

Curriculum Units and Learning Outcomes

Content Area: Algebra II Accelerated	Grade Level: 10, 11
Unit 3: Exponentials, Logarithms, Limits, and Matrices	
Unit Summary: Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation.	
Massachusetts Standards: ● F-IF.C C. Analyze functions using different representations. 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F-IF.C C. Analyze functions using different representations. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. ● F-IF.C C. Analyze functions using different representations.	

10. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, or exponential.

- F-BF.A

A. Build a function that models a relationship between two quantities.

1. Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic) that describes a relationship between two quantities.
- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

- F-LF.A

A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function (of the form $f(x) = b^x + k$) in terms of a context.

- PC.A-BF

B. Build new functions from existing functions.

4. Find inverse functions.
- b. (+) Verify by composition that one function is the inverse of another.
- c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

- N-VM Matrix Quantities

C. Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a Commutative operation, but still satisfies the Associative and Distributive properties.

Enduring Understandings:

- Investigates and identifies the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. The characteristics of exponential functions and their representations are useful in solving real-world problems.
- How are exponential and logarithmic functions related?
- The characteristics of logarithmic functions and their representations are useful in solving real-world problems.
- Limits are used to help understand the nature of a curve and its approach to a point.
- Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- Distinguish between situations that can be modeled with linear functions and with exponential functions
- For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e , evaluate the logarithm using technology.

Essential Questions:

- How do exponential functions model real-world problems and their solutions?
- How do you apply the compound interest formulas and when?
- How do logarithmic functions model real-world problems and their solutions?
- How are exponential and logarithmic functions related?

Students will demonstrate KNOWLEDGE of:

- Inverse Logarithmic Functions
- Exponential Growth and Decay
- Introduction to Limits - concavity, continuity, slope-increasing function with decreasing slope, approaching limits, end behavior written as limits.
- One Sided Limits, find limits algebraically, find limit graphically.
- Find limits numerically, piecewise functions and limits
- Infinite Limits
- Right Triangle Trigonometry: trig ratio, inverse trig ratio, angle of elevation, angle of depression

Students will be SKILLED at:

- Expressing the concept of continuity in terms of limits.
- Students should know the definitions of and conditions in which the types of continuities exist • Removable, jump, infinite, oscillating
Students should know and be able to apply the properties of continuous functions.
- Write and evaluate exponential equations in logarithmic form
- Translating parent logarithmic functions
- Simplifying and expanding logarithms • Change of base formula • Logarithmic applications • PH Scale • Bacteria
- Solving exponential and logarithmic equations • Different bases • With a graph or table • Modeling with exponential equations
- Evaluate the natural logarithmic function and its inverse

- The exponential function $y = b^x$ is one-to-one, so its inverse $x = b^y$ is a function.
To express "y as a function of x" for the inverse, write $y = \log_b x$.
- Converting between exponential and logarithmic form.
- Logarithms can be used to solve exponential equations; and conversely, exponents can be used to solve logarithmic equations.
- Solving equations using logs and exponents.
- The function $y = e^x$ and $y = \ln x$ are inverse functions.
- Evaluating natural logarithms.
- Add, Subtract, Multiply Matrices, Scalar

- Solve exponential and logarithmic equations including real world models.
- Solve equations using natural logarithms.
- Model exponential growth and decay • Sketch graphs of exponentials (with translations) • Solve problems using e, the natural base • Write and evaluate logarithmic expressions • Graph logarithmic functions (with translations)
- Expand and simplify logarithms using the properties of logarithms • Solve exponential equations • Solve logarithmic equations •
- Solve problems involving $\ln$, the natural log
- Evaluate and simplify matrices Add, Subtract, Multiply Matrices, Scalar

Estimated Duration: 8 weeks