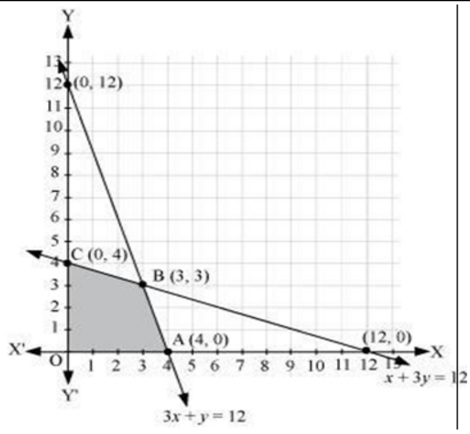
PRE-BOARD EXAM 2023-24
CLASS: XII
MATHEMATICS

TIME: 3 hrs. **MARKING SCHEME** **Max Marks: 80**

QUESTION NUMBER	ANSWER/SOLUTION	MARKS
1	c	1
2	c	1
3	b	1
4	b	1
5	c	1
6	d	1
7	c	1
8	d	1
9	a	1
10	c	1
11	a	1
12	d	1
13.	a	1
14	b	1
15	a	1
16	d	1
17	a	1
18	a	1
19	a	1
20	c	1
21	$cos^{-1}\left(cos\left(\frac{7\pi}{6}\right)\right) = cos^{-1}\left(cos\left(2\pi - \frac{5\pi}{6}\right)\right)$ $= cos^{-1}\left(cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$	1 1
22	$A = \pi r^2$ $dA/dr = 2\pi r$ at $r = 6$ $dA/dr = 12\pi$ cm^2/cm OR $f'(x) = 3x^2 - 6x + 4$	½ ½ 1 1

	$= 3(x-1)^2 + 1 > 0$, in every interval of R. Therefore, the function f is increasing on R.	1
23	$2x + 3y = \sin y$. $2 + 3\,dy/dx = \cos y\,dy/dx$ $dy/dx = 2/(\cos y - 3)$	1 1
24	$\frac{2}{a} = \frac{-3}{6} = \frac{4}{-8}$ a= -4 OR $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$ and $\vec{a} - \vec{b} = -4\hat{i} + (7-\lambda)\hat{k}$ $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ will be orthogonal if, $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$ i.e., if, $-24 + (49 - \lambda^2) = 0 \Rightarrow \lambda^2 = 25$ i.e., if, $\lambda = \pm 5$	1 1 1 1
25	Direction of the required line = Direction of given line $\vec{b} = 2\hat{i} - 5\hat{j} + 3\hat{k}$ $\vec{r} = \vec{a} + \lambda\vec{b}$ Required equation of line $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(2\hat{i} - 5\hat{j} + 3\hat{k})$	1 1
26	$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$ Solving A = 3/5, B = 2/5, C = 1/5 $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int\frac{dx}{(x+2)} + \frac{1}{5}\int\frac{2x\,dx}{(x^2+1)} + \frac{1}{5}\int\frac{dx}{((x^2+1)}$ $= \frac{3}{5}\log\|x+2\| + \frac{1}{5}\log\|(x^2+1)\| + \frac{1}{5}\tan^{-1}x + c$	1 1 1
27	Putting $U = x^{\cos x}$ and $V = (\cos x)^{\sin x}$ Finding $\frac{dU}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$ Finding $\frac{dV}{dx} = (\cos x)^{\sin x}(\cos x \log\cos x - \sin x \tan x)$ Writing the value of $\frac{dy}{dx}$ OR Finding $y_1 = \frac{2\tan^{-1}x}{1+x^2} \Rightarrow (1+x^2)y_1 = 2\tan^{-1}x$ Again differentiating & obtaining the result	1 1 1 1.5 1.5
28	Getting $\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ This is a homogeneous diff. eq. So let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Putting in (i) and getting $-\frac{2}{v^2}dv = \frac{1}{x}dx$	1 1

