The Pigeonhole Principle

If n pigeonholes are occupied by n+1 or more pigeons, then at least one pigeonhole is occupied by greater than one pigeon. Generalized pigeonhole principle is: – If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

Examples 1–3 show how the pigeonhole principle is used.

EXAMPLE 1 Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays. ◀

EXAMPLE 2 In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet. ◀

EXAMPLE 3 How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Solution: There are 101 possible scores on the final. The pigeonhole principle shows that among any 102 students there must be at least 2 students with the same score. ◀

PIEGON HOLE PRINCIPLE

If (N + 1) or more objects are placed into N boxes then there is atleast one box containing two or more objects.

Example : If 6 colour are used to paint 37 home. Show that at least 7 home of them will be of same colour.

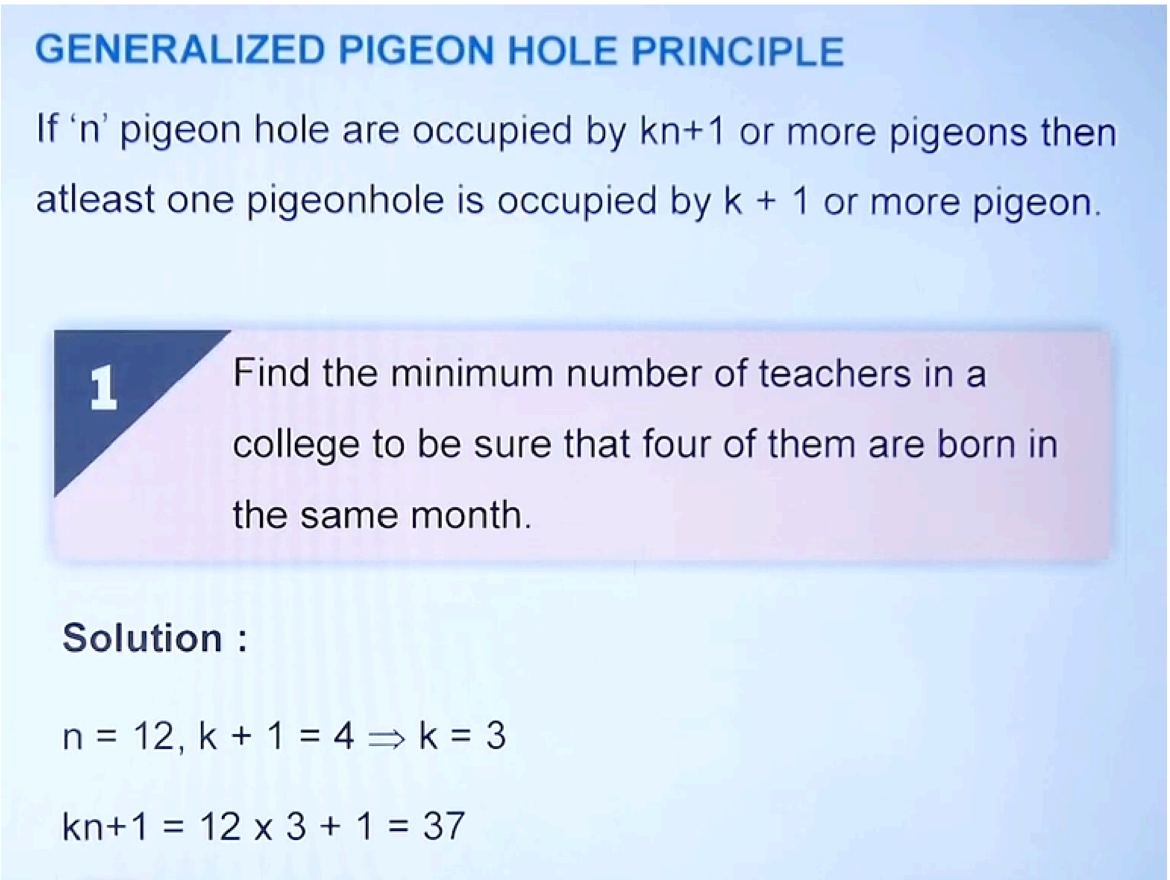
Solution :

$$\frac{37}{6} = 6$$

$\Rightarrow$ 6 home each of 6 colour.

