 - Vector Functions (10 blocks)
Unit 4 - Partial Derivatives (12 blocks)

**3rd Marking Period:**

Unit 5 - Multiple Integrals: Double Integrals and Their Applications (13 blocks)
Unit 6 - Multiple Integrals: Triple Integrals (9 blocks)

**4th Marking Period:**

Unit 7 - Vector Calculus: Line Integrals and Surface Integrals (11 blocks)
Unit 8 - Vector Calculus: Green's, Stokes', and Divergence Theorems (11 blocks)

| **Unit 1 - Parametric Equations and Polar Coordinates** | **Duration** |
|---|---|
| Summary: Building on the foundation of plane curves where y is a function of x or x is a function of y or y is implicitly defined as a function of x, this unit introduces two new methods for describing curves. Some curves, such as the cycloid, are best handled when both x and y are given in terms of a third variable t, called the parameter. Other curves, such as the cardioid, have their most convenient description when we use a new coordinate system, called the polar coordinate system. We can use calculus to compute motion, arc lengths, and areas for polar and parametric equations. | 9 Blocks |

| **Enduring Understandings** | **Essential Questions** |
|---|---|
| ● Functions and relations can be represented using vectors, parametric equations, and polar coordinates.<br>● Vectors, parametric equations, and polar coordinates are useful in solving real-world problems.<br>● Parametric equations provide more information about mathematical relations over time.<br>● Vectors are used to describe quantities that have both magnitude and direction.<br>● Polar equations may be more useful in certain situations than rectangular equations. | ● Why are functions and relations represented by parametric equations?<br>● Why are functions and relations represented by polar equations?<br>● What are some advantages of using parametric equations for graphing?<br>● How do you find the length of a curve given in parametric form?<br>● How do you find the area enclosed by a polar curve?<br>● What considerations are needed when analyzing planar curves given in polar form?<br>● How can technology help when investigating graphs of parametric and polar curves?<br>● How do you compute velocity and acceleration of vector-valued functions?<br>● What advantages exist when using parametric, polar, and vector functions to analyze curves? |

| **NJSLS for Unit** |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |
| |

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br>● Use parametric equations for plane curves and space curves (10.1).<br>● Use and convert between parametric and symmetric equations for a straight line (10.1).<br>● Find a tangent line at a point on a parametric curve; compute the length of a parametric curve (10.2).<br>● Compute velocity, unit tangent and acceleration vectors along a parametric curve (10.2).<br>● Graph, locate, and operate with polar coordinates (10.3).<br>● Find areas and lengths in polar coordinates (10.4).<br>● Graph and work with conic sections (10.5).<br>● Graph and work with conic sections in polar coordinates (10.6). | Calculus Early Transcendentals 9E Stewart Chapter 10:<br>● Discovery Project pg 672: Running Circles Around Circles<br>● Problems Plus Chapter 10, page 722<br><br>MIT Open Courseware:<br>● Problem Set 1<br><br>**Assessments**<br>Benchmark:<br>● Progress checks<br><br>Formative:<br>● Classwork<br>● Teacher observations<br>● Entrance/exit tickets<br><br>Summative:<br>● Mid-unit quizzes<br>● Topic tests<br><br>Alternative:<br>● End-of-unit project | ● Mathispower4u video resources<br>● OCW Scholar Sessions 1<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing<br>● Stewart Calculus Instructor's Guide (GDrive) |

| **Unit 2 - Vectors and the Geometry of Space** | **Duration** |
|---|---|
| Summary: In this unit, we introduce vectors and coordinate systems for three-dimensional space, setting the stage for studying calculus of curves in space and of functions of two variables whose graphs are surfaces in space. We will look at vectors and how they provide particularly simple descriptions of lines and planes in space. | 13 Blocks |

| **Enduring Understandings** | **Essential Questions** |
|---|---|
| ● Represent position and motion in space using vectors.<br>● Relation of vectors to their dot product and cross product.<br>● Relation of two-space to three-space via vectors.<br>● Use of vectors to represent planes in space. | ● How much work is needed to pull a sled up an incline?<br>● How can we use vectors to analyze the flight of a plane?<br>● How much torque is being applied to the fulcrum of a crank shaft?<br>● How can we use vectors to write equations of plane?<br>● How can the equations of functions in three dimensions be constructed visually?<br>● How can we create equations for three dimensional space?<br>● What are the defining characteristics for quadric surfaces and how are they determined?<br>● What is the right hand rule for vectors and how does it apply in determining a solution? |

