MEE 342

Gearbox, Compound Reverted Double-Reduction

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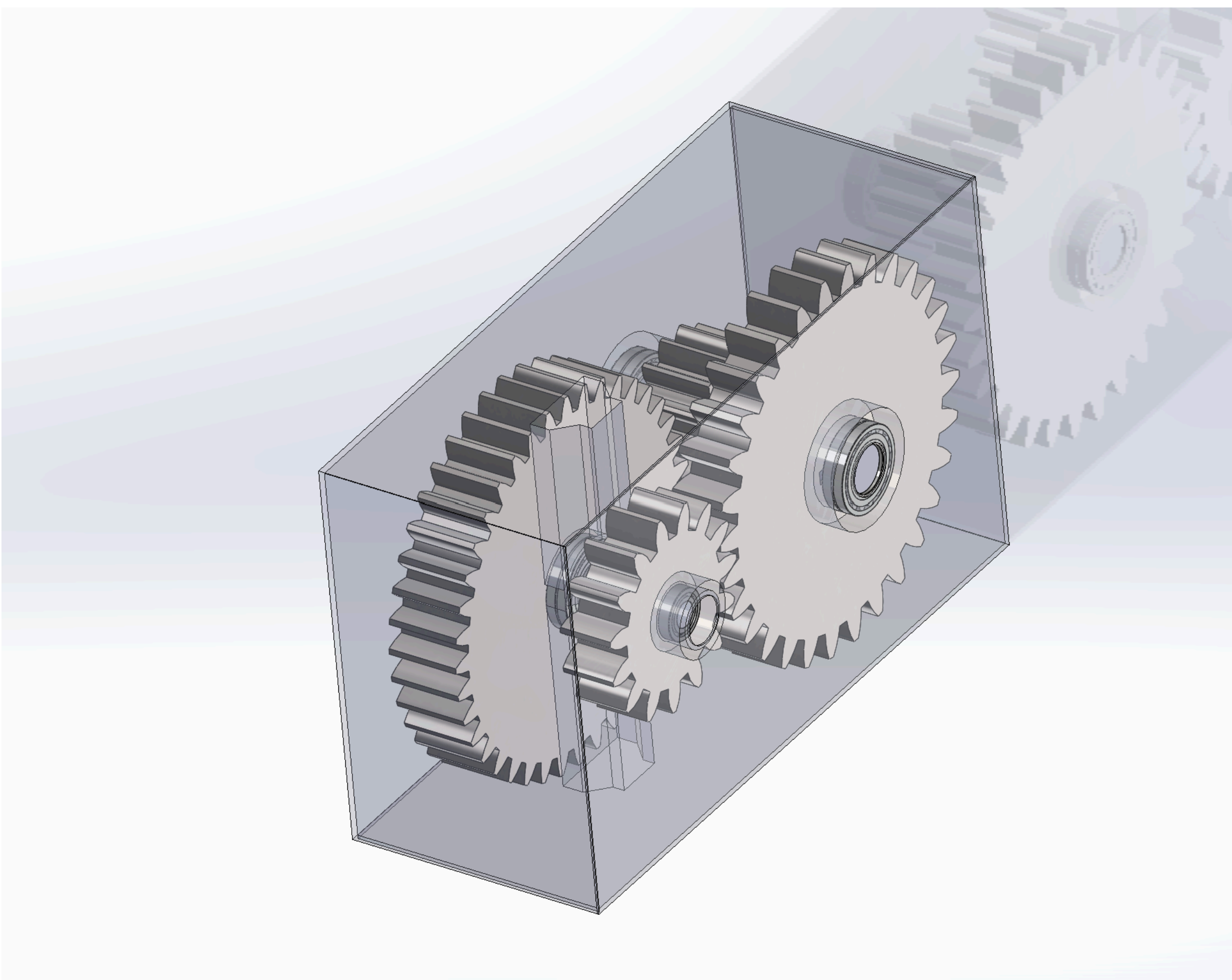

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Introduction

The point of this project is to create a double reduction gear chain to achieve a desired rotational speed when given an input. There are many different ways to go about creating this reduction. For this scenario, what is sought out to create a variety of different parts to optimize this gear chain. The design includes 4 gears, 3 shafts, 6 bearings, and 4 keys. There are many variables that are involved in the optimization of each part, with different factors of safety and optimization techniques. Most of the calculations for this study were conducted in an excel sheet that can be found in the Appendix.

Design Parameters

Design Parameters	Requirement	Specification
Gear Fatigue load	99% Reliability	10^8 cycles
Design Life	Maximize	5 years
Gear surface factor of safety	Maximize	$\eta \geq 1.2$
Gear bending factor of safety	Maximize	$\eta \geq 1.5$
Key factor of safety	Maximize	$\eta = 2$
Shaft factor of safety	Maximize	$\eta_f \geq 2, \eta_y \geq 2$
Bearing reliability	99% Reliability	$\eta = 1$
Yield and Fatigue factors of safety	90% Reliability	$\eta \geq 2$

Table 1

There are many factors that can be changed in this system. The goal will be to optimize every part of the system while minimizing the amount of material and space used. This system should have a cycle life of 5 years. One goal that is factored is that a key must be the weakest point of the system because it is the cheapest option to fix or replace.

Gear design

Given Values #88			
P (hp)	d_s (in)	n_{in} (rpm)	n_{out} (rpm)
125	15	1000	5000

Table 2

Gear	Teeth	Pitch Diameter (in)	m_g	n (rpm)	F (in)	$\sigma_{b,all}$	$\sigma_{c,all}$	HB_c
1	40	21.4287	2.5	1000	4	2334.59	31970.9	120.21
2	16	8.5714	2.5	2500	4	3236.34	50169.3	188.69
3	30	20	2	2500	3	1719.95	31924.0	120.03
4	15	10	2	5000	3	2494.37	45307.9	170.39

Table 3

$$N_{P,\min} = \frac{2k}{(1+2m_G)\sin^2\phi}\left[m_G + \sqrt{{m_G}^2 + (1+2m_G)\sin^2\phi}\right]$$

$$N_{G,\max} = \frac{{N_P}^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi}$$

$$m_G = \frac{N_G}{N_P} \text{ (Gearing Ratio)}$$

The process started with analyzing the gear train. The desired output was a gear ratio of 1:5. To achieve this, the gear ratio was broken into a ratio of 1:2 and 1:2.5. The parameters of the gears being spur gears with a pitch of 20^o because it is common in industry. First the minimum number of teeth was found and for each ratio. Once you have the minimum, you can solve for the resulting teeth on the other gear. The gear would also have full depth teeth. The safety factor was 1.5 for bending and 1.2 for the surface.

Gear	W_t(lbf)	Qv	Cp	Km
A	735.29	7	2300	1.2198
B	735.29	7	2300	1.2018
C	315.12	5	2300	1.2035
D	315.12	5	2300	1.1967

Table 4

$P = \frac{N}{d}$ $W_t = 33000\frac{H}{V}(lbf)$ $V = \frac{\pi d n}{12}$

$$\sigma_{b,all} = \begin{cases} \frac{S_t}{S_F}\frac{Y_N}{K_T K_R} & \text{US Customary Units} \\ \frac{S_t}{S_F}\frac{Y_N}{Y_\theta Y_Z} & \text{SI Units} \end{cases}$$

$$\sigma_{c,all} = \begin{cases} \frac{S_C}{S_H}\frac{Z_N C_H}{K_T K_R} & \text{US Customary Units} \\ \frac{S_C}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z} & \text{SI Units} \end{cases}$$

The system will be a uniform driven machine and a power source of 125 hp. The gear used for this project were commercial gears with uncrowned teeth. This will be an enclosed unit with only external gears spinning. The material decided to be used was grade 2 hardened steel. This was used to solve for Brinell Hardness for both the allowable bending and torsional stress. As a result, the material chosen for the gears is 1050 cold drawn steel.

