Unit	A6
Title	Exponentials II
Target Standards	**N.Q.2, A.SSE.1, A.SSE.2, A.SSE.3c, A.SSE.4, A.CED.1, A.CED.2, A.REI.11, F.IF.4, F.IF.6, F.IF.7e, F.IF.8b, F.BF.1a, F.BF.1b, F.LE.1, F.LE.2, F.LE.5, N.RN.1**
Mathematical Goals	
The story before this unit (including prior knowledge)	Students have worked with geometric sequences, and understand they change by a constant ratio over a constant interval. They are able to write both recursive and closed equations for them. Students understand the difference between a linear and exponential function, can recognize situations and tables described by each, and know that an exponential function will always overtake a linear function. They know that an exponential relationship gets out of control in one direction, and approaches a value asymptotically in the other direction. They have solved exponential equations of the form $a*b^x = c$ by graphing. They can construct an exponential function given a graph, description of a relationship, or two input output pairs (including in a table). Given an exponential equation, they can interpret its parameters in a context. They can also fit a simple exponential function to a scatterplot. Every exponential function until now has only involved the domain on the integers.
The part of the story happening in this unit	Students will revisit exponential functions, and should again be asked to construct exponential functions given a graph, description of a relationship, or two input-output pairs (including a table). They will be able to recognize when a quantity is growing by a constant percent, and write and interpret percent growth and decay functions. To model a context, students should be required to combine exponential functions with other functions using arithmetic operations, as well as translate and dilate an exponential function using both the equation and the graph. They should solve simple equations by rewriting terms so they have the same base. Students learn to fit an exponential function to scatterplot from data, with a domain not in the integers. Students will build on previous work with geometric sequences to derive a formula for the sum of a geometric series.
The story after this unit	Students are proficient at graphing, analyzing, solving, and modeling exponential situations. Students are comfortable with exponentials from a functions viewpoint. This lays the foundations for success in calculus. A later unit introduces logarithms as the solutions to exponential equations.

UNIT FLOW SUMMARY

UNIT xxx (xxx days)	**Title**
Section 0 (1 day)	**Diagnostic Pre-Unit Assessment**

Section 1 (1-3 days)	**Analyze and Create Simple Exponential Models (hook/umbrella activity)**
Section 2 (1 - 2 days)	**Understand and Explain Properties of Exponents**
Section 3 (1 - 2 days)	**Create and Compare Real-Life Exponential Models**
Section 4 (3 days)	**Analyze Real-Life Exponential Models**
Section 5 (2 days)	**Geometric Sequences and Series**
Section 6 (1 - 2 days)	**Motivate Exponential Equations**
Section 7 (1 day)	**Summative Assessment**

Section 0: 1 day	**Diagnostic Pre-unit assessment**
Pre-Unit Assessment Targets	Diagnose students' ability to ● create basic exponential data F.LE.2 ● create an exponential model for a simple context F.BF.1 ● apply exponent rules N.RN.1
Sample Activity	Unit A7 Pre-Assessment

Section 1: 1 - 3 days	**Analyze and Create Simple Exponential Models (hook/umbrella activity)**
Mathematical Goal	Create and analyze simple exponential arising from real-word data.
Narrative overview of section (and how the standards are achieved)	Students have been introduced to exponential models in A2. This unit starts with some real world contexts to motivate and remind students why exponential models are useful. The activities in this introductory section all have contexts that can be modeled as exponentials with integer domains (i.e. sequences) F.LE.2 This provides review of previous experience as needed for students.
Sample Activity 1.1	Mathalicious Xbox Xponential, Mathalicious

