Compact binary evolution, rates and population modelling

https://icts.res.in/program/gws2022

Lectures will be blackboard style with discussion. Tutorials are planned to follow these (each suggested idea would fill the time, the suggestion is not to fit all activities into a session)

1. Forming merging binaries

- Source of GWs: twiddle physics and frequency evolution
- Parameters and inspiral timescale
- The problem of forming close binaries
- Different formation channels
- Characteristic properties (masses, spins, redshift)

Tutorial ideas:

1. Calculate Peters & Mathews inspiral timescale as a function of separation, and compare with radius of stars at different phases in their evolution. This should highlight that some process is needed to shrink binaries to be close enough to merge through gravitational-wave emission, after they have evolved through their giant phase. Extension: compare with Roche lobe radius (using Eggleton approximation), to see when stars would undergo mass transfer?
2. Download model data from https://zenodo.org/record/4448170#.YmFY03_MKV4 and plot the parameters for different models. Where possible, explain key features of models. Extension: plot GW observations in comparison.

2. Physics of binary evolution

- Mass transfer (stability)
- Common envelope physics
- Impact of metallicity
- Evolution with cosmic time

Tutorial ideas:

1. Calculate whether mass transfer in a binary shrinks and orbit or expands it as a function of mass ratio. Extension: extend from the conservative case, to the case where some mass is lost from the system (probably best done numerically)
2. Make a Monte Carlo code to simulate a simple population of binaries: (i) draw primary mass from IMF, (ii) draw secondary mass assuming a uniform distribution for mass ratio,

(iii) map to remnant mass using a back-of-the-envelope power-law scaling (ignoring things like mass transfer), (iii) draw separation from Opik's law. Extension: calculate delay time distribution (works well if idea 1 was done for Tutorial 1), and factor in formation time from star formation rate, to get merger distribution as a function of redshift.

3. Introduction to GW data analysis

- Three stages of data analysis: detection, parameter estimation, population inference. Population inference builds on all steps
- Basics of detection (false alarm rates, ranking statistics)
- Introduction to Bayes theorem
- The parameter estimation likelihood, and the nature of posterior samples
- Population inference as hierarchical inference (Bayes again)

Tutorial ideas:

1. Example of using Bayes theorem. I would recommend the exercise from section 3.1 of https://www.inference.org.uk/itprnn/book.pdf This is not GW-related, but it is nice to point people to the book, the problem is solvable in the time, and there is a link to the selection effects we'll discuss next lecture. If students figure out the problem formulation quickly, they can be asked to write an MCMC to infer the posterior distribution on lambda. If finding more difficult, they can reconstruction the probability analytically on a grid as it's only 1D.
2. Download posterior samples for some GWs, and make some plots. The point of this would be to familiarise themselves with the data format, and also concepts such as 90% credible intervals (and how these are not unique i.e. symmetric, versus one-sided versus highest-probability density). Plotting intervals in terms of different parameterizations (e.g. total mass versus chirp mass) would also highlight how the posterior density transforms (maximum a posteriori points are coordinate dependent)

4. Population inference

- Hierarchical inference
- Derivation of hierarchical inference with selection effects

Tutorial ideas:

1. Perform a simple hierarchical inference. Using a code like GWPopulation might make this easier. A suggestion could be fitting a line to sets of normally distributed data points at different x values, where the mean of the Gaussians varies linearly with y. The population parameter would be the gradient and intercept of the line, as well as the standard deviation for the points.An extension would be to make the standard deviation a function of x too.

2. Download posterior samples for GWs, and reweight with a population prior (the population prior would be fixed, as performing a hierarchical inference would be too much to do in the time). Plot the results. This could work well if combined with idea 2 for Tutorial 1/2 and idea 2 for Tutorial 3, as a model from Tutorial 1/2 could be used as the prior, and the the students would already have experience with posteriors from Tutorial 3.

5. Review of LVK results

- Introduce models used, and discuss results
- Highlight extensions to cosmology
- Discuss complications when it comes to testing GR (if time is available).

Tutorial ideas:

1. Download hyperposterior samples from Populations paper (or Cosmology paper), and make corner plots of model parameters. Explain any correlations found (see Appendix A of https://arxiv.org/abs/2005.00023 for the sort of thing)
2. Download hyperposterior samples, and use them to draw a population of binaries. A simple task would be to just start drawing from, say the 1D m1 distribution. This activity is harder than it sounds because the format of the data release is not straightforward.
3. Compile notes on topics from course to present one idea of interest (I know this is less hands on, but I'm thinking everyone will be tired at this point, and could benefit from revision)