Sb = HBb x 102 + 16400 Sc = HBc x 349 + 34300

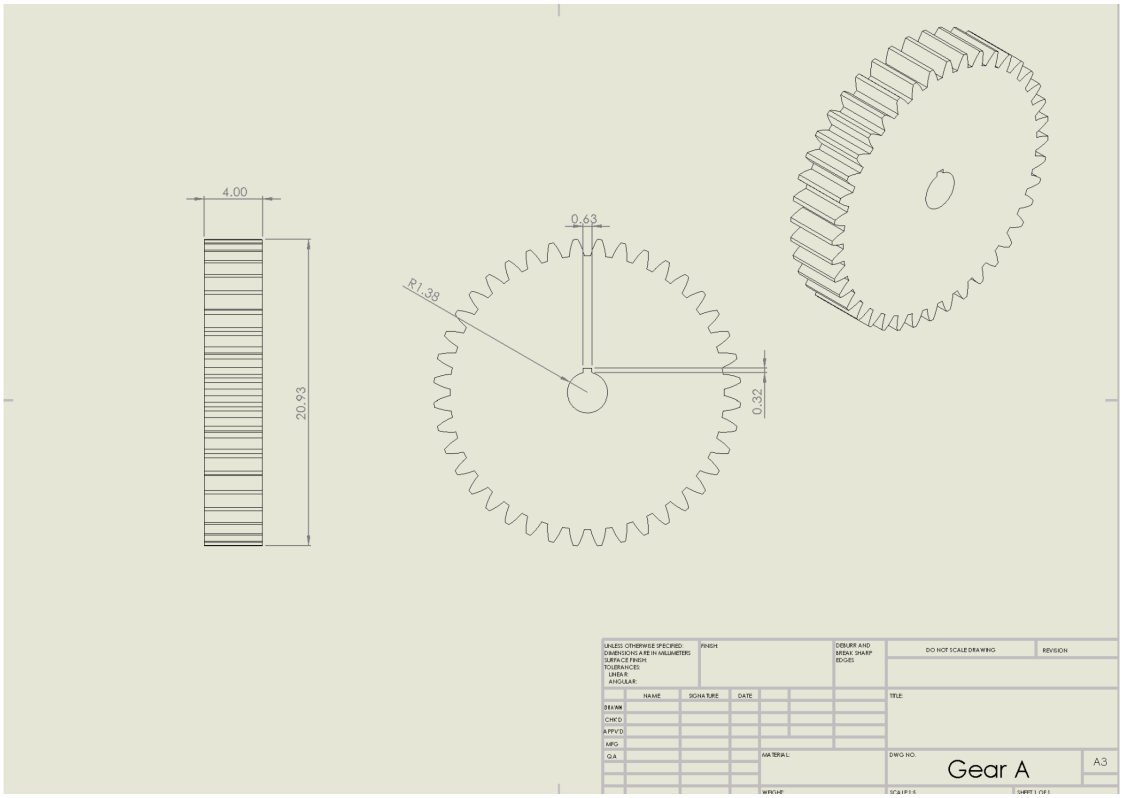

Figure 1: Drive gear

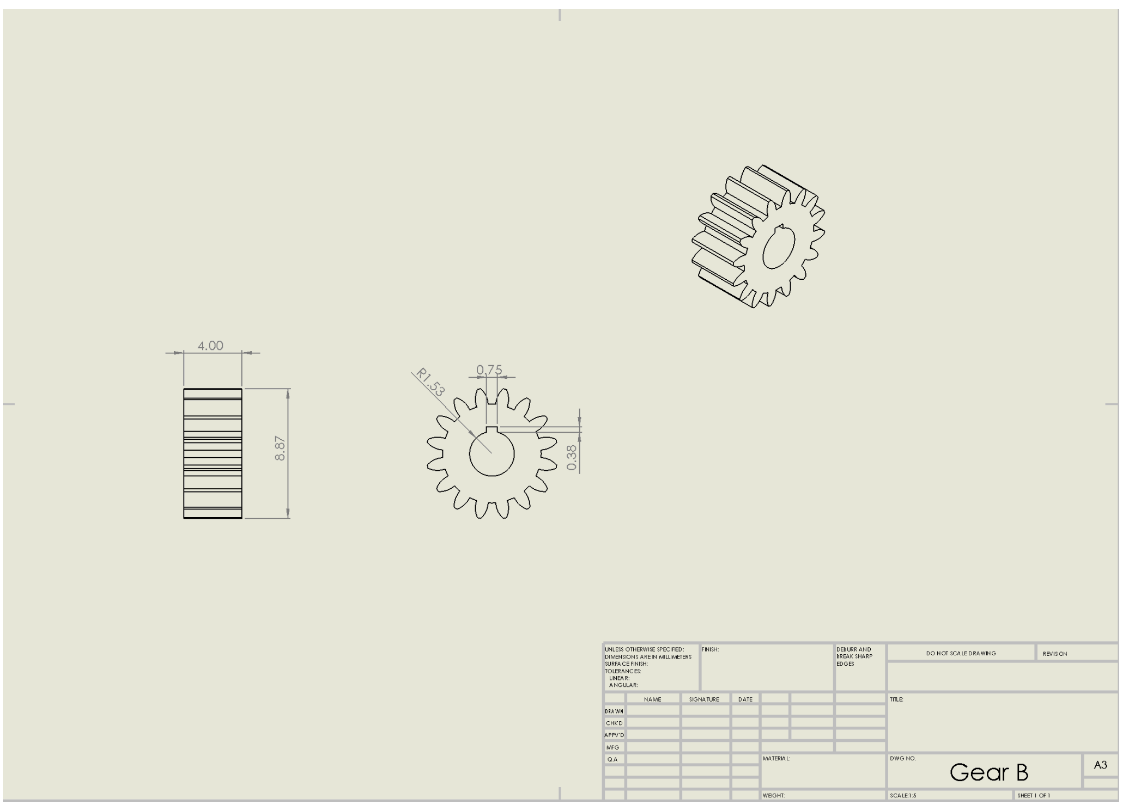

Figure 2: Second gear

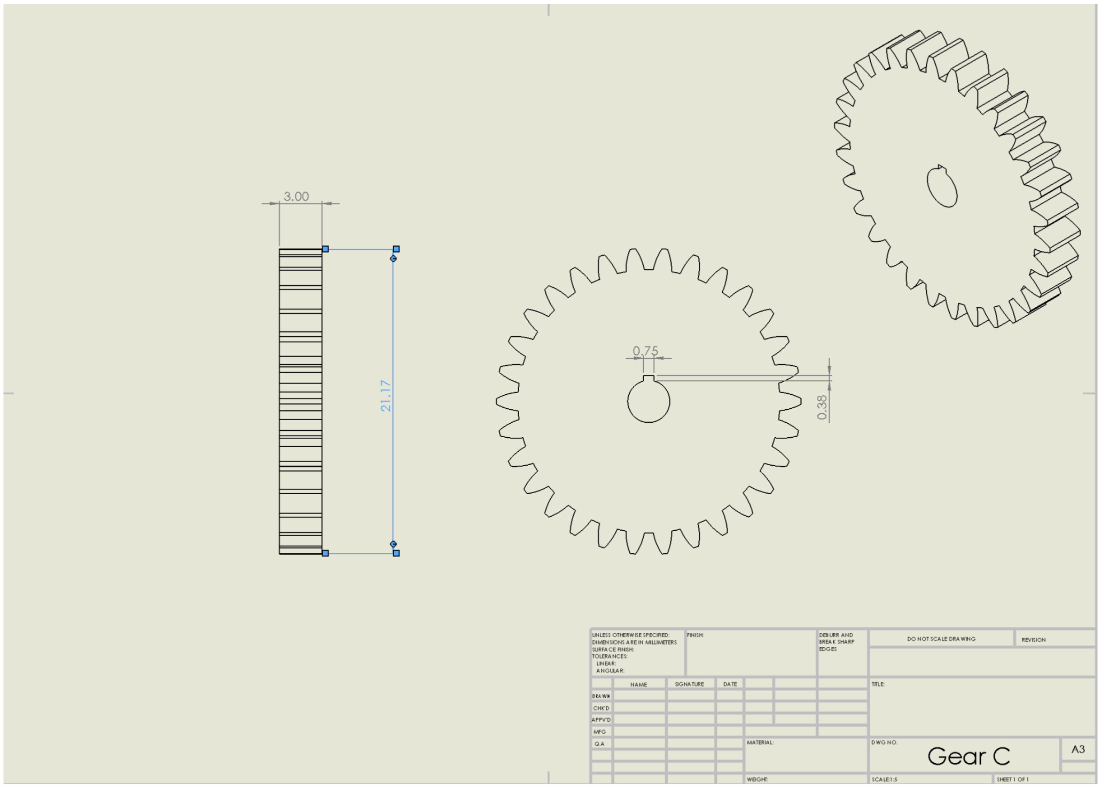

Figure 3: Third gear

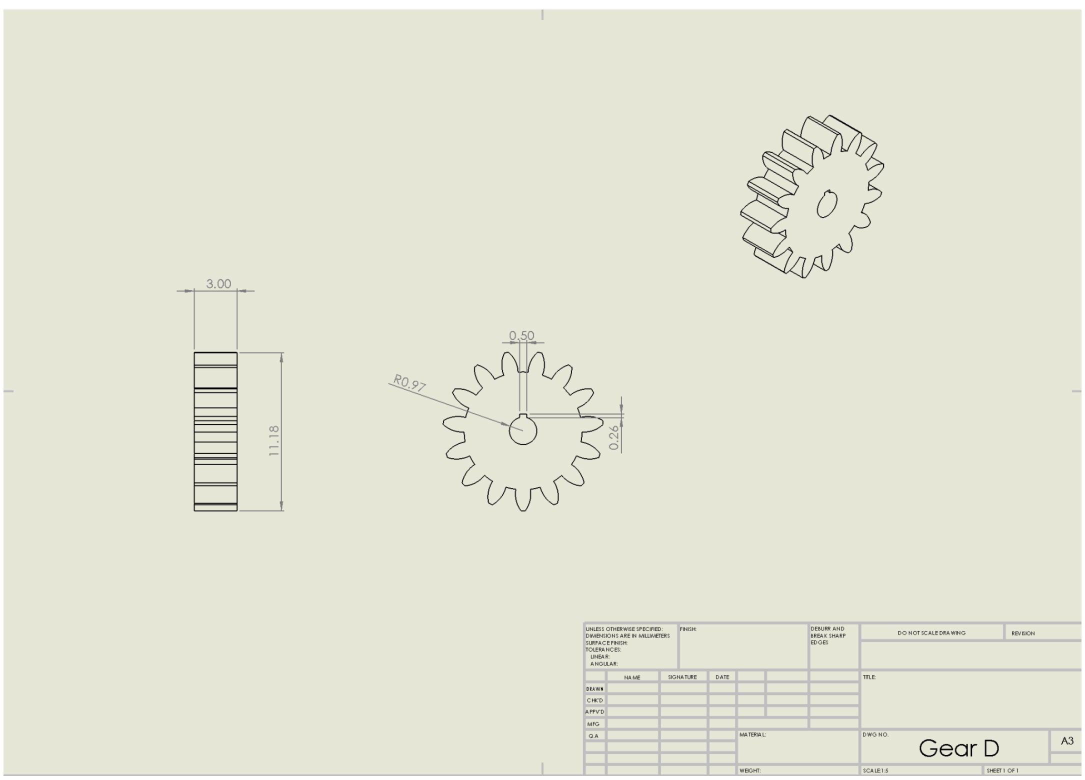

Figure 4: Output gear

Shaft Design

Reaction Forces at the Bearings (lbf)				Reaction moments at the Bearings (lbf in)			
	;Shaft A	Shaft B	Shaft C		;Shaft A	Shaft B	Shaft C
F_{ax}	133.81	211.74	57.34	M_{ax}	401.43	635.24	143.37
F_{bx}	133.81	170.57	57.34	M_{bx}	401.43	511.72	143.37
F_{ay}	367.64	569.65	157.56	M_{ay}	1102.94	1708.95	393.90
F_{by}	367.64	480.77	157.56	M_{by}	1102.94	1442.31	393.90
F_{a}	391.24	607.73	167.67	M_{a}	1173.72	1823.20	419.18
F_{b}	391.24	510.13	167.67	M_{b}	1173.72	1530.39	419.18

Table 5

Shaft A

$\Sigma F_x = 0 : F_{ax} + F_{bx} = W_{ta}$

$\Sigma F_y = 0 : F_{ay} + F_{by} = W_{ta}\, x\, tan(20)$

$\Sigma M_{ax} = 0 : F_{bx}\, x\, 6 = W_{ta}\, x\, 3$

$\Sigma M_{ay} = 0 : F_{by}\, x\, 6 = W_{ta}\, x\, 3\, x\, tan(20)$

Shaft B

$\Sigma F_x = 0 : F_{ax} + F_{bx} = W_{tb} + W_{tc}$