	WHAT: Computer scientist Gordon Moore made a prediction in the 1960's that computer processor speeds would double every two years. When it comes to the processors in video game consoles, how good was that prediction? In *Xbox Xponential*, students write and graph a doubling function based on Moore's Law, analyze what it would imply for game consoles, research and create a scatterplot of how console speeds have actually changed over time, graph and interpret a function that best fits the scatterplot, and compare the predicted speed and actual speed functions to analyze the accuracy of Moore's prediction. F.LE.2, F.LE.5, N.Q.2,MP4 **WHY:** The goal of the preview lesson is to activate students' prior knowledge about exponential functions, give students an overview of some of the deeper ideas they will encounter related to exponential functions, and provide a context in which to articulate their understanding. This can serve as a diagnostic pre-unit assessment as well.
Sample Activity 1.2	The Shrinking Dollar, Dan Meyer **WHAT:** Students examine what happens when you photocopy a dollar bill and reduce the image size, then take the reduced image and photocopy it again, etc. Students then create an exponential model to describe the size of the sequence of images. F.LE.2 , N.Q.2,MP4 **WHY**: This activity provides a concrete context where the quantities can be easily measured (size of image). This activity is a fairly straightforward modeling task because the domain is whole numbers (i.e. number of reductions) so the model is a sequence and the base of the exponential is the scale factor.
Sample Activity 1.3	Bouncing Ball Lab, Classic Activity as Interpreted by Mrs. Gemmill **WHAT:** The Bouncing Ball lab allows students to predict how high various balls will bounce compared to their initial starting heights. It presents an opportunity for students to make predictions, collect data, and potentially use technology to attempt fitting an exponential graph to their data. F.LE.2 , N.Q.2, MP4, MP6 **WHY:** This is a classic example. It fits in this section because the domain is whole numbers (number of bounces). The data will be slightly messier than the previous activity, so that students will need to grapple with precision of measuring the bounce heights and engage in analyzing the accuracy of the model and its predictions. This is also an opportunity for students to interpret the base of the exponential function in terms of exponential decay. F.IF.8b In addition, students can use the context of the situation to interpret key features of the graph, such as the y-intercept, and can think critically about where the graph no longer matches the context, in terms of the long term behavior. F.IF.4, F.IF.7e
Target Standards	F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

	N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
Mathematical Practices	MP4, MP6

Section 2: 1 - 2 days	**Understand and Explain Properties of Exponents**
Mathematical Goals	• Use the properties of exponents to extend the definition of b^x N.RN.1 • Explain and justify the properties of exponents N.RN.1
Narrative overview of section (and how the standards are achieved)	This section starts off with students having an opportunity to expand their existing understanding of integer exponents to rational exponents. N.RN.1 Although by this point students have seen expressions like $2^{1/2}$, they have not systematically developed the idea of b^x where x is any real number. Students have seen and used basic exponents prior to this point. Now they can revisit these concepts and develop structures and patterns for negative and rational exponents. They should be able to construct and communicate arguments and explanations of the properties of exponents. MP3, MP4
Sample Activity 2.1	Extending the Definition of Exponents, Variation 2, Illustrative Mathematics **WHAT:** The Extending the Definition of Exponents, Variation 2 task examines a bacteria population that is doubling every hour. The data starts with friendly numbers, but then the problem asks students to think about what is happening every half-hour and every third-hour, leading to a richer idea about exponential growth while also necessitating rational exponents. NR.N.1, F.LE.2,MP8 **WHY:** The purpose of this task is to motivate a need for rational exponents arising in a real-life context. The following task decontextualizes this to investigate the same extension of the definition from a more abstract perspective.

Sample Activity 2.2	Evaluating Exponential Expressions, Illustrative Mathematics **WHAT**: The goal of this task is to use properties of exponents for whole numbers (in particular the fact that $(a^n)^m = a^{nm}$ in order to explain how expressions with fractional exponents are defined N.RN.1 . There are some situations where fractional exponents result in whole number answers: for example, $4^{1/2} = \sqrt{4} = 2$. **WHY:** The purpose of this task is to reason abstractly MP2 about generalizing the rules of exponents. This contrasts with the previous task where this generalization is contextualized.
Target Standards	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Mathematical Practices	MP2, MP3, MP4, MP8

Section 3: 1 - 2 days	**Create and Compare Real Life Exponential Models**
Mathematical Goals	• Distinguish between situations that can be modeled with linear and exponential functions F.LE.1i • Compare rates of change of linear and exponential functions F.IF.6 • Find solutions and intersections of linear and exponential functions A.REI.11 • Construct exponential functions given a description or data from a table F.LE.2
Narrative overview of section (and how the standards are achieved)	In these cases the students are not given a model, rather the purpose is for them to use given data or the structure of a situation to create the model MP4. This includes determining the parameters of an exponential model and comparing exponential and linear models F.LE.1. There are opportunities for students to create exponential models from data given in either a table or in a scatterplot. F.LE.2
Sample Activity 3.1	Basketball Rebounds, Illustrative Mathematics **WHAT:** The Basketball Rebounds task uses the requirements set by the International Basketball Federation regarding the required rebound height of a basketball. They use this information to find a formula for the height of the ball, given the number of bounces. **WHY:** This task provides a real-life context that students can use the given information to build an exponential function to