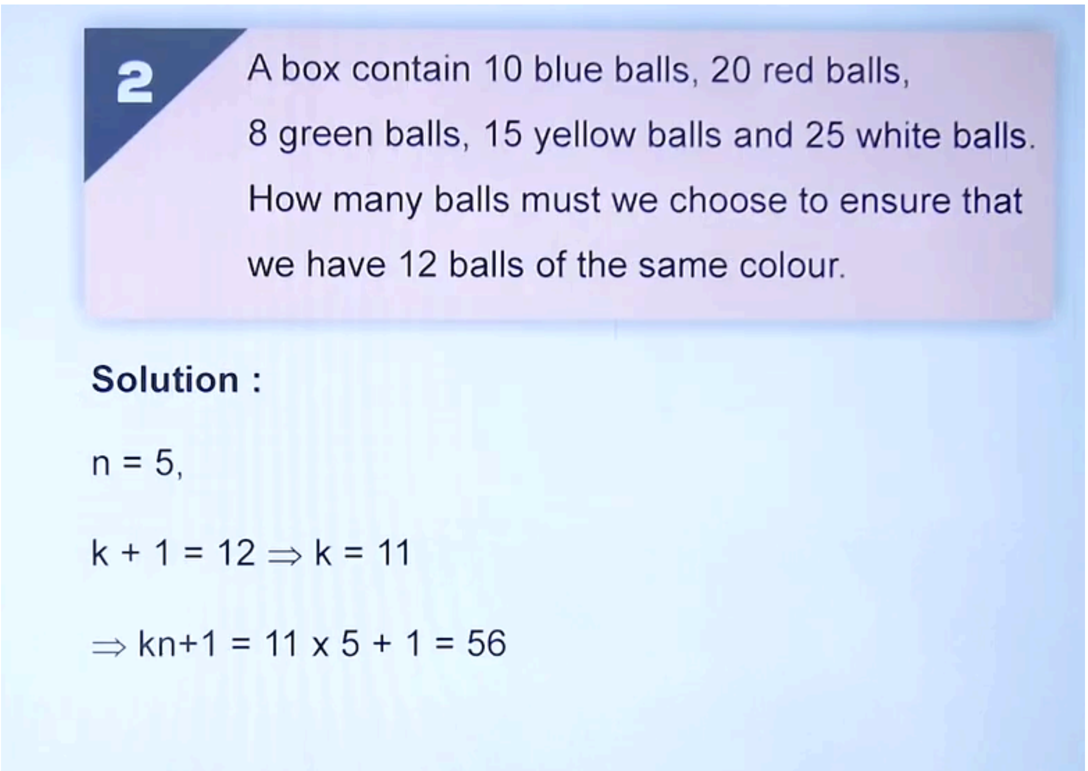
but remainder 1 will be a colour form the 6.

$\Rightarrow$ 7 home may have a same cycle.



Example1: Find the minimum number of students in a class to be sure that three of them are born in the same month.

Solution: Here n = 12 months are the Pigeonholes
And k + 1 = 3
K = 2



3 Prove that among 1,00,000 people there are two who are born on same time.

Solution :

Let $P = \{P_1, P_2, \ldots P_{1,00,000}\}$ be the set of people.

Let $H = \{H_1, H_2, \ldots H_{24}\}$ be the set of all hours in day

By extended pigeon hole principle, there are atleast

$\left[\frac{|P|}{|H|}\right]$ person during the same hour.

$$= \left[\frac{100000}{24}\right] = 4167$$

Now $Q = \{Q_1, Q_2, \ldots, Q_{4167}\}$ and $M = \{M_1, M_2, \ldots, M_{60}\}$

Then number of person born in same minute are atleast

$$= \left[\frac{4167}{60}\right] = 70$$

Again $R = \{R_1, R_2, \ldots, R_7\}$ & $S = \{S_1, S_2, \ldots, S_{60}\}$

Then number of person born in same second are atleast

$$= \left[\frac{70}{60}\right] = 2$$