$\Sigma F_y = 0 : F_{ay} + F_{by} = W_{tb}\, x\, tan(20) + W_{tc}\, x\, tan(20)$

$\Sigma M_{ax} = 0 : F_{bx}\, x\, 13 = W_{tb}\, x\, 4 + W_{tc}\, x\, 10.5$

$\Sigma M_{ay} = 0 : F_{by}\, x\, 13 = W_{tb}\, x\, 4\, x\, tan(20) + W_{tc}\, x\, tan(20)\, x\, 10.5$

Shaft C

$\Sigma F_x = 0 : F_{ax} + F_{bx} = W_{td}$

$\Sigma F_y = 0 : F_{ay} + F_{by} = W_{td}\, x\, tan(20)$

$\Sigma M_{ax} = 0 : F_{bx}\, x\, 5 = W_{td}\, x\, 2.5$

$\Sigma M_{ay} = 0 : F_{by}\, x\, 5 = W_{td}\, x\, 2.5\, x\, tan(20)$

$F = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$

$M = \sqrt{M_x^{\ 2} + M_y^{\ 2}}$

The shafts have been designed to resist the torsional and shear forces that will be caused by the gears and the bearings. Holes were also designed for the keys to fit into. The drive and output gears are also designed for the gear to sit in the center of the shaft for simplicity. The shafts went through many different iterations until I got the final product. The shafts must be designed to allow the gears to slide onto the shaft so the thickest points must be the center of the shaft. The shaft must also be designed so the key will fit into the cut outs.