	model the bounce height of a basketball as a function of the number of bounces. F.BF.1a, F.LE.2
Sample Activity 3.2	Boiling Water, Illustrative Mathematics **WHAT:** The Boiling Water task uses technology to have students find good linear models that work of a small range of elevations, but these fail over broader ranges. In this case exponential models perform much better. **WHY:** This task gives real-life data on boiling point and elevation. Under small ranges of elevation the data can be reasonably modeled using a linear function, however over broader ranges, the data requires an exponential F.LE.1 This task gives data in a table, and also a scatter plot which the students can use to create the models F-LE.2
Sample Activity 3.3	Population and Food Supply, Illustrative Mathematics **WHAT:**In this task students construct and compare linear and exponential functions and find where the two functions intersect. F-LE.2, A-REI.11. They do so by comparing the exponential growth of a population with the linear growth of a food supply, and answer questions regarding when food will run out. **WHY:** One purpose of this task is to demonstrate that exponential functions grow faster than linear functions even if the linear function has a higher initial value and even if we increase the slope of the line. By comparing the functions and their graphs, students can see that the slope of an exponential function continues to increase, while the slope of a linear function stays the same. F.IF.6
Target Standards	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
Mathematical Practices	MP4

Section 4: 3 days	Analyze Real-Life Exponential Models
Mathematical Goals	• Analyze situations where the model is known or given, for example debt or population growth F.BF.1 • Create and solve exponential equations A.CED.1
Narrative overview of section (and how the standards are achieved)	Students will analyze various contexts such as credit card debt, mortgages, loan, population growth, etc. In these cases *the model is known or given* (e.g. simple vs compound interest). The goal is for them to use these models to calculate and estimate meaningful quantities. For example, how long will it take to pay off a credit card debt paying the minimum monthly payment?
Sample Activity 4.1	Compounding With a 5% Interest Rate, Illustrative Mathematics **WHAT:** The Compounding With a 5% Interest Rate task looks at how the frequency of interest being compounded affects the balance of a savings account. Students continue to compound more and more frequently and realize that the frequency of compounding can only have so much of an impact. F.BF.1.a **WHY**: This task develops reasoning behind the general formula for balances under continuously compounded interest. While this task itself specifically addresses the standard (F-BF), building functions from a context, a auxiliary purpose is to introduce and motivate the number e, which plays a significant role in the (F-LE) domain of tasks.
Sample Activity 4.2	Loan Ranger, Mathalicious **WHAT:** Loan Ranger is a contextualization of percent growth, and alternative compounding intervals, in the context of credit card debt. Students compare the consequences of not paying balances on two cards with different interest rates, observe the effect of adding monthly compounding to the model, compare the effect of paying the minimum vs paying more both in how much interest is paid, and in how long it takes to pay off the debt. **WHY:** This activity gives students an opportunity to evaluate the recursive process of credit card debt, and turn it into a closed-form equation. F.BF.1.a While the calculations are not difficult, credit card debt is a great real-world connection for students to contextualize exponential functions and to apply percent growth and decay to a very relevant topic MP4. This also gives students a chance to construct and evaluate exponential functions in a rich context, and make connections between the graphs, equations and implications within that specific context. F.BF.1b, A.CED.1, F.LE.2
Target Standards	F.BF.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.1.b Combine standard function types using arithmetic operations. For example, build a function that models the

	temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Mathematical Practices	MP4

Section 5: 2 days	**Geometric Sequences and Series**
Mathematical Goals	• Analyze situations that involve geometric sequences and series • Derive the formula for the sum of a finite geometric series A.SSE.4
Narrative overview of section (and how the standards are achieved)	Analyze situations that involve summing a exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts.
Sample Activity 5.1	A Lifetime of Savings, Illustrative Mathematics **WHAT:** The A Lifetime of Savings task gets at the heart of how the total length of time makes a big impact in an investment, and how a small amount of savings can grow very large over a given time. It also ties in what students already know about exponential functions and finances, and ties these topics to geometric series. **WHY:** The purpose of this instructional task is to give students an opportunity to construct and find the value of a geometric series in a financial literacy context. A.SSE.4 The task assumes that students have already developed the formula for a geometric series themselves; having them recognize the need for this formula (and look up if necessary) allows them to engage in MP 5 (use appropriate tools strategically). The task also provides students with an opportunity to look for and express regularity in repeated reasoning. MP 8 This task also asks students to interpret the variables in the future value formula in the context of the problem. A-SSE.1
Sample Activity 5.2	Course of Antibiotics, Illustrative Mathematics **WHAT:** The Course of Antibiotics task has students use data about a dosage of medicine, including dosage times and how much medicine remains in the body, and develop a geometric series to determine how much medicine is in the body at a given time. It is a good problem for basic practice of using data to generate a series. and how to sum the series.

