Curve Fitting with Geogebra

Curve fitting (or "regression" if we want to get all technical) is the art of finding a function that matches some data reasonably well.

In your science classes, you have often performed *linear* regressions on experimental data, in order to find a linear relationship between two variables. However, not all all relationships are linear.

Today you will learn how to use Geogebra to perform a **polynomial regression**, meaning that you will find a polynomial curve of a given degree that closely matches some data. (Actually, linear regression can be thought of as a polynomial regression with degree = 1). I will also briefly outline the other types of regression Geogebra can do: **exponential** and **sinusoidal**.

The data we will be using will be visual: it will be the x- and y-values of different points along a literal curve. This will allow us to find a function whose graph approximates the original curve.

Furthermore, since all four basic types of regression produce functions with easy-to-find derivatives and antiderivatives, we will be able to use our regression curves to solve problems involving both optimization and area.

As an example, I will be showing you how I found a function to represent the underside of the Gateway Arch in St. Louis. I will be using a 4th-degree (quartic) polynomial regression (meaning, the largest power of x in my function will be 4). Even though the original shape is not a polynomial curve (it's actually a catenary) this will give me a "good-enough" match to work with.

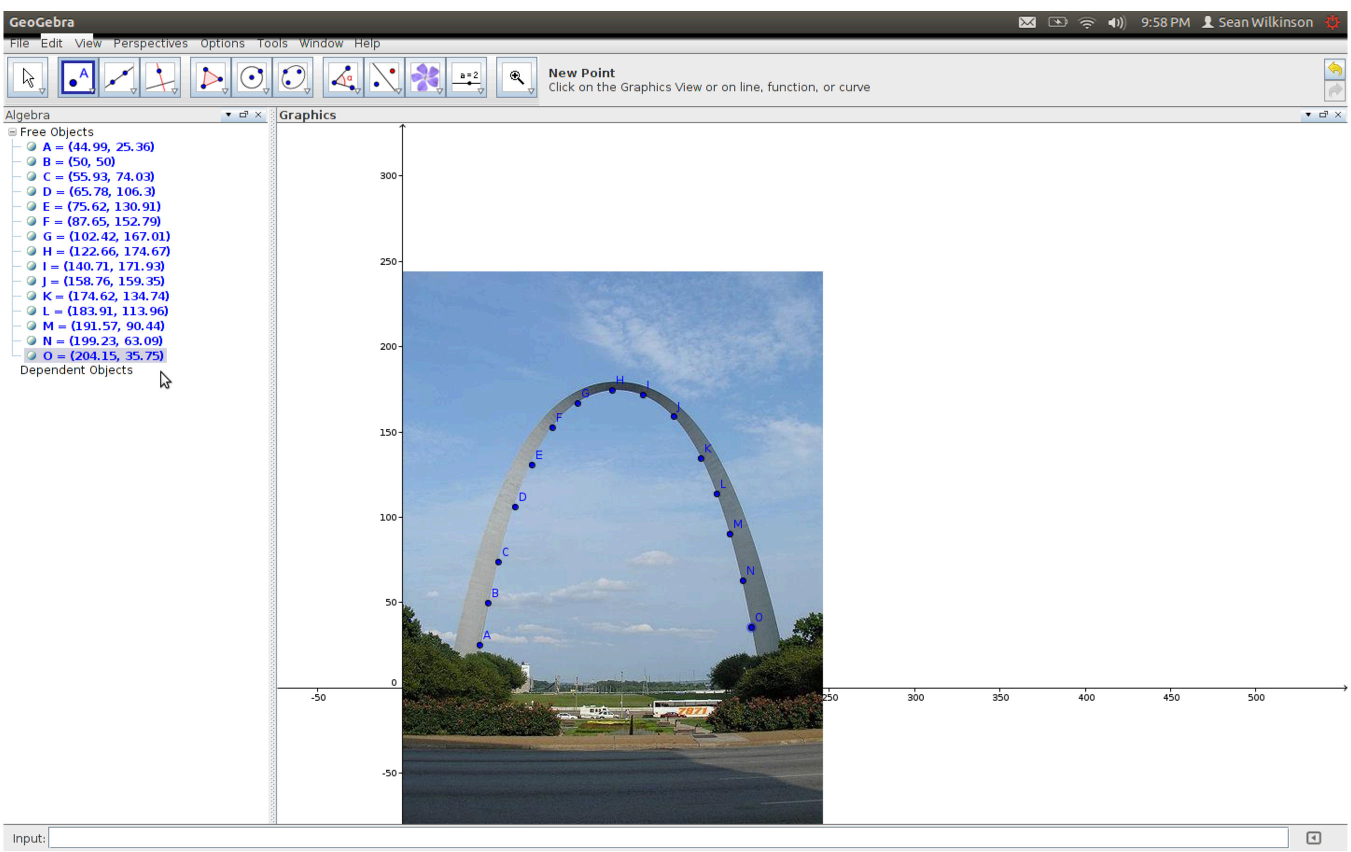
This is the curve I want to find a function for.