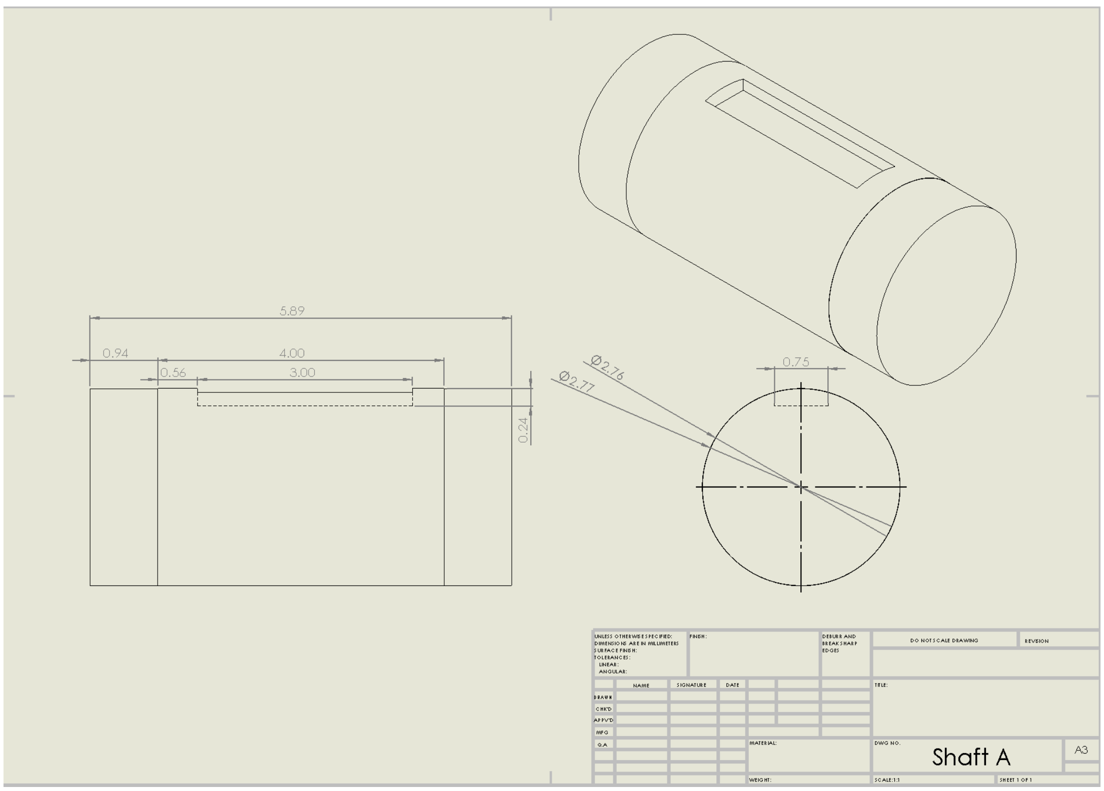

Figure 5: First shaft

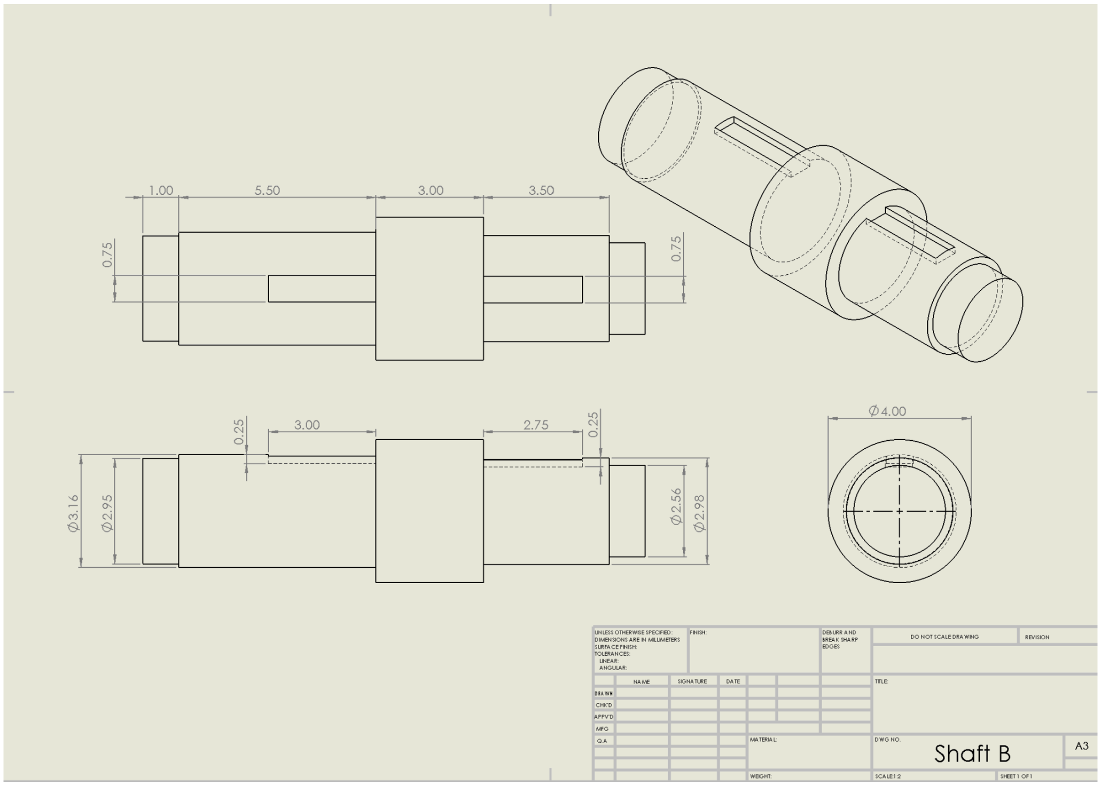

Figure 6: Second shaft that holds Gear B & Gear C

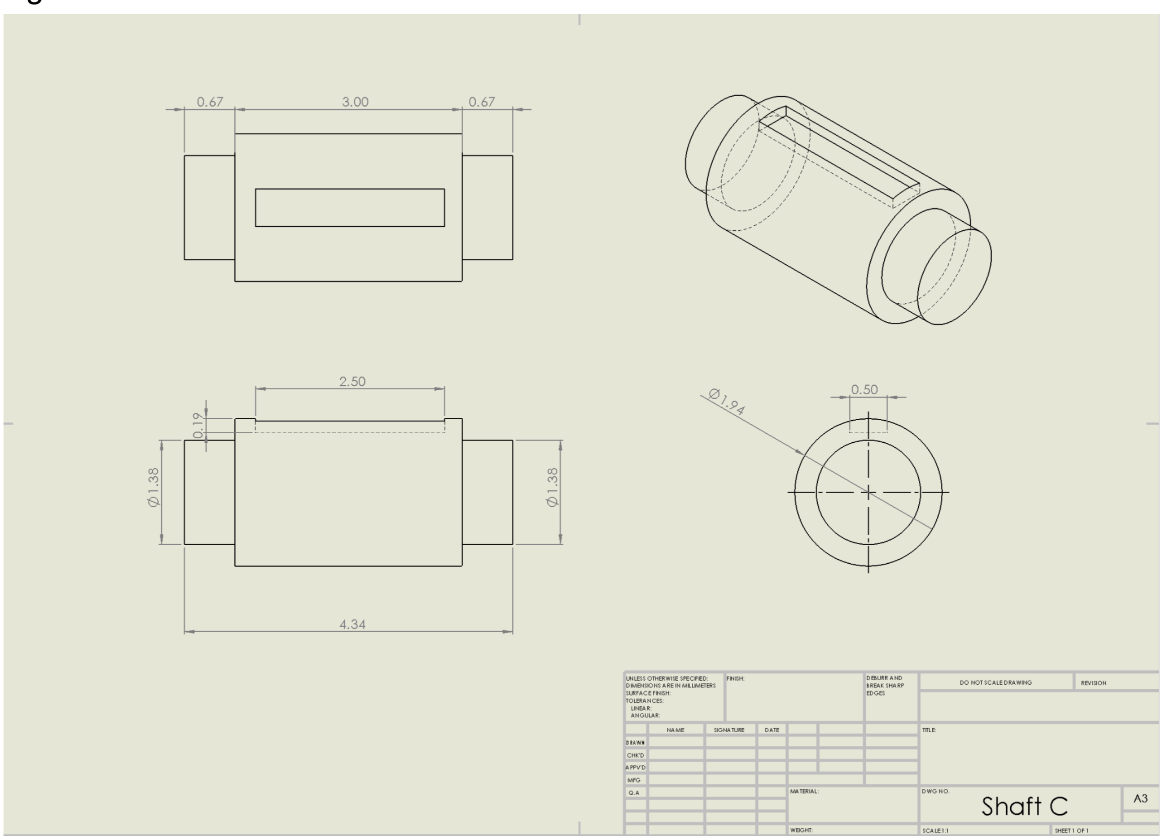

Figure 7: Third shaft

Bearing Design

Bearing Calculations						
	N (rpm)	Reaction Force	Applicatio n Factor	Bore (mm)	OD (mm)	Width (mm)
Bearing A 1	1000	391.25	1.5	70	125	24
Bearing A 2	1000	391.25	1.5	70	125	24
Bearing B 1	2500	607.75	1.5	75	130	25
Bearing B 2	2500	510.15	1.5	65	120	23
Bearing C 1	5000	167.68	1.5	35	72	17
Bearing C 2	5000	167.68	1.5	35	72	17

Table 6

The bearings took many different iterations until the final product. They were designed with sharp shoulder filets. The rating life is the number of revolutions that 90% of all bearings reach or exceed before failing. The rating life for most bearings in industry is 10^6cycles. A design life of 5 years was used for these bearings. Straight rolled, deep groove bearing, were opted for in this final design. The inner ring was rotating in the bearing. The reliability was 0.99 for all the bearings as well. The design created allowed the use of the same 2 bearings for shaft A & shaft C. The shafts were designed with an Sy of 420 and and Sut of 560.