	Integrating we get $-\left(-\frac{2}{v}\right) = \log log\,\vert x\vert + c$ Putting $\frac{2x}{y} = \log log\,\vert x\vert + c$ is the required solution OR $\frac{dy}{dx} + ysec^2x = \frac{tanx}{cos^2x}$ $I.F = e^{tanx}$ $ye^{tanx} = \int t.e^t dt + c \quad put\ t = tanx$ $y = (tanx - 1) + ce^{-tanx}$	1 1 1 1
29	$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx \quad \ldots(1)$ $\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}\, dx$ [Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx \quad \ldots(2)$ Adding (1) and (2), we have $2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\right) dx$ $2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\right) dx \quad \Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx \quad 2I = [x]_0^{\frac{\pi}{2}} \quad \Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$ OR Writing I = $\int_0^{\pi/4} \frac{Sin\ x + \cos x}{25 - 16 + 16\sin 2x} dx$ Taking 16 common & putting 1-sin 2x=(sin x – cos x)2. Substitution of sin x – cos x = t & limit change Integrating & getting the result $\frac{1}{40}\log 9$	 1 1 1 1 1 1
30.	Maximize Z = 17.5x + 7y … (1) subject to the constraints, $x + 3y \le 12$ … (2) $3x + y \le 12$ … (3) $x, y \ge$ 0 … (4) The feasible region determined by the system of constraints is as follows.	1



The corner points are A (4, 0), B (3, 3), and C (0, 4). The values of Z at these corner points are as follows. 1.5

Corner point	$Z = 17.5x + 7y$	
O(0, 0)	0	
A(4, 0)	70	
B(3, 3)	73.5	→ Maximum
C(0, 4)	28	

0.5

The maximum value of Z is 73.50 at (3, 3).

31

A={(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

B={(1,2),(2,1),(2,2),(2,3),(3,2),(4,2),(2,4),(5,2),(2,5),(6,2),(2,6)} 1

$A \cap B$ = (5,2) (2,5) 1

P(A/B)= $\frac{2}{11}$ 1

32

Reflexive :$|a - a| = 0$, which is divisible by 4, $\forall\ a \in A$

$\therefore (a, a) \in R,\ \forall\ a \in A$ ∴ R is reflexive 1.5

Symmetric : Let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 4

$\Rightarrow |b - a|$ is divisible by 4 ($\because |a - b| = |b - a|$)

$\Rightarrow (b, a) \in R$ ∴ R is symmetric

Transitive : Let $(a, b),\ (b, c) \in R$

$\Rightarrow |a - b| \& |b - c|$ are divisible by 4 1.5

$$\Rightarrow a - b = \pm 4m,\ b - c = \pm 4n,\ m, n \in Z$$

Adding we get, $a - c = 4\ (\pm m \pm n)$

$\Rightarrow |a - c|$ is divisible by 4 $\therefore (a, c) \in R$ 2

∴ R is transitive

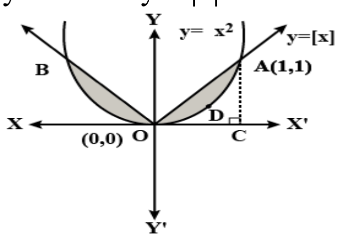
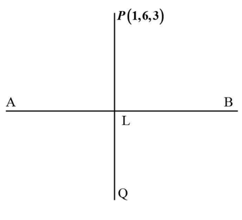
OR