| **NJSLS for Unit** |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem. |

- 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br><br>● Work with three-dimensional coordinate systems (12.1).<br>● Create, operate with, and graph vectors (12.2).<br>● Use the parallelogram law to add geometric vectors (12.2).<br>● Resolve geometric vectors into components parallel to coordinate axes (12.2).<br>● Perform the operations of vector addition and scalar multiplication, and interpret them geometrically (12.2).<br>● Use the dot product to calculate magnitude of a vector, angle between vectors and projection of one vector on another (12.3).<br>● Find and use direction angles and direction cosines of a vector (12.3).<br>● Use and interpret geometrically the standard equation for a plane (12.3).<br>● Use the cross product; interpret the cross product geometrically and as the area of a parallelogram (12.4).<br>● Interpret the vector triple product as volume of a parallelopiped (12.4).<br>● Work with equations of lines and planes (12.5).<br>● Recognize cylinders and quadric surfaces from their Cartesian equations (12.6).<br>● Use cylindrical and spherical coordinates, and convert among these two and | Calculus Early Transcendentals 9E Stewart Chapter 12:<br>● Discovery Project pg 865: The Geometry of a Tetrahedron<br>● Discovery Project pg 874: Putting 3D in Perspective<br>● Problems Plus Chapter 12, page 887<br><br>MIT Open Courseware:<br>● Problem Set 1<br><br>**Assessments**<br>Benchmark:<br>● Progress checks<br><br>Formative:<br>● Classwork<br>● Teacher observations<br>● Entrance/exit tickets<br><br>Summative:<br>● Mid-unit quizzes<br>● Topic tests<br><br>Alternative:<br>● End-of-unit project | ● Mathispower4u video resources<br>● OCW Scholar Sessions 1<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing<br>● Stewart Calculus Instructor's Guide (GDrive) |

| | | |
|---|---|---|
| rectangular coordinates (12.6). | | |

| Unit 3 - Vector Functions | Duration |
|---|---|
| Summary: Building on the foundation of real-valued functions, this unit looks at functions whose values are vectors. Such functions are needed to describe curves and surfaces in space. This unit also uses vector-valued functions to describe the motion of objects through space. This unit uses them to drive Kepler's laws of planetary motion. | 10 Blocks |

| Enduring Understandings | Essential Questions |
|---|---|
| ● Extend the concepts of limits and continuity to vector-valued functions.<br>● Analyze speed and velocity in space using vector-valued functions.<br>● Determine the calculus of vector-valued functions. | ● How can vector-valued functions be used to predict speed and velocity in space?<br>● How can vector-valued functions be used in athletics?<br>● How do we model three-dimensional structures using vector-valued functions? |

| NJSLS for Unit |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br>● Identify vector functions and space curves (13.1).<br>● Find derivatives and integrals of vector functions (13.2).<br>● Find arc length and curvature of vector functions (13.3).<br>● Find normal and binomial vectors (13.3) | Calculus Early Transcendentals 9E Stewart Chapter 13:<br>● Applied Project pg 925: Kepler's Laws<br>● Problems Plus Chapter 13, page 930<br><br>MIT Open Courseware:<br>● Problem Set 3 | ● Mathispower4u video resources<br>● OCW Scholar Sessions 1<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing |

| | | |
|---|---|---|
| ● Work with motion in space to create and operate with velocity and acceleration vectors (13.4).<br>● Find velocity, speed, and acceleration of a particle given a positon vector (13.4).<br>● Use Keplar's Laws of Planetary Motion (13.4).<br><br>14.4 Motion in Space: Velocity and Acceleration 906<br>*(Applied Project-Kepler's Laws)* 916 | Interdisciplinary Science: MS-PS3-1: In lessons on comparing distance-time graphs to speed-time graphs students will construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.<br><br>**Assessments**<br>Benchmark:<br>● Progress checks<br><br>Formative:<br>● Classwork<br>● Teacher observations<br>● Entrance/exit tickets<br><br>Summative:<br>● Mid-unit quizzes<br>● Topic tests<br><br>Alternative:<br>● End-of-unit project | ● Stewart Calculus Instructor's Guide (GDrive) |

| **Unit 4 - Partial Derivatives** | **Duration** |
|---|---|
| Summary: Physical quantities often depend on two or more variables. Unit 4 focuses on partial derivatives, using functions of several variables in order to extend the basic ideas of differential calculus to such functions. | 12 Blocks |

