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B.Sc. (Hons.) Math (Semester – 3rd)
NUMBER THEORY
Subject Code: BMATS1-323
Paper ID: [22131212]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Find the remainder when 15! is divided by 17.

b. If gcd $(a,30)=1$, show that 60 divides a^4+59.

c. Solve linear congruence $34x \equiv 60 (\mathrm{mod}\, 98)$.

d. Calculate $\phi(n)$ for $n=5040$

e. If $a \mid c$ and $b \mid c$ and $(a,b)=1$, then show that $ab \mid c$.

f. Give an example to show that the converse of Fermat's theorem is not true

g. Use Euler's theorem to confirm that for any integer $n \geq 0$ $51 \mid 10^{32n+9}-7$.

h. State and prove the Chinese remainder theorem.

i. State Gauss theorem.

j. Define Twin primes with examples.

Section – B **(5 marks each)**

Q2. State and Prove Wilson's Theorem

Q3. State and prove Mobius Inversion formula.

Q4. Determine all solutions in integers of the following Diophantine equation:
$221x+35y=11$

Q5. Solve the linear congruence 9x = 21 (mod30)

Q6. Show that $\sigma(n)$ is an odd integer iff n is either a perfect square or twice a perfect square.

Section – C **(10 marks each)**

Q7. Prove that every integer n>1 can be represented as a product of prime factors in one and only one way

Q8. If p is a prime and $d \mid (p-1)$, then prove that $x^d-1 \equiv 0 (\mathrm{mod}\, p)$ has exactly d solutions.

Q9. Find all integers a and b satisfying the equations $(a,b)=10$ and $[a,b]=100$