MARKING SCHEME FOR NORMAL DISTRIBUTION MARKING SCHEME

1.

5 (a)	$P(X < 38.8) = P\left(Z < \frac{38.8-40}{4}\right)\left[= P\left((Z < -0.3)\right]\right.$	M1
	$= 1 - 0.6179 = 0.3821$ *	A1*
		(2)
(b)	$P(\text{Qualify}) = 1-(0.3821)^3$ or $1-0.3821+0.3821\times(1-0.3821)+0.3821^2\times(1-0.3821)$	M1
	$[= 0.9442]$	
	$P(X > 44) = P\left(Z > \frac{44-40}{4}\right)\left[= P\left((Z > 1)\right]\right.$	M1
	$[= 1 - 0.8413] = 0.1587$	A1
	$P(X > 44 \text{ on 2nd attempt} \mid \text{Qualify}) = \frac{0.3821\times'0.1587'}{'0.9442'}$	M1
	0.06422... awrt 0.0642	A1
		(5)
	Notes	**Total 7**

2.

(a)	$P(X > \mu + 2k) = 0.2$ or $P(X < \mu + 2k) = 0.8$	or	$P(X < \mu - 2k) = 0.2$ or $P(X > \mu - 2k) = 0.8$	M1
	$\frac{\mu + 2k - \mu}{6} = 0.8416$	or	$\frac{\mu - 2k - \mu}{6} = -0.8416$	M1 A1
	$k = 2.5248...$		awrt 2.52	A1
				(4)
(b)	$P\left(Y > \frac{3}{2}\mu\right) \Rightarrow P\left(Z > \frac{\frac{3}{2}\mu - \mu}{\sigma}\right) \Rightarrow P\left(Z > \frac{\frac{1}{2}\mu}{\sigma}\right)$			M1
	$\mu = \frac{3}{2}\sigma^2 \Rightarrow P\left(Z > \frac{'\frac{1}{2}\left(\frac{3}{2}\sigma^2\right)'}{\sigma}\right)\left[= P\left(Z > \frac{3}{4}\sigma\right)\right]$ or $\sigma = \sqrt{\frac{2\mu}{3}} \Rightarrow P\left(Z > \frac{\frac{1}{2}\mu}{\sqrt{\frac{2\mu}{3}}}\right) = \left[P\left(Z > \frac{1}{2}\sqrt{\frac{3\mu}{2}}\right)\right]$ or $\frac{\frac{1}{2}\mu}{\sigma} = k$ and $2\mu = 3\sigma^2$			M1
	$'\frac{3}{4}\sigma' = 1.5$ or $\frac{1}{2}\sqrt{\frac{3\mu}{2}} = 1.5$ or $3\sigma^2 = 6\sigma$			M1
	$\mu = 6$ only, $\sigma = 2$ only			A1 A1
				(5)
	Notes			**Total 9**

3.

5 (a)	$X \sim \mathrm{N}(210, 25^2)$	
	$\mathrm{P}(X < 240) = \mathrm{P}\left(Z < \frac{240-210}{25}\right)\left[= \mathrm{P}(Z < 1.2)\right]$	M1
	$= 0.8849$*	A1*
		(2)
(b)	$\mathrm{P}(190 < X < 240) = 0.8849 - \mathrm{P}\left(Z < \frac{190-210}{25}\right)\left[= 0.8849 - \mathrm{P}(Z < -0.8)\right]$	M1
	$0.8849 - 0.2119 = 0.673$ awrt 0.673	A1
		(2)
(c)	$\frac{210+k-210}{25} = 1.96$ or $\frac{210-k-210}{25} = -1.96$	M1 B1
	$k = 49$ awrt 49	A1
		(3)
(d)	$\mathrm{P}(X < S) = 0.15 \Rightarrow \frac{S-210}{25} = -1.0364$	M1 B1
	$S = 184.09$ awrt 184	A1
		(3)
(e)	$Y \sim \mathrm{N}(\mu,\ \sigma^2)$	
	$\mathrm{P}(Y < 152) = 0.05 \Rightarrow \frac{152-\mu}{\sigma} = -1.6449$	M1 A1
	$\mathrm{P}(Y > 180) = 0.40 \Rightarrow \frac{180-\mu}{\sigma} = 0.2533$	A1
	$28 = 1.8982\sigma$	dM1
	$\sigma = 14.75...$ and $\mu = 176.26...$	A1
		(5)
	Notes	**Total 15**

4.