| **Enduring Understandings** | **Essential Questions** |
|---|---|
| ● Compute rates of change of functions of several variables.<br>● Use gradient and normal vectors describe characteristics of curves.<br>● Determine extrema of functions of several variables.<br>● Lagrange multipliers as a method to solve boundary (constraint) problems. | ● How can real-life problems be characterized by functions of several variables?<br>● How do the concepts from single variable calculus apply and take form in multivariable calculus?<br>● What is a function of several variables?<br>● How do the concepts and rules of differentiation for functions of one variable extrapolate to functions of several variables?<br>● How do you determine maximum, minimum and saddle points for functions of several variables?<br>● What is the gradient of a function?<br>● How does the method of Lagrange Multipliers work? |

| **NJSLS for Unit** |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| **Student Learning Outcomes** | **Student Learning Activities and Assessments Aligned to Student Learning Outcome(s)** | **Suggested Materials & Resources** |
|---|---|---|
| Students will be able to:<br>● Represent a function of two variables as the graph of a surface; sketch level curves (14.1). | Calculus Early Transcendentals 9E Stewart Chapter 14:<br>● Discovery Project pg 1019 Quadratic Approximations and Critical Point<br>● Applied Project pg 1030: Hydro-Turbine | ● Mathispower4u video resources<br>● OCW Scholar Sessions 2<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources |

| | | |
|---|---|---|
| • Work with functions of several variables (14.1).<br>• Find limits and continuity given functions with several variables (14.2).<br>• Calculate partial derivatives and the gradient (14.3).<br>• Use the gradient to find tangent planes, directional derivatives and linear approximations (14.4).<br>• Interpret the gradient geometrically (14.4).<br>• Use the Chain Rule (14.5).<br>• Find directional derivatives and the gradient vector (14.6).<br>• Find and classify critical points of functions, using the second derivative test (14.7).<br>• Find maximum and minimum values for a function defined on a closed, bounded, planar set (14.7).<br>• Use Lagrange Multipliers (14.8). | Optimization<br>• Problems Plus Chapter 14, page 1035<br><br>MIT Open Courseware:<br>• Problem Set 4<br>• Problem Set 5<br>• Problem Set 6<br><br>**Assessments**<br>Benchmark:<br>• Progress checks<br><br>Formative:<br>• Classwork<br>• Teacher observations<br>• Entrance/exit tickets<br><br>Summative:<br>• Mid-unit quizzes<br>• Topic tests<br><br>Alternative:<br>• End-of-unit project | • Wolfram Mathematica CAS<br>• CalcPlot3D<br>• Geogebra 3D Graphing<br>• Stewart Calculus Instructor's Guide (GDrive) |

| **Unit 5 - Multiple Integrals: Double Integrals and Their Applications** | **Duration** |
|---|---|
| Summary: Here we extend the idea of a definite integral to double integrals of functions of two variables. These ideas are then used to compute volumes, masses, and centroids of more general regions than we were able to consider earlier in the study of Calculus.We also use double integrals to calculate probabilities when two random variables are involved. We will see that polar coordinates are useful in computing double integrals over some types of regions. | 13 Blocks |

| **Enduring Understandings** | **Essential Questions** |
|---|---|
| ● Double integrals can be used to evaluate complicated volumes, masses, probabilities etc.<br>● Polar coordinates are useful in evaluating certain types of integrals.<br>● Computer algebra systems (CAS) can aid in visualizing surfaces/volumes and evaluating difficult integrals. | ● How can double integrals be used to solve complex problems?<br>● When is using polar coordinates helpful in evaluating integrals?<br>● How can manipulating double integrals help simplify integration?<br>● How are computer algebra systems (CAS) utilized in solving real-life problems? |

| **NJSLS for Unit** |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| **Student Learning Outcomes** | **Student Learning Activities and Assessments Aligned to Student Learning Outcome(s)** | **Suggested Materials & Resources** |
|---|---|---|
| Students will be able to:<br>● State the definition of the integral of a function over a rectangle. (15.1)<br>● Use iterated integrals to evaluate integrals over planar regions, and to calculate volume. (15.2)<br>● Build on elementary integration techniques | Calculus Early Transcendentals 9E Stewart Chapter 15:<br>● (Discovery Project-Volumes of Hyperspheres) pg. 1095<br>● Problems Plus Chapter 15.<br><br>MIT Open Courseware:<br>● Problem Set 7 | ● Mathispower4u video resources<br>● OCW Scholar Sessions 3A<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing |

| | | |
|---|---|---|
| to evaluate multiple integrals efficiently. (15.2)<br>● Set up and evaluate double integrals in polar coordinates. (15.3)<br>● Use double and triple integrals to compute moments, center of mass, and moments of inertia. (15.4)<br>● Set up and evaluate integrals to compute surface area. (15.5)<br>● Set up and evaluate triple integrals in Cartesian coordinates. (15.6)<br>● Use cylindrical and spherical coordinates (15.7)<br>● Change coordinates from rectangular to cylindrical or spherical or the reverse. (15.7)<br>● Set up and evaluate triple integrals in cylindrical and spherical coordinates. (15.7, 15.8)<br>● Change the order of variables in multiple integrals. Carry out change of variables in multiple integrals. (15.9) | **Assessments**<br>Benchmark:<br>● Progress checks<br><br>Formative:<br>● Classwork<br>● Teacher observations<br>● Entrance/exit tickets<br><br>Summative:<br>● Mid-unit quizzes<br>● Topic tests<br><br>Alternative:<br>● End-of-unit project | ● Stewart Calculus Instructor's Guide (GDrive) |

| Unit 6 - Multiple Integrals: Triple Integrals | Duration |
|---|---|
| Summary: Here we extend the idea of a definite integral to triple integrals of functions of three variables. These ideas are then used to compute volumes, masses, and centroids of more general regions than we were able to consider earlier in the study of Calculus. We will introduce two new coordinate systems in three-dimensional space – cylindrical coordinates and spherical coordinates – that greatly simplify the computation of triple integrals over certain commonly occurring solid regions | 9 Blocks |

| Enduring Understandings | Essential Questions |
|---|---|
| ● Triple integrals can be used to evaluate complicated volumes, masses, probabilities etc.<br>● Cylindrical and Spherical coordinates can greatly simplify the computation of triple integrals.<br>● Computer algebra systems (CAS) can aid in visualizing surfaces/volumes and evaluating difficult integrals. | ● How can triple integrals be used to solve complex problems?<br>● How can manipulating triple integrals help simplify integration?<br>● How are computer algebra systems (CAS) utilized in solving real-life problems? |

| NJSLS for Unit |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br>● Use double and triple integrals to compute moments, center of mass, and moments of inertia. (15.4)<br>● Set up and evaluate integrals to compute surface area. (15.5) | Calculus Early Transcendentals 9E Stewart Chapter 15:<br>● (Discovery Project-Volumes of Hyperspheres) pg. 1095<br>● (Discovery Project-The Intersection of Three<br>● Cylinders) pg. 1101<br>● (Applied Project-Roller Derby) pg. 1108 | ● Mathispower4u video resources<br>● OCW Scholar Sessions 3A<br>● Calculus Early Transcendentals 9E Stewart Chapter 15 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D |

| | | |
|---|---|---|
| • Set up and evaluate triple integrals in Cartesian coordinates. (15.6)<br>• Use cylindrical and spherical coordinates (15.7)<br>• Change coordinates from rectangular to cylindrical or spherical or the reverse. (15.7)<br>• Set up and evaluate triple integrals in cylindrical and spherical coordinates. (15.7, 15.8)<br>• Change the order of variables in multiple integrals. Carry out change of variables in multiple integrals. (15.9) | • Problems Plus Chapter 15.<br><br>MIT Open Courseware:<br>• Problem Set 10<br><br>**Assessments**<br>Benchmark:<br>• Progress checks<br><br>Formative:<br>• Classwork<br>• Teacher observations<br>• Entrance/exit tickets<br><br>Summative:<br>• Mid-unit quizzes<br>• Topic tests<br><br>Alternative:<br>• End-of-unit project | • Geogebra 3D Graphing<br>• Stewart Calculus Instructor's Guide (GDrive) |

| Unit 7 - Vector Calculus: Line Integrals and Surface Integrals | Duration |
|---|---|
| Summary: Here we study the calculus of vector fields. (These are functions that assign vectors to points in space.) In particular, we define line integrals (which can be used to find the work done by a force field in moving an object along a curve). Then we define surface integrals (which can be used to find the rate of fluid flow across a surface). The connections between these new types of integrals and the single, double, and triple integrals that we have already met are given by the higher-dimensional versions of the Fundamental Theorem of Calculus: Green's Theorem, Stokes' Theorem, and the Divergence Theorem. | 11 Blocks |