Step 1. Insert an Image into Geogebra.

- Open Geogebra
- View → Grid
- Options → Rounding → Choose a level of precision appropriate for your task. This will affect the precision of the function that we get at the end. For now, let's use 5 decimal places.
- ABC → Insert Image. A dialogue box will open, asking you to choose an image file.
- Click where you want the bottom-left corner of the image to go (usually the origin) (you can move it later if you want).
- Zoom to the scale you want. The image will stay the same size, but the axes will change scale. If you know the length of the object you are using, try to let 1 gridline = 1 cm, or 1 m.

Step 2. Tell the computer where your curve is.

- Choose the "Add Point" tool and add points (by eyeballing) along the curve you want to find a function for.
  Here, I have added 15 points to the curve, which Geogebra has labelled A *through* O.



Step 3. Choose a type of regression and use it!

- There are four basic types of regression that Geogebra knows: linear, polynomial, sinusoidal, and exponential. It is up to YOU, not the computer, to decide which of these is the most appropriate to use. Once you have decided which form of regression to use, you may enter one of the following commands in the bar at the bottom of your screen:

    - If you think that your shape can be most accurately modelled by a **linear** function, use the `FitLine[]` command. Inside the square brackets, make a list of all of the different points you want to use, separated by commas.
      EXAMPLE: `FitLine[A,B,C,D,E]` will choose a line of best fit for the points A,B,C,D, and E.

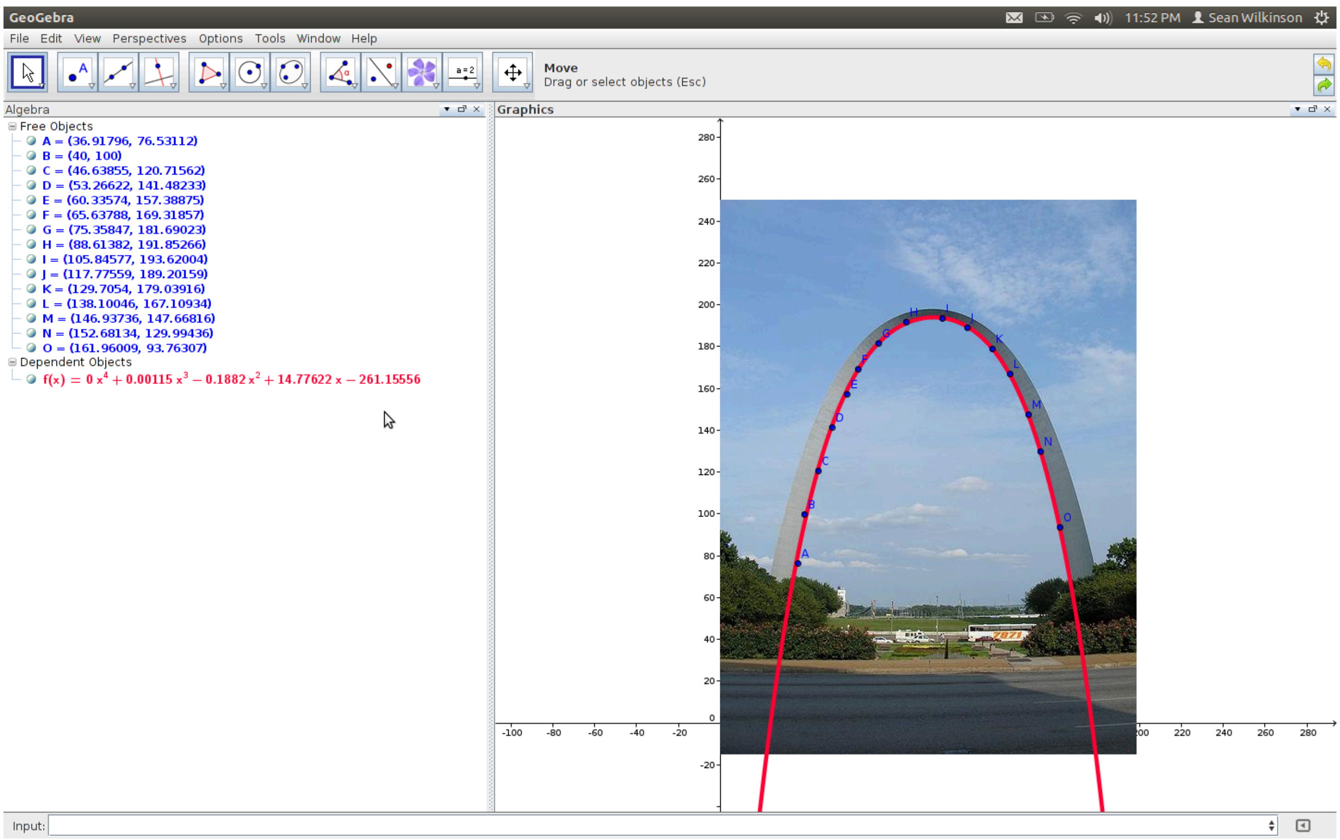
    - If you think that your shape can be most accurately modelled by a **polynomial**, use the `FitPoly[]` command. Unlike `FitLine`, `FitPoly` takes two arguments. The first is a list of points, encased in curly braces. Then type a comma. The second is the degree of the polynomial (for a quadratic, choose "2"; for a cubic, choose "3"; etc.)
      EXAMPLE: `FitPoly[{A,B,C,D,E,F,G,H}, 4]` will choose the quartic polynomial that best fits the the points A,B,C,D, E, F, and G.
    - **How do I choose what degree to use for a polynomial regression?** First, as always, use the underlying structure of what you are modelling as your guide. If you think that a cubic polynomial makes the most sense for the shape you want to approximate, use a cubic polynomial.
      Next, be aware that the higher the degree of your polynomial, the more closely it will be able to fit your points (*prove it!*). However, if you make your polynomial's degree too close to the number of points you have, you will find that it gets overly wavy (*why?*). Also, Geogebra will not allow you to choose a degree that is equal to or greater than the number of points you are using (*why?*).
      If you're not happy with your results, try adding more points.