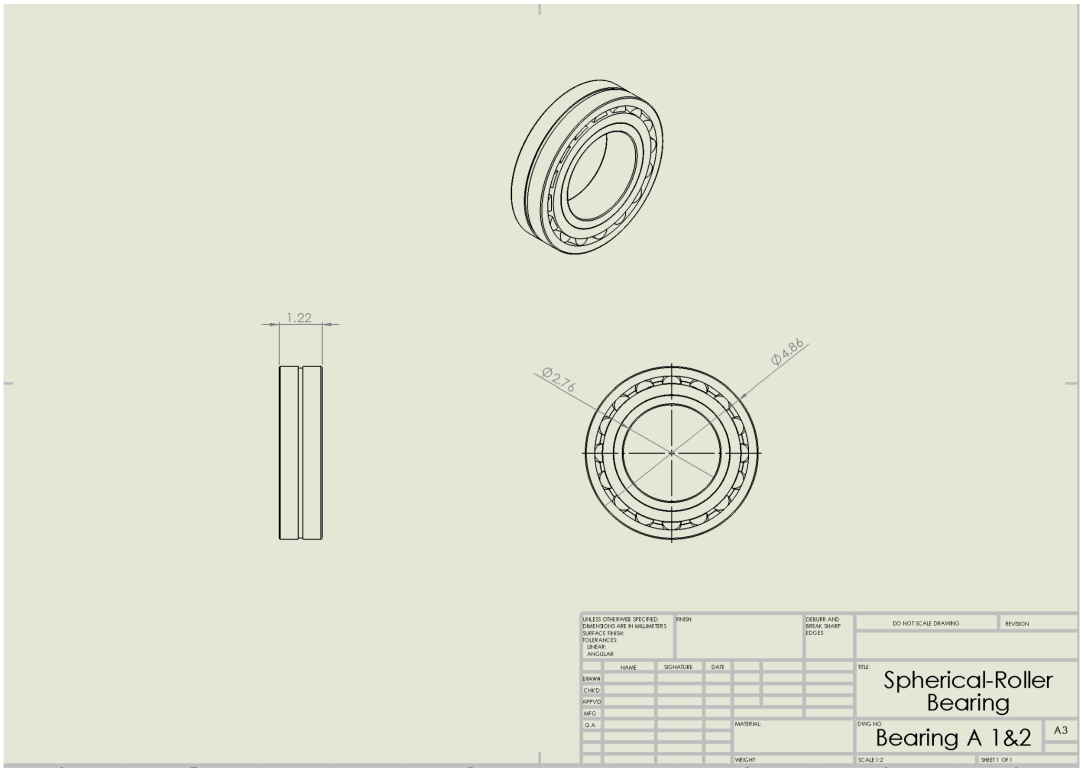

Figure 8: Bearings for the first shaft

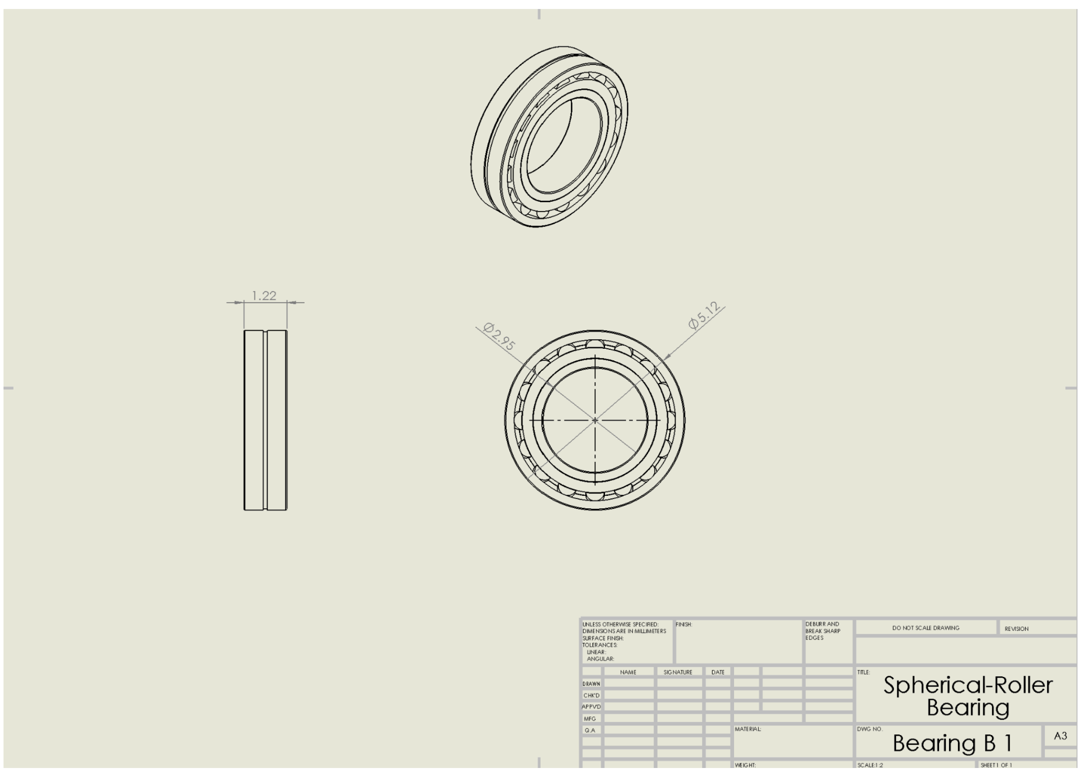

Figure 9: Bearings for the left part of the second shaft

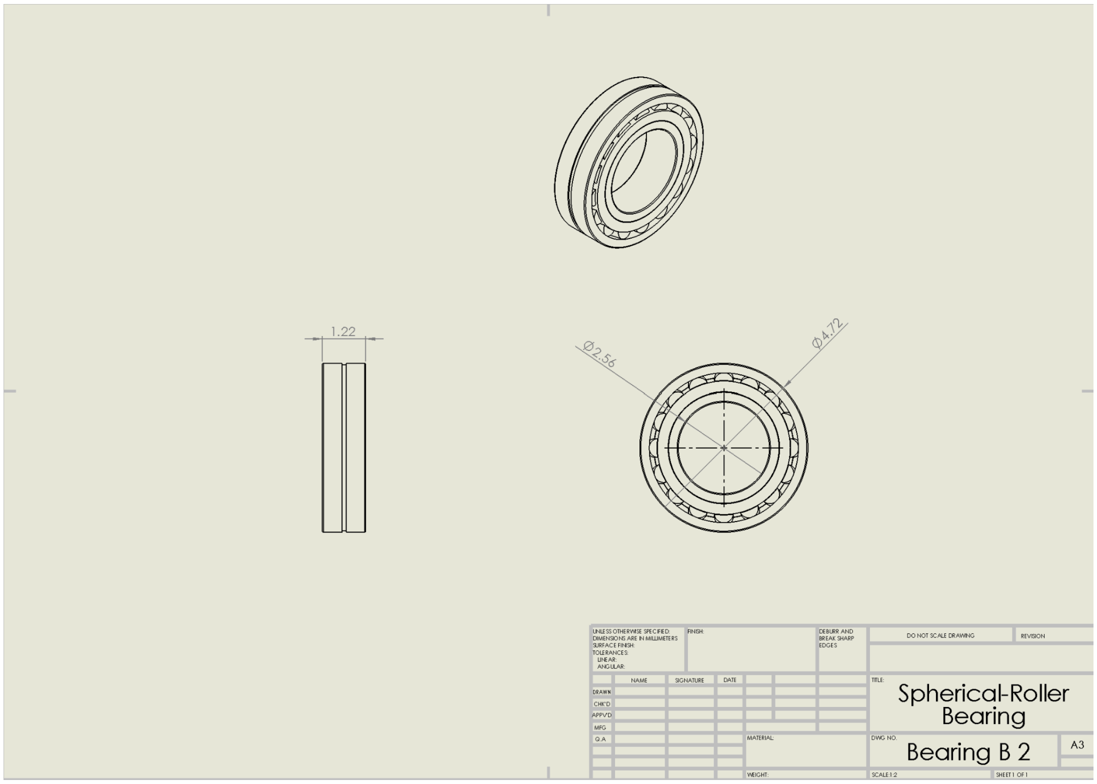

Figure 10: Bearings for the right part of the second shaft

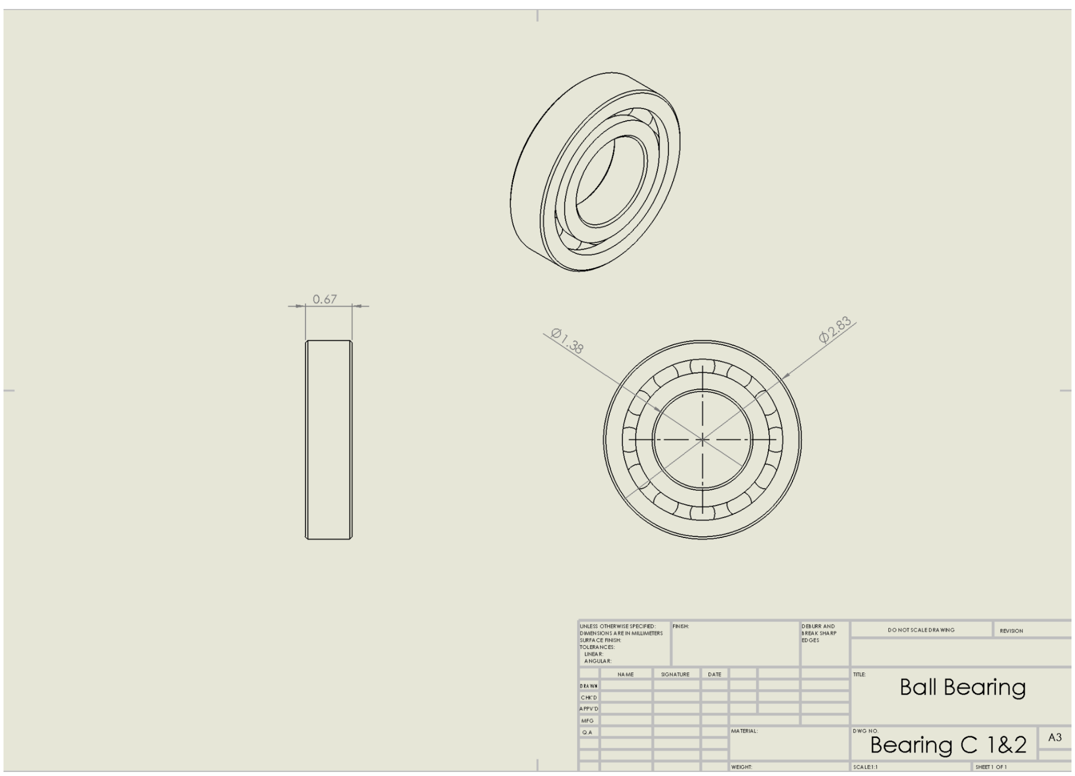

Figure 11: Bearings for the first shaft

Keys design

$$F_t = \frac{T}{r} \qquad \tau = \frac{F_t}{Lxw} \qquad C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)\left[\ln\left(\frac{1}{R_D}\right)\right]^{1/b}} \right]^{1/a}$$

$$\sigma' = \frac{F_t}{L}\sqrt{\frac{4}{h^2} + \frac{3}{w^2}} = 30000$$

The keys were designed with the lowest factor of safety compared to the rest of the system so they are the failure point of the entire system. The forces can be taken from the torsion caused by the gears and the shaft radius where the gears sit. I assigned my factor of safety to be nkey = 2 for the key while all other factors of safeties were n > 2 The keys used were square shaped. The shear max yield will be 54000 psi the first key and 30000 for the other 3.