(a)(i)	$P(J>510) = P\left(Z > \frac{510-500}{25}\right)$ or $P(Z>0.4)$	M1
	$= 1-0.6554 \Rightarrow 0.3446$ *	A1*
		(2)
(ii)	$\frac{d-500}{25} = -1.4$ (calc $-1.3997\ldots$)	M1B1
	$d = 465$ (calc 465.007)	dA1
		(3)
(b)	$(1-0.3446)^5$	M1
	$= 0.1209\ldots$ **awrt 0.121**	A1
		(2)
(c)	$\frac{r-520}{k} = -1.0364$	M1A1
	$\frac{3r-800-520}{k} = 2.5758$	M1A1
	$-240 = (3\times -"1.0364k") - "2.5758"k$ or $\frac{r-520}{"-1.0364"} = \frac{3r-1320}{"2.5758"}$ oe	ddM1
	$k = 42.216\ldots$ **awrt 42**	A1
	$r = 476.246\ldots$ **awrt 476**	dA1
		(7)
	Notes	**Total 14**

5.

. (a)	$P(W > 70) = P\left(Z > \frac{70-80}{8}[=-1.25]\right)$	M1
	$= P(Z > -1.25)$ or $P(Z < 1.25)$	A1
	$= 0.8944$ awrt **0.894**	A1
		(3)
(b)	$P(W < k) = 0.85$ or $P(W > k) = 0.15$	B1
	$\pm\left(\frac{k-80}{8}\right) =$ **1.0364**	M1 B1
	$k = 88.29\ldots$ awrt **88.3**	A1
		(4)
(c)	$P(W < 66) = P\left(Z < \frac{66-80}{8}[=-1.75]\right)[= 0.0401$ (calc 0.040059…)]	M1
	$0.25 \times P(Z < -1.75)[= 0.010025$ (calc 0.0100147...)] or $0.25 \times (1 - P(Z < 1.75))$	dM1
	$\frac{y-80}{8} =$ **−2.32**(63)	M1 A1
	$y = 61.389\ldots$ awrt **61.4**	A1
		(5)
	Notes	**Total 12**

6.

(a)	$P(V > 104.9) = P\left(Z > \frac{104.9-100}{2.5}\right)$	M1
	$= 1 - 0.975$	M1
	$= 0.0250$ 0.025 or awrt 0.0250	A1
		(3)
(b)	Expected number $= 150 \times$ "0.025"	M1
	$= 3.75$ awrt 3.75	A1
		(2)
(c)	$[P(V > v \mid V < 104.9) =] \frac{[P(v < V < 104.9)]}{P(V < 104.9)} = 0.2801$	M1
	$P(v < V < 104.9) = (1 - $"0.025"$) - P(V < v)$	M1
	$P(V < v) = (1 - $"0.025"$) - (1 - $"0.025"$) \times 0.2801$ [=0.7019] oe	dM1
	$\frac{v-100}{2.5} = 0.53$	M1A1
	$v = 101.325$ awrt 101.32 or awrt 101.33	A1
		(6)
	Notes	**Total 11**

7.

. **(a)**	$P(X<37) = P\left(Z<\frac{37-40}{2.4}\right) = P(Z<-1.25)$	M1
	$= 1 - 0.8944$; $= 0.105649\ldots$ awrt **0.106**	M1; A1 (3)
(b)	P(one value is greater than 32) $= \sqrt{0.16}$ [=0.4]	M1
	$\frac{32-m}{2.4} = 0.2533$	M1 B1
	$m = 31.392\ldots$ awrt **31.4**	A1 (4)
(c)	$P(Y<0) = P\left(Z<\frac{0-4}{8}\right) = P(Z<-0.5)$ [= 0.3085]	M1
	Let X be the number of negative values	
	$P(X \geqslant 1) = 1 - P(X=0)$ oe	M1
	$= 1 - (0.6915)^5$	M1
	$= 0.84188\ldots$ awrt **0.842**	A1 (4)
		[11]
	Notes	

8.

(a)	$H \sim N(25.1, 5.5^2)$	
	$P(H > 30.8) = P\left(Z > \frac{30.8 - 25.1}{5.5}\right)$ or $P(Z > 1.03636\ldots)$	M1
	$= 1 - 0.8508$	M1
	$= 0.1492$ or better (calc: 0.1500…)	A1cso
		(3)
(b)	[$P(H < y) = 0.05$ implies] $\frac{y - 25.1}{5.5} = -1.6449$	M1B1
	$y = 16.053\ldots$ so range is awrt **16.1** ~ **30.8**	A1
		(3)
(c)(i)	$P(H < d) = 0.05 + 0.2 + 0.3$ [= 0.55]	M1
	$\frac{d - 25.1}{5.5} = 0.13$ (Calc 0.12566)	M1
	$d = 0.13 \times 5.5 + 25.1 = 25.815$ (25.791… calc)	A1cso
		(3)
(ii)	$P(H < m) = 0.05 + 0.2$ [= 0.25]	M1
	$\frac{m - 25.1}{5.5} = -0.67$ (Calc 0.674489)	M1M1
	m = awrt **21.4**	A1
		(4)
(d)	Height = $2 \times$ "m" + $3 \times 25.8 + 3 \times 30.8$ [+8]	M1
	= 220.6 awrt **221** (cm)	A1
		(2)
		[15]
	Notes	

9.