1

1.5

1.5

1.5

	R = {(L_1, L_2): L_1 is parallel to L_2} R is reflexive as any line L_1 is parallel to itself i.e., (L_1, L_1) ∈ R. Now, let (L_1, L_2) ∈ R. ⇒ L_1 is parallel to L_2 ⇒ L_2 is parallel to L_1. ⇒ (L_2, L_1) ∈ R ∴ R is symmetric. Now, let (L_1, L_2), (L_2, L_3) ∈ R. ⇒ L_1 is parallel to L_2. Also, L_2 is parallel to L_3. ⇒ L_1 is parallel to L_3. ∴ R is transitive. Hence, R is an equivalence relation. The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$. Slope of line $y = 2x + 4$ is $m = 2$ It is known that parallel lines have the same slopes. The line parallel to the given line is of the form $y = 2x + c$, where $c \in \mathbf{R}$. Hence, the set of all lines related to the given line is given by $y = 2x + c$, where $c \in \mathbf{R}$.	1
33	Finding AB = $8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = 8I$ Writing given system as BX= C so that X = $B^{-1}C = \frac{1}{8}AC$ Putting values and getting sol as x= 3, y=-2, z= -1	1.5 1.5 2
34	y = x^2 and y = |x|  Required area =2{area of OACO – area of ODACO} $\int_0^1 x\,dx - \int_0^1 x^2\,dx$ = 1/3 sq. units **OR** The line y = 3x + 2 meets x-axis at x = -2/3 and its graph lies below x-axis for x $\in \left(-1, \frac{-2}{3}\right)$ and above x-axis for x $\in \left(\frac{-2}{3}, 1\right)$ The required area = Area of the region ACBA + Area of the region ADEA $= \left\|\int_{-1}^{\frac{-2}{3}} (3x+2)\,dx\right\| + \left\|\int_{\frac{-2}{3}}^{1} (3x+2)\,dx\right\|$ = 1/6 + 25/6 = 13/3 sq units	2 1 2 2 1 2
35	Let ***P*** (**1, 6, 3**) be the given point, and let **'** ***L*'** be the foot of the perpendicular from **'** ***P*** **'** to the given line ***AB*** (as shown in the figure below). The coordinates of a general point on the given line are given by 	

	$\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$, λ is a scalar, i.e., $x=\lambda$, $y=2\lambda+1$ and $z=3\lambda+2$ Let the coordinates of L be $(\lambda, 2\lambda+1, 3\lambda+2)$. So, direction ratios of PL are $\lambda-1, 2\lambda+1-6$ and $3\lambda+2-3$, *i.e.* $\lambda-1, 2\lambda-5$ and $3\lambda-1$. Direction ratios of the given line are **1, 2** and **3,** which is perpendicular to PL . Therefore, $(\lambda-1)1+(2\lambda-5)2+(3\lambda-1)3=0 \Rightarrow 14\lambda-14=0 \Rightarrow \lambda=1$ So, coordinates of L are **(1, 3, 5).** Let $Q(x_1, y_1, z_1)$ be the image of $P(1,6,3)$ in the given line. Then, L is the mid-point of PQ. Therefore, $\frac{(x_1+1)}{2}=1$, $\frac{(y_1+6)}{2}=3$ and $\frac{(z_1+3)}{2}=5$ $\Rightarrow x_1=1, y_1=0$ and $z_1=7$ Hence, the image of $P(1,6,3)$ in the given line is $(1,0,7)$. **Now, the distance of the point $(1,0,7)$ from the y - axis is $\sqrt{1^2+7^2}=\sqrt{50}$ *units*.**	1 1 1 1 1
36	(i) x =2πr r = x/2π m. (ii) The length of the wire will be needed to fence the squared garden = 112/(4+π) m.	1.5 2.5
37	(i) x=4 (ii) Maximum height=8cm (iii) Height after 2 days=6cm OR x=1	1 1 2

38.	A: he will come by cab B: he will come by metro C: he will come by bike D: he will come by other means E: HE arrives late P(A) = 0.3, P(B) = 0.2, P(C) = 0.1, P(D) = 0.4 P(E/A) = 0.25, P(E/B) = 0.3 P(E/C) = 0.35, P(E/D) =0.1 i)P(B/E) = $\frac{0.2\times0.3}{0.3\times0.25+0.2\times0.3+0.1\times0.35+0.4\times0.1} = \frac{6}{21} = 2/7$ ii) P(D/E) $= \frac{0.4\times0.1}{0.3\times0.25+0.2\times0.3+0.1\times0.35+0.4\times0.1} = \frac{4}{21}$	1 1.5 1.5