Key	w (in)	h (in)	Keyway Depth (in)	Ft	L (in)
A	.75	.5	.25	5700.311	1.95
B	.75	.5	.25	1996.929	1.229
C	.75	.5	.25	2114.896	1.302
D	.5	.375	.1875	1620.311	1.373

Table 7

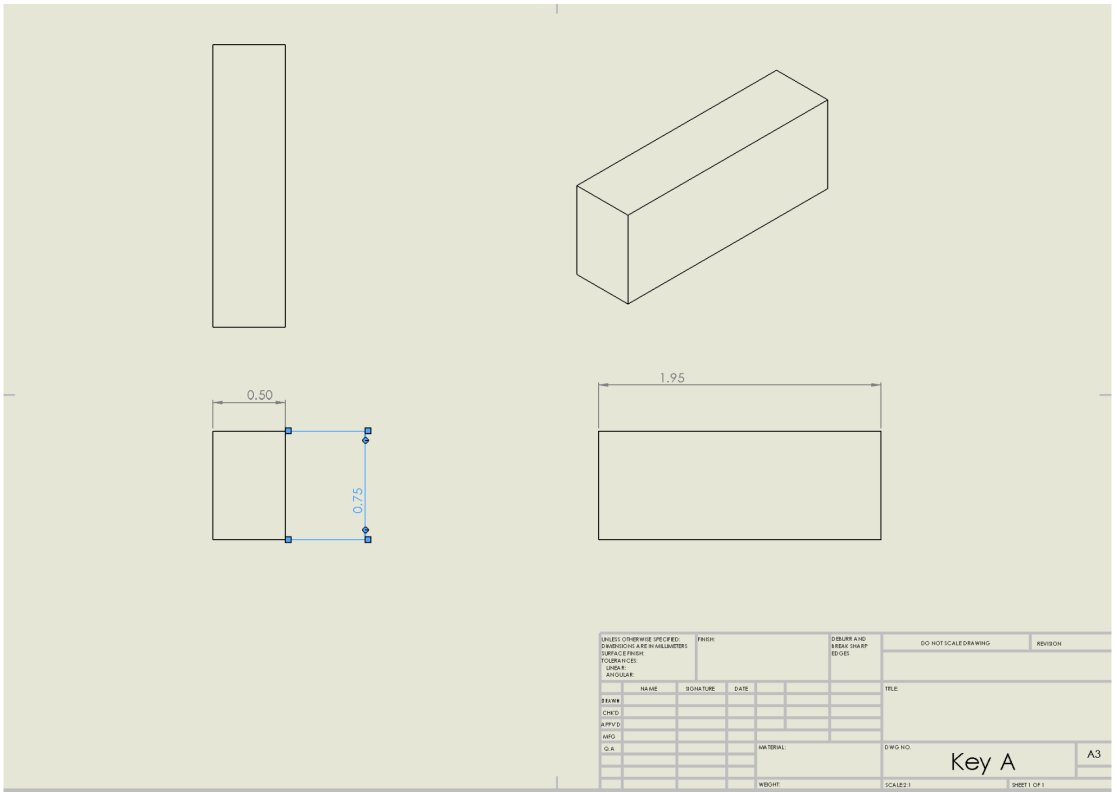

Figure 12: Key for gear A

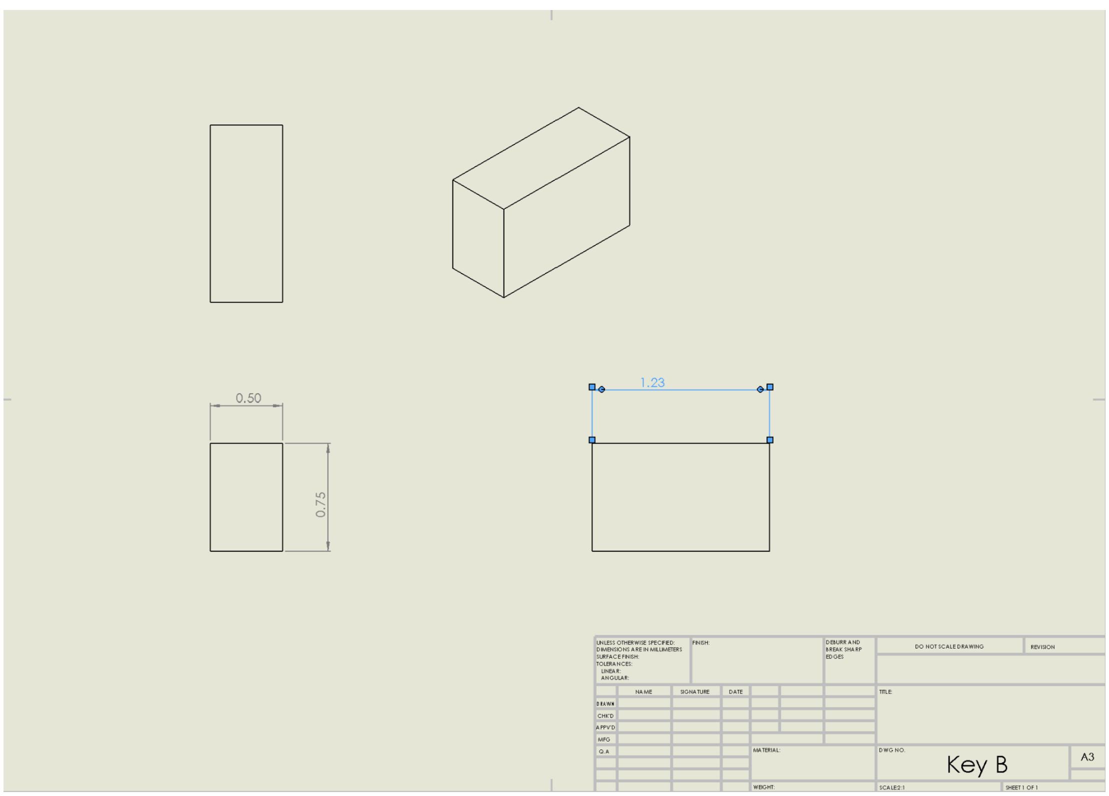

Figure 13: Key for gear B

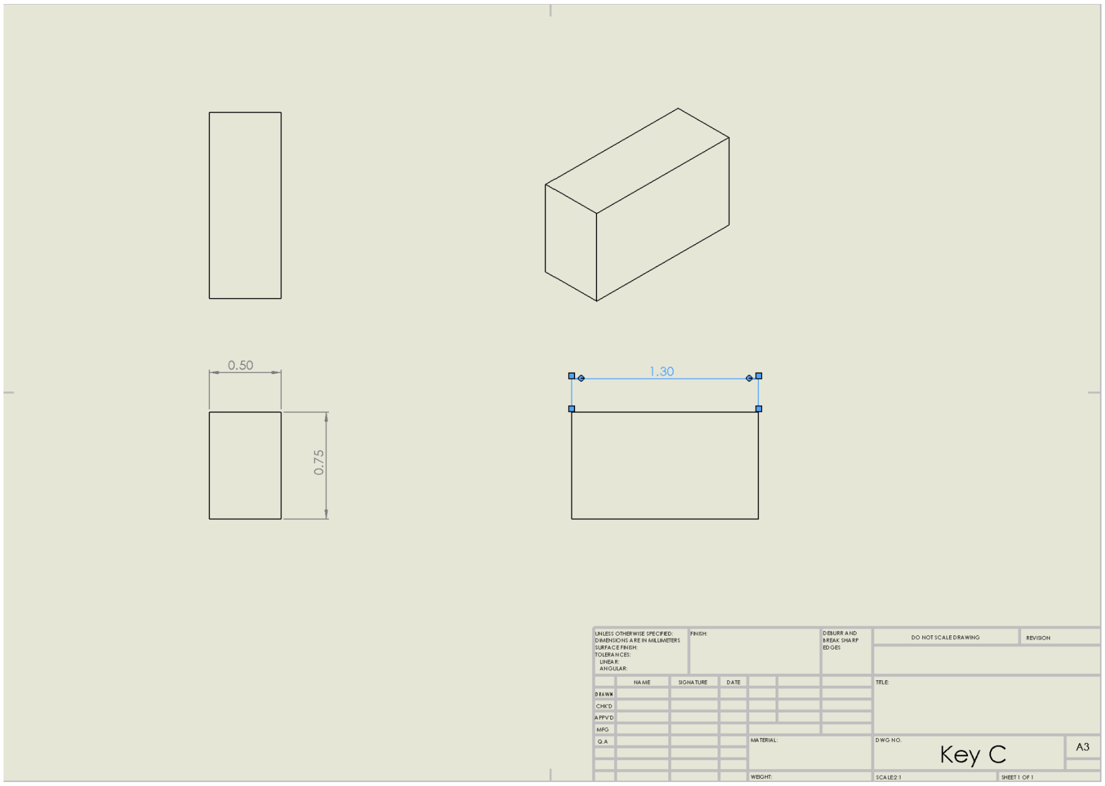

Figure 14: Key for gear C

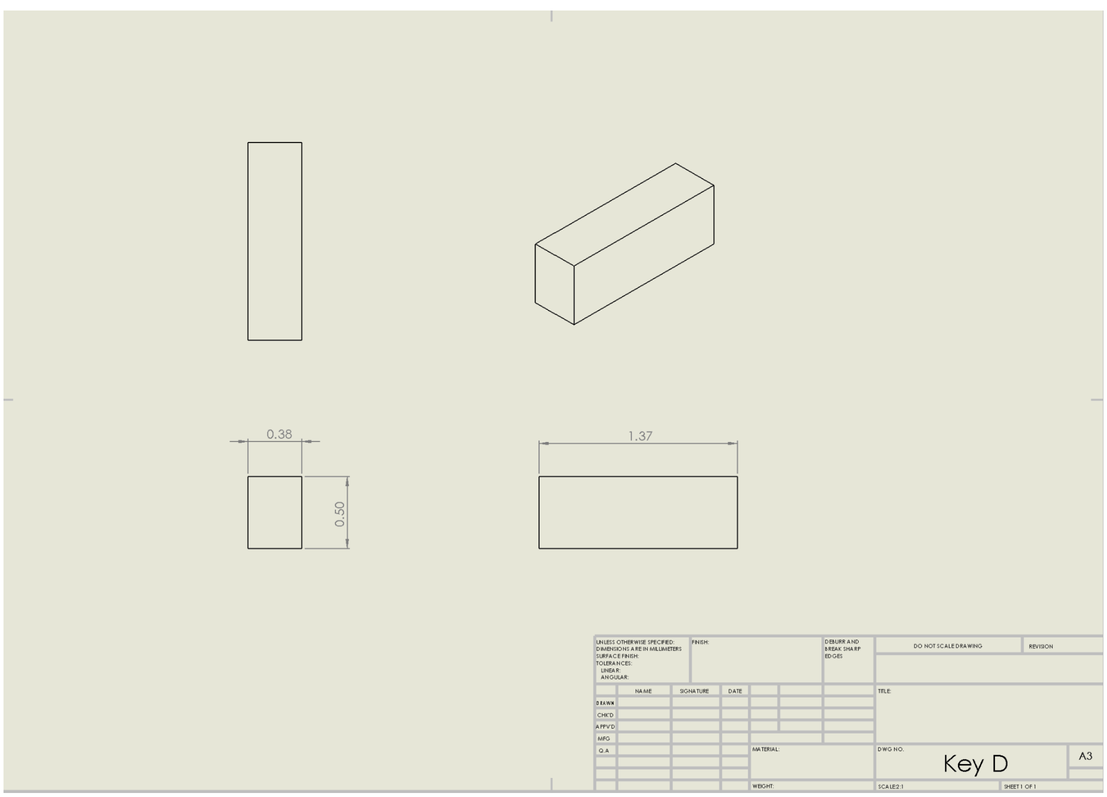

Figure 15: Key for gear D

Case design

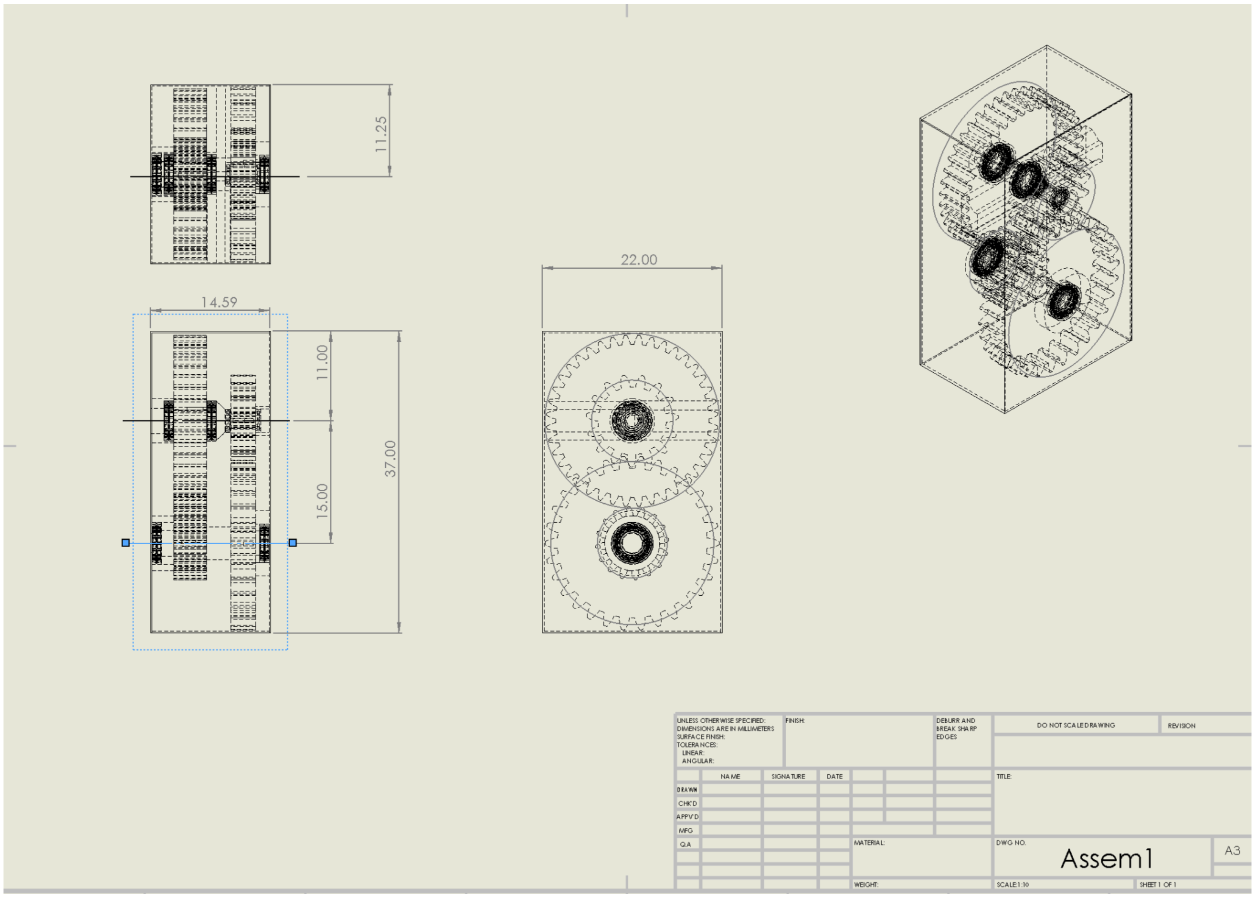

Figure 16: Assembly

ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Shaft A		1
2	Shaft B		1
3	Inch - Spur gear 2DP 40T 20PA 4FW --- S40N3.0H2.0L2.75S1		1
4	Inch - Spur gear 2DP 16T 20PA 4FW --- S16N3.0H2.0L3.0625S1		1
5	Shaft C		1
6	Inch - Spur gear 1.5DP 15T 20PA 3FW --- S15N3.0H2.0L1.9375S1		1
7	Inch - Spur gear 1.5DP 30T 20PA 3FW --- S30N3.0H2.0L2.9375S1		1
8	2721T11	Spherical-Roller Bearing	2
9	2721T12	Spherical-Roller Bearing	1
10	2721T9	Spherical-Roller Bearing	1
11	5972K317	Ball Bearing	2
12	Key A		1
13	Key D		1
14	Key B		1
15	Key C		1
16	Case 2		1
17	Case 1		1

