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Total No. of Questions: [09]

B.Sc. (Hons.) Chemistry (Semester: 4th)

MATHEMATICS-IV

Subject Code: BMATH5-401

Paper ID: 19131629

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.

b) Give the Fourier series representation of a periodic function $f(x)$ defined in $(-c, c)$.

c) Find the Fourier cosine transform of $f(x) = \{x, 0 < x < 1\ 2 - x, 1 < x < 2\ 0, x > 2$

d) Using Parseval identities, prove that $\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$.

e) If the Fourier sine transform of $f(x)$ is $1 - \frac{\cos cos\, \pi x}{x^2\pi^2}$, for $0 \leq x \leq \pi$.

f) Find the Laplace transform of $f(t) = \{\frac{t}{a},\ 0 < t < a\ 1,\ t \geq a$.

g) Evaluate the integral $\int_0^\infty \frac{\sin sin\, mt}{t} dt$.

h) Find the Laplace transform of $f(x) = \{\sin sin\, \omega t,\ 0 < t < \frac{\pi}{\omega}\ 0,\ \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$.

i) Find the Laplace transform of $f'(t) = e^{-3t}(2\cos cos\, 5t - 5\sin sin\, 5t)$.

j)Find the Fourier series expansion of $f(x) = \{-x,\ -\pi < x < 0\ x, 0 < x < \pi$.

Section – B **(5 marks each)**

Q2. Obtain the Fourier series for $f(x) = e^x$ in the interval $0 < x < 2\pi$.

Q3. Solve the integral equation $\int_0^\infty f(\theta) \cos cos\, \alpha\theta\, d\theta = \{1 - \alpha,\ 0 \leq \alpha < 1\ 0,\ \alpha > 1$.

Hence evaluate $\int_0^\infty dt$.

Q4. Using unit step function, find the Laplace transform $f(x) = \{\sin sin\, t, 0 \leq x < \pi\ \sin sin\, 2t, \pi \leq t < 2\pi\ \sin sin\, 3t,\ t \geq 2\pi$.

Q5. Find the Fourier cosine and sine transform of xe^{-ax}.

Q6. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{s^2+a^2}\right\}$.

Section – C **(10 marks each)**

Q7. Obtain the Fourier expansion of $x \sin sin\, x$ as a cosine series in the interval $(0, \pi)$.

Q8. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty x \frac{\sin sin\, mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$, $m > 0$.

Q9. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t$ with $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$.