	WHY: The purpose of this task is to present a real-world application of finite geometric sequences in a tangible context -- the amount of medicine that remains in a body over time. A.SSE.4 It is also an opportunity for students to use repeated reasoning as they try to find a pattern that will result in a formula. MP8
Sample Activity 5.3	Triangle Series, Illustrative Mathematics **WHAT:** The Triangle Series task applies the same concepts of a geometric series and sums, but in a geometric context. **WHY:** This task provide an opportunity for students to apply geometric series in geometric contexts A.SSE.4. It is an opportunity to reason abstractly. MP2
Sample Activity 5.4	Cantor Set, Illustrative Mathematics **WHAT:** The Cantor Set task is more practice with geometric series and sums, but might be conceptually more abstract for students. **WHY:** This task provide an opportunity for students to apply geometric series in geometric contexts A.SSE.4. It is an opportunity to reason abstractly. MP2
Target Standards	A.SSE.1 Interpret expressions that represent a quantity in terms of its context. A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★
Mathematical Practices	MP2, MP5, MP8

Section 6: 1 - 2 days	**Motivate Exponential Equations**
Mathematical Goals	• Create and solve exponential equations A.CED.2 • Manipulate exponential expressions A.SSE.2, A.SSE.3 • Compare exponential functions, models and graphs F.LE.1
Narrative overview of section (and how the standards are achieved)	Students will have had occasions to solve simple exponential equations arising in the previous sections, but the functions and models have been the primary focus. This sections moves the attention to the solving and manipulating of exponential equations. Students use inspection and basic properties to solve exponential equations.

Sample Activity 6.1	Exponential Functions, Illustrative Mathematics **WHAT**: This task requires students to use the fact that the value of an exponential function f(x)=ab^x increases by a multiplicative factor of b when x increases by one. Students use properties of exponential functions and their graphs to make comparisons. F.LE.1 **WHY:** This task intentionally omits specific values for c and d in order to encourage students to use this fact instead of computing the point of intersection, (p,q), and then computing function values to answer the question.
Sample Activity 6.2	Forms of Exponential Expressions, Illustrative Mathematics **WHAT:** The Forms of Exponential Expressions task compares the same exponential function in four different forms and asks students to show that they are the same function. A.SSE.2 Students are also asked when each form might be useful, based on what information is wanted. A.SSE.3. **WHY:** This task is good practice for manipulation of exponential equations. MP7
Sample Activity 6.3	Uranium 238, Illustrative Mathematics **WHAT:** The goal of this task is to represent an exponential relationship by an equation and identify, using knowledge of the context and the structure of the equation, possible graphs for the equation. The teacher may wish to provide more possible graphs to highlight different aspects of the exponential equation. In addition, the teacher will need to make sure that students understand the context, that is that they understand that the half-life is the amount of time it takes for half of the Uranium 238 to decay. A.CED.2 WHY:
Target Standards	A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. A.SSE.3 Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
Mathematical Practices	MP.7

Section 7: 1 day	**Summative Assessment**
Unit Assessment Targets	Assess students' ability to ● Demonstrate an understanding of the meaning of a rational exponent N-RN.1 ● Construct exponential functions given a description or data from a table F-LE.2 ● Manipulate exponential expressions A.SSE.3 ● Use the formula for the sum of a finite geometric series to solve a problem A-SSE.4 ● Create and analyze simple exponentials arising from real-word data.F-LE.5
Sample Assessment Items	A6 Final Assessment Sample Items PARCC Summative Assessment

	1.1	1.2	1.3	2.1	2.2	3.1	3.2	3.3	4.1	4.2	5.1	5.2	5.3	5.4	6.1	6.2	6.3
N.Q.2	x	x	x														
A.SSE.1											x						
A.SSE.2																x	
A.SSE.3c																x	
A.SSE.4											x	x	x	x			
A.CED.1										x							
A.CED.2																	x
A.REI.11								x									
F.IF.4			x														

F.IF.6								x									
F.IF.7e			x														
F.IF.8b			x														
F.BF.1a						x			x	x							
F.BF.1b										x							
F.LE.1							x								x		
F.LE.2	x	x	x	x		x	x	x		x							
F.LE.5	x																
N.RN.1				x	x												

	1.1	1.2	1.3	2.1	2.2	3.1	3.2	3.3	4.1	4.2	5.1	5.2	5.3	5.4	6.1	6.2	6.3
MP.1																	
MP.2					x								x	x			
MP.3																	
MP.4	x	x	x							x							
MP.5											x						
MP.6			x														
MP.7																x	
MP.8				x							x	x					