| Enduring Understandings | Essential Questions |
|---|---|
| ● Vector fields can be used to model real world phenomena<br>● Line Integral can be used to solve fluid flow, forces, E&M problems | ● How do vector fields model real world phenomena?<br>● How are line integrals evaluated? |

| NJSLS for Unit |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br>● Sketch vector fields. (16.1)<br>● Find the gradient vector field of a function. (16.1)<br>● Set up and evaluate line integrals of scalar functions or vector fields along curves. (16.2)<br>● Recognize conservative vector fields, and apply the fundamental theorem for line integrals of conservative vector fields. (16.3) | Calculus Early Transcendentals 9E Stewart Chapter 16:<br>● Writing Project-Three Men and Two Theorems<br>● Problems Plus Chapter 16.<br><br>MIT Open Courseware:<br>● Problem Set 8 | ● Mathispower4u video resources<br>● OCW Scholar Sessions 3B<br>● Calculus Early Transcendentals 9E Stewart Chapter 16 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing<br>● Stewart Calculus Instructor's Guide (GDrive) |

| **Unit 8 - Vector Calculus: Green’s Theorem, Stoke’s Theorem and the Divergence Theorem** | **Duration** |
|---|---|
| Summary: Here we study the calculus of vector fields. (These are functions that assign vectors to points in space.) In particular, we define line integrals (which can be used to find the work done by a force field in moving an object along a curve). Then we define surface integrals (which can be used to find the rate of fluid flow across a surface). The connections between these new types of integrals and the single, double, and triple integrals that we have already met are given by the higher-dimensional versions of the Fundamental Theorem of Calculus: Green’s Theorem, Stokes’ Theorem, and the Divergence Theorem. | 11 Blocks |

| **Enduring Understandings** | **Essential Questions** |
|---|---|
| ● Green’s theorem can translate difficult line integrals into more simple double integrals, and vice versa.<br>● Stokes’ theorem gives a relation between line integrals and surface integrals.<br>● Divergence theorem relates the flux of a vector field through the closed surface to the divergence of the field in the volume enclosed. | ● What is the significance of Green’s Theorem?<br>● How is Stokes’ Theorem used in application?<br>● How is the Divergence Theorem used in application? |

| **NJSLS for Unit** |
|---|
| ● Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. (MP.5)<br>● Collaborate and apply a design process to solve a simple problem from everyday experiences. (8.2.2.D.1)<br>● Use digital tools and online resources to explore a problem or issue. (8.1.2.E.1)<br>● Apply appropriate academic and technical skills. (CRP2.)<br>● Communicate clearly and effectively and with reason. (CRP4.)<br>● Utilize critical thinking to make sense of problems and persevere in solving them. (CRP8.)<br>● 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.<br>● 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.<br>● 9.4.8.TL.3: Select appropriate tools to organize and present information digitally. |

| Student Learning Outcomes | Student Learning Activities and Assessments Aligned to Student Learning Outcome(s) | Suggested Materials & Resources |
|---|---|---|
| Students will be able to:<br>● State and apply Green's Theorem. (16.4)<br>● Calculate the curl and divergence of a vector field. (16.5)<br>● Set up and evaluate integrals over parametric surfaces. (16.6)<br>● Set up and evaluate surface integrals; compute surface area and the flux of a vector field through a surface. (16.7)<br>● State and apply Stokes' Theorem. (16.8)<br>● State and apply the Divergence Theorem. (16.9) | Calculus Early Transcendentals 9E Stewart Chapter 16:<br>● Writing Project-Three Men and Two Theorems<br>● Problems Plus Chapter 16.<br><br>MIT Open Courseware:<br>● Problem Set 9<br>● Problem Set 11<br>● Problem Set 12<br><br>**Assessments**<br>Benchmark:<br>● Progress checks<br><br>Formative:<br>● Classwork<br>● Teacher observations<br>● Entrance/exit tickets<br><br>Summative:<br>● Mid-unit quizzes<br>● Topic tests<br><br>Alternative:<br>● End-of-unit project | ● Mathispower4u video resources<br>● OCW Scholar Sessions 3B<br>● Calculus Early Transcendentals 9E Stewart Chapter 16 and resources<br>● Wolfram Mathematica CAS<br>● CalcPlot3D<br>● Geogebra 3D Graphing<br>● Stewart Calculus Instructor's Guide (GDrive) |