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B.Sc. (Hons.) Chemistry (Semester: 4th)

MATHEMATICS-IV

Subject Code: BMATH5-401

Paper ID: 19131629

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.

b) Give the Fourier series representation of a periodic function $f(x)$ defined in $(-c, c)$.

c) Find the Fourier cosine transform of $f(x) = \{x, 0 < x < 1\ 2 - x, 1 < x < 2\ 0, x > 2$

d) Using Parseval identities, prove that $\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$.

e) If the Fourier sine transform of $f(x)$ is $1 - \frac{\cos cos\ \pi x}{x^2\pi^2}$, for $0 \le x \le \pi$.

f) Find the Laplace transform of $f(t) = \{\frac{t}{a},\ 0 < t < a\ 1,\ t \ge a.$

g) Evaluate the integral $\int_0^\infty \frac{\sin sin\ mt}{t} dt$.

h) Find the Laplace transform of $f(x) = \{\sin sin\ \omega t,\ \ 0 < t < \frac{\pi}{\omega}\ 0,\ \ \ \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}.$

i) Find the Laplace transform of $f'(t) = e^{-3t}(2\cos cos\ 5t - 5\sin sin\ 5t)$.

j) Find the Fourier series expansion of $f(x) = \{-x,\ \ -\pi < x < 0\ x, 0 < x < \pi.$

Section – B **(5 marks each)**

Q2. Obtain the Fourier series for $f(x) = e^{x}$ in the interval $0 < x < 2\pi$.

Q3. Solve the integral equation $\int_0^\infty f(\theta) \cos cos\ \alpha\theta\ d\theta = \{1 - \alpha,\ 0 \le \alpha < 1\ 0,\ \ \ \ \alpha > 1.$

Hence evaluate $\int_0^\infty dt$.

Q4. Using unit step function, find the Laplace transform $f(x) = \{\sin sin\ t, 0 \le x < \pi\ \ \sin sin\ 2t, \pi \le t < 2\pi\ \ \sin sin\ 3t,\ \ \ t \ge 2\pi.$

Q5. Find the Fourier cosine and sine transform of xe^{-ax}.

Q6. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{S^2+a^2}\right\}$.

Section – C **(10 marks each)**

Q7. Obtain the Fourier expansion of $x \sin sin\, x$ as a cosine series in the interval $(0, \pi)$.

Q8. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^{\infty} x \frac{\sin sin\, mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0.$

Q9. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t$ with $y(0) = 1,\ y'(0) = 0,\ y''(0) = -\ 2.$