    - If you think that your shape can be most accurately modelled by a **sinusoidal** function, use the `FitSin[]` command.
      The syntax for `FitSin[]` is the same as the syntax for `FitLine[]`.

    - If you think that your shape can be most accurately modelled by an **exponential** function, use the `FitExp[]` command.
      The syntax for `FitSin[]` is the same as the syntax for `FitLine[]`.

Step 4. Examine your results in the geometry and algebra windows.

- You should see a new function listed in the algebra window, and the corresponding curve in the geometry window.
- Inspect the curve visually: is it a good match for your shape? Or do you need to make some adjustments? (See previous page).
- *Pro Tip!* If you are having trouble distinguishing the regression curve from lines in the image, then double-click on the curve, select “Object Properties,” choose the “colour” tab, and make it a funky colour that contrasts from the image. You can also increase the line thickness in the “style” tab.
- Once you are satisfied with your curve, read off its equation from the algebra window. Now you have something you can do calculus with!



Here, I have entered the command `FitPoly[{A,B,C,D,E,F,G,H,I,J,K,L,M,N,O}, 4]`

The fit is pretty good! Notice that my coefficient on x^4 is displaying as 0. It isn’t actually 0, but we can’t tell the difference with only 5 decimal points. Feel free to use Options → Rounding at any point to see more decimal points in your coefficients.

Play it Loud!

(You will need a **computer**. You may work in **groups** of up to 4 people).

While there are many different aspects of guitar design that affect the sound of the instrument, one of the most obvious variables is size: all other things being equal, the larger a guitar's body, the louder it will be.

Here is an image of different guitar designs[1] (it is the same as the image above, but rotated onto its side).   **Your task is to place the guitars in order from quietest to loudest.** You may assume that the depth of each instrument is identical and that there are no other confounding variables. (ALERT: both of those assumptions are **false** in the real world!)

To do this, you will need to know how to **perform a polynomial regression with geogebra** and how to **use antidifferentiation to calculate the area between the graphs of two polynomial functions**.

There are two guitars that will give you more trouble than the others. (*Which ones are they? And why?*) If you wish, you may choose to ignore those two.

*(Note to the guitar illiterate: the body of a guitar is *just* the wide curvy part with the circular hole in it. That's the part where the sound resonates. Also, you do **not** need to subtract the circular hole from the area).*

*If you are working in a group, on different computers, make sure that you are all using the same scale!

[1] Temporary url: http://wp.me/a2RoU2-6C