(a)	$P(W < 120) = P\left(Z < \frac{120-165}{35}\right)$	M1
	$= P(Z < -1.2857\ldots) = 1 - 0.9015$ or $1 - 0.9007285\ldots$	M1
	$= 0.09927\ldots$ = awrt **0.0985~0.0994**	A1
		(3)
(b)	e.g. $P(W > x) = \frac{1}{3}$ gives $\frac{x-165}{35} = \pm 0.43$ (calculator 0.430727…)	M1B1
	Limits 149.9245… to 180.0754… awrt **150** to **180**	A1, A1
		(4)
(c)	$P(W < 200 \mid W > \text{“180”})$ or $\frac{P(\text{"180"} < W < 200)}{P(W > \text{"180"})}$ or $\frac{1}{3}$	M1
	$= \frac{0.8413(44739...) - \frac{2}{3}}{\frac{1}{3}}$	A1 (num)
	$= 0.52403\ldots$ **(0.523~0.5264)**	A1
		(3)
(d)	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}; \times 3!$	M1;M1
	$= \frac{2}{9}$	A1
		(3)
		[13 marks]
	Notes	

10.

(a)	[W = weight of a package delivered to factory $W \sim N(18, 5.4^2)$]	
	$P(W < 18) = P\left(Z < \frac{10-18}{5.4}\right)$ or $P(Z < -1.481\ldots)$	M1
	$= 1 - 0.9306$ (calc: 0.069239…)	M1
	$= 0.0694$ **[0.0692, 0.0694]**	A1
		(3)
(b)	[$P(W > j) = 0.15$ implies] $\frac{j-18}{5.4} = 1.0364$	M1B1
	$j = 23.596\ldots$ awrt **23.6**	A1
		(3)
(c)	[$P(W > 18 \mid W <$ "23.59…") =] $\frac{P(18 < W < "23.6")}{P(W < "23.6")}$	M1
	$= \frac{0.5-0.15}{0.85}$ or $\frac{0.85-0.5}{0.85}$; $= \frac{0.35}{0.85}$	M1;A1
	$= \frac{35}{85} = \frac{7}{17}$ or allow awrt **0.412**	A1
		(4)
(d)	$0.85^2 \times 0.15^2 \times 6$	M1dM1
	$= 0.0975375$ awrt **0.0975**	A1
		(3)
		[13 marks]

11.

(a)	[D = distance achieved] $P(D > 4.3) = P\left(Z > \frac{4.3-3.8}{0.9}\right)$ or $P(Z > 0.555\ldots)$	M1	
	$= 1 - 0.7123$ (tables)	M1	
	$= 0.2877$ (tables) or $0.289257\ldots$ (calc) awrt **0.288** or awrt **0.289**	A1	(3)
(b)	$\frac{d-3.8}{0.9} = -0.8416$ (calc $-0.84162123\ldots$)	M1 ; B1	
	$d = 3.0425\ldots$ awrt **3.04**	A1	(3)
(c)	$P(D > g \mid D > 4.3) = \frac{P(D > g)}{P(D > 4.3) \text{ or (a)}}$ $\left[= \frac{1}{3}\right]$ (o.e.)	M1	
	$\therefore P(D > g) = \frac{1}{3}(a) = 0.096419\ldots$	A1ft (o.e)	
	$\frac{g-3.8}{0.9} = 1.302228\ldots$	dM1	
	so $g = 4.97200..$ awrt **4.97** or awrt **4.98**	A1	(4)
(d)	P(no gold medals) $= \left(\frac{2}{3}\right)^3$	M1	
	P(at least one gold) $= 1 - \left(\frac{2}{3}\right)^3$	M1	
	$= \frac{19}{27}$	A1	(3)
			[13]

12.