Figure 17: Bill of materials

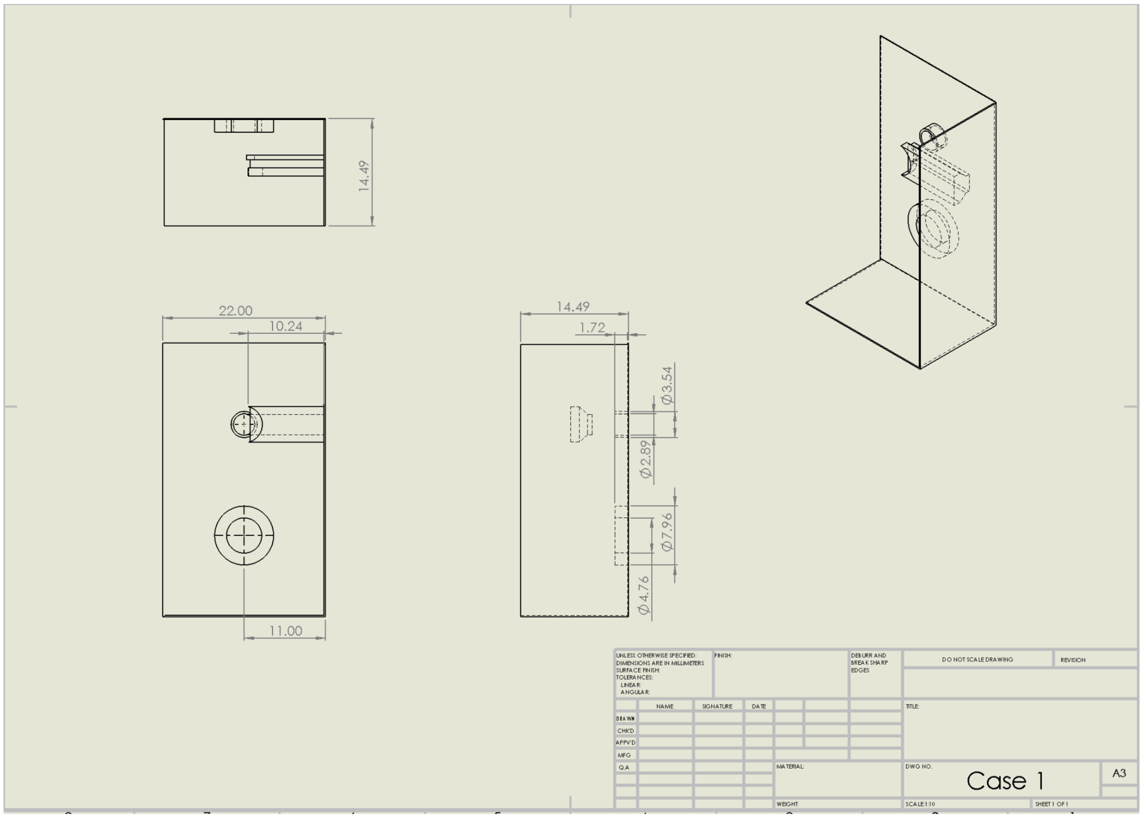

Figure 18: Half of the casing

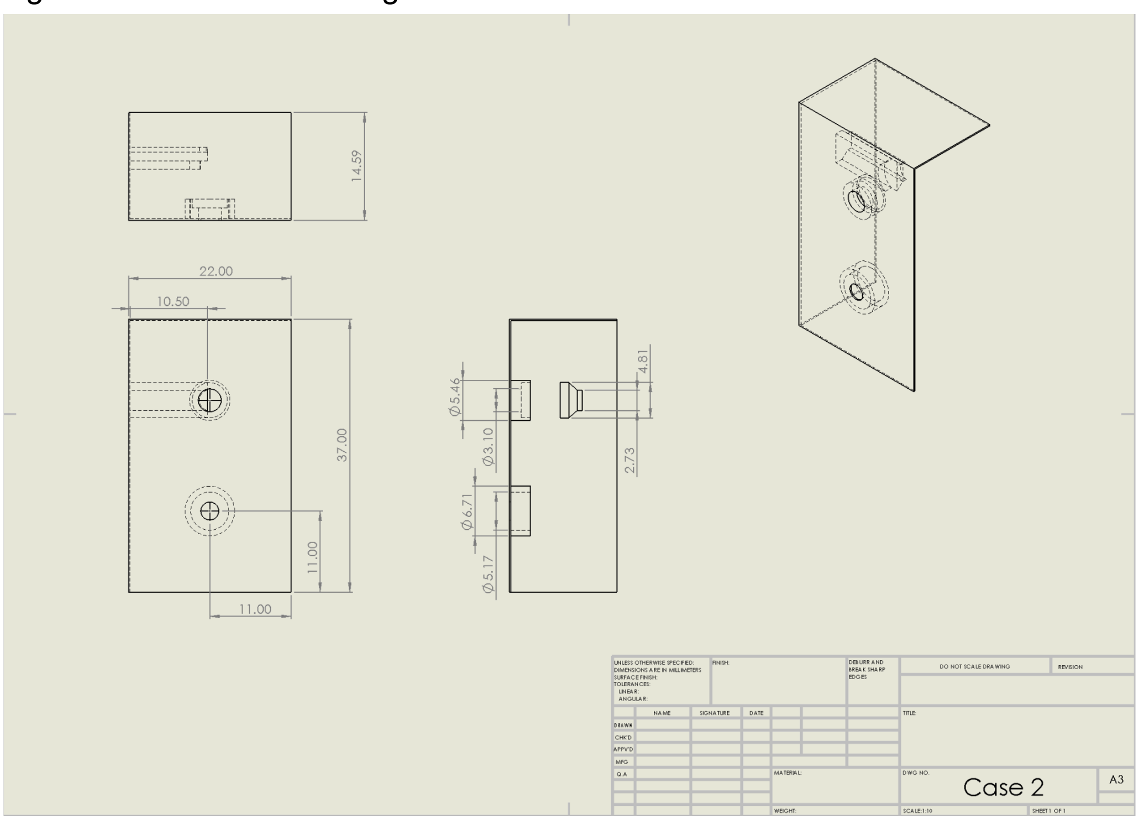

Figure 19: Second half of the casing

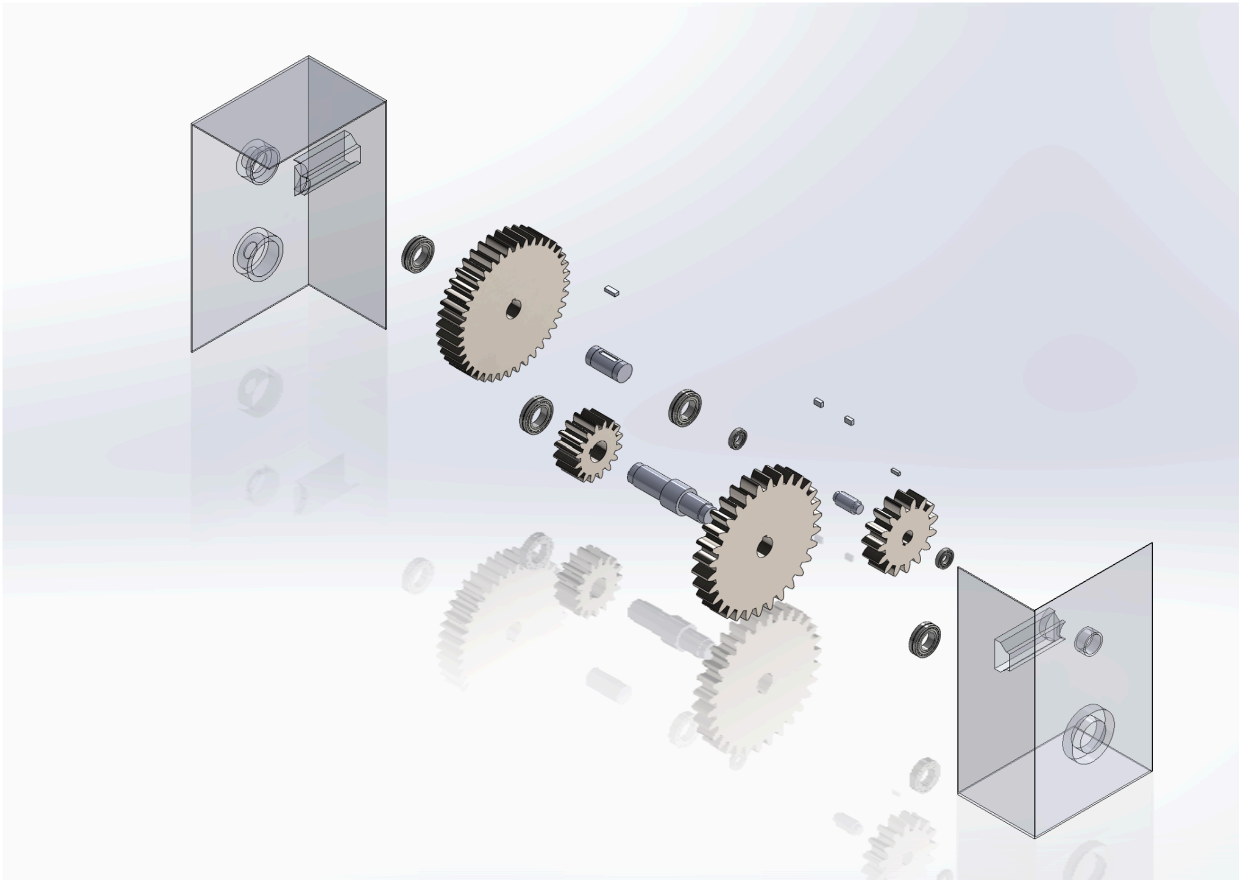
Figure 20: Exploded view of assembly

Conclusion

There is so much freedom in the variables that can be changed in gear trains. The goal was to create a gear system that could withhold a large resistance while also minimizing the material used. This experiment gave the opportunity to explore this optimization process.

An error that kept occurring was that the bearings were requiring a larger diameter then the diameter the gear nearest to it was sitting on. There were many iterations created before settling on this design. The gears and shaft are much wider than the first iteration. This changed the material used for most of the variables while solving, however this allowed the geometry to physically fit together.

Appendix

This is the excel sheet with all calculations

https://docs.google.com/spreadsheets/d/1zONR_xymv25fl4jB50-yLIkx-6DxbYDOAOgV
gLSkYfQ/edit?usp=sharing