(a)	$P(X<40)=P\left(Z<\frac{40-42}{5}\right)=P(Z<-0.4)$	M1
	$=1-0.6554$	M1
	$=0.3446$ awrt **0.345**	A1
		(3)
(b)	P(Qualify) = $1-("0.3446")^3$ or $(1-"0.3446")+("0.3446")(1-"0.3446")+("0.3446")^2(1-"0.3446")$ [q = 0.9590… full calc: 0.9590867…]	M1
	$P(X>45)=P\left(Z>\frac{45-42}{5}\right)=P(Z>0.6)$	M1
	= 1 – 0.7257 or 0.2743 allow 1 – awrt 0.726 or awrt 0.274	A1
	$P(X>45 \text{ on 3rd throw} \mid \text{in final})=\frac{"0.3446"^2\times"0.2743"}{"0.959"}$	dM1
	$=\frac{"0.0326..."}{"0.9590..."}$ (calc: 0.033952…)	
	= awrt **0.034**	A1
		(5)
		Total 8

13.

(a)	$X \sim N(170, 16^2)$		
	$P(X > 190) = P\left(Z > \frac{190-170}{16}\right)\left[= P(Z > 1.25)\right]$		M1
	$[= 1 - 0.8944] = 0.1056$	awrt 0.106	A1
			(2)
(b)	$P(X > d) = 0.9$		
	$\frac{d-170}{16} = -1.2816$ or $\frac{170-d}{16} = 1.2816$	(Calc value ±1.28155…)	M1 A1
	$d = 149.494...$	awrt 149	dA1
			(3)

		Notes	Total 5
		NB correct answers with no working scores no marks	
(a)	**M1**	For standardising using 190, 170 and 16 (May be implied by 1.25)	
	A1	awrt 0.106 Do not ISW	
(b)	**M1**	For standardising and setting = z value, where $1 < \|z\| < 2$	
	A1	A fully correct standardisation = a correct compatible z value to 4 dp or better.	
	dA1	Dependent on previous A1 149.5 or awrt 149	

14.

(a)	$\frac{162-\mu}{\sigma} = -1.2816$ (Calculator gives –1.28155…) or $\frac{175-\mu}{\sigma} = 1.04$ (Calculator gives 1.03987…)	M1 A1 A1
	$\mu - 1.2816\sigma = 162$	
	$\mu + 1.04\sigma = 175$	
	$2.3216\sigma = 13$	dM1
	$\sigma = 5.5995...$ awrt 5.6 $\mu = 169.1764...$ awrt 169	A1
		(5)
b)	$Q_1 = 208.26$ or $Q_3 - Q_1 = 13.48$	B1
	$221.74 + 1.5(221.74 - 'Q_1')[= 241.96]$ or $'Q_1' - 1.5(221.74 - 'Q_1')[= 188.04]$	M1
	Probability of an outlier / **1 – Probability of not an outlier**	
	$[P(B > '241.96') =]$ $P\left(Z > \frac{'241.96' - 215}{10}\right)[= P(Z > 2.70)]$ or $[P(B < '188.04') =]$ $P\left(Z < \frac{'188.04' - 215}{10}\right)[= P(Z < -2.70)]$ or	$[P('188.04' < B < '241.96') =]$ $P\left(\frac{'188.04' - 215}{10} < Z < \frac{'241.96' - 215}{10}\right)$ $[= P(-2.70) < Z < P(2.70)]$	dM1
	$= 0.0035$ (Calculator gives 0.0034883…)	$= 0.993$ (Calculator gives 0.99298…)	
	$P(\text{Outlier}) = 2 \times '0.0035'$	$P(\text{Outlier}) = 1 - '0.993'$	M1
	$= 0.007$ (Calculator gives 0.006976…) awrt 0.007	$= 0.007$ (Calculator gives 0.007017…) awrt 0.007	A1
			(5)

15.

(a)(i)	$P(S > 640) = P\left(Z > \frac{640-700}{50}\right)$	M1
	awrt **0.885**	A1 (2)
(ii)	$675 < S < 725$	M1
	$P(675 < S < 725) = P(S < 725) - P(S < 675)$ **or** use of symmetry to find correct area	M1
	$P(S < 725) = P\left(Z < \frac{725-700}{50}\right)$ or $P(S < 675) = P\left(Z < \frac{675-700}{50}\right)$	M1
	$P(-0.5 < Z < 0.5) = 0.6915 - (1 - 0.6915)$ or $1-2\times0.3085$ or $2\times(0.6915-0.5)$	A1
	$= 0.383$ awrt **0.383**	A1
		(5)
b)(i)	$\frac{680-\mu}{\sigma} = 1.5$ $\frac{599-\mu}{\sigma} = -0.5244$	M1A1 A1
		(3)
(ii)	$(680-\mu)-(599-\mu) = 1.5\sigma-(-0.5244)\sigma$	M1
	$(81 = 2.0244\sigma)$	
	$\sigma = 40.01185$ $\mu = 619.98....$ awrt **40** (to 2sf) awrt **620** (to 3sf)	A1A1
		(3)
		Total 13

16.

(a)	$P(X<18) = P\left(Z < \pm\left(\frac{18-15}{2}\right)[=\pm 1.5]\right)$	M1
	$= 0.9332$ awrt 0.933	A1
		(2)
(b)	$\frac{x-15}{2} = 0.2533$	M1B1
	$x = 15.506\ldots$ awrt 15.5	A1
		(3)
(c)	$P(T > \mu - 10) = 0.975$	M1
	$\frac{(\mu \pm 10) - \mu}{\sigma} = \pm 1.96 \Rightarrow \sigma = \frac{10}{1.96}[=5.10\ldots]$	M1
	$P(T > \mu - 5) = P\left(Z > \frac{\mu - 5 - \mu}{"5.10\ldots"}[=-0.98]\right)[=0.836\ldots]$	M1
	$P(T > \mu - 5 \mid T > \mu - 10) = \frac{"0.836\ldots"}{"0.975"}$	M1
	$= 0.8579\ldots$ awrt 0.858	A1
		(5)
	Notes	**Total 10**

17.

(a)	$X \sim N(170, 16^2)$		
	$P(X > 190) = P\left(Z > \frac{190-170}{16}\right)\left[= P(Z > 1.25)\right]$		M1
	$[= 1 - 0.8944] = 0.1056$	awrt 0.106	A1
			(2)
(b)	$P(X > d) = 0.9$		
	$\frac{d-170}{16} = -1.2816$ or $\frac{170-d}{16} = 1.2816$	(Calc value $\pm 1.28155\ldots$)	M1 A1
	$d = 149.494\ldots$	awrt 149	dA1
			(3)

		Notes	Total 5
		NB correct answers with no working scores no marks	
(a)	**M1**	For standardising using 190, 170 and 16 (May be implied by 1.25)	
	A1	awrt 0.106 Do not ISW	
(b)	**M1**	For standardising and setting = z value, where $1 < \lvert z \rvert < 2$	
	A1	A fully correct standardisation = a correct compatible z value to 4 dp or better.	
	dA1	Dependent on previous A1 149.5 or awrt 149	

18.

3 (a)	$\frac{162-\mu}{\sigma} = -1.2816$ (Calculator gives –1.28155…) or $\frac{175-\mu}{\sigma} = 1.04$ (Calculator gives 1.03987…)	M1 A1 A1
	$\mu - 1.2816\sigma = 162$	
	$\mu + 1.04\sigma = 175$	
	$2.3216\sigma = 13$	dM1
	$\sigma = 5.5995...$ awrt 5.6 $\mu = 169.1764...$ awrt 169	A1
		(5)
(b)	$Q_1 = 208.26$ or $Q_3 - Q_1 = 13.48$	B1
	$221.74 + 1.5(221.74 - '\mathrm{Q}_1')[= 241.96]$ or $'\mathrm{Q}_1' - 1.5(221.74 - '\mathrm{Q}_1')[= 188.04]$	M1
	Probability of an outlier / **1 – Probability of not an outlier**	
	$\left[\mathrm{P}(B > '241.96') = \right]$ $\mathrm{P}\left(Z > \frac{'241.96' - 215}{10}\right)\left[= \mathrm{P}(Z > 2.70)\right]$ or $\left[\mathrm{P}(B < '188.04') = \right]$ $\mathrm{P}\left(Z < \frac{'188.04' - 215}{10}\right)\left[= \mathrm{P}(Z < -2.70)\right]$ or	$\left[\mathrm{P}('188.04' < B < '241.96') = \right]$ $\mathrm{P}\left(\frac{'188.04' - 215}{10} < Z < \frac{'241.96' - 215}{10}\right)$ $\left[= \mathrm{P}(-2.70) < Z < \mathrm{P}(2.70)\right]$	dM1
	$= 0.0035$ (Calculator gives 0.0034883…)	$= 0.993$ (Calculator gives 0.99298…)	
	$\mathrm{P(Outlier)} = 2 \times '0.0035'$	$\mathrm{P(Outlier)} = 1 - '0.993'$	M1
	$= 0.007$ (Calculator gives 0.006976…) awrt 0.007	$= 0.007$ (Calculator gives 0.007017…) awrt 0.007	A1
			(5)