Calculus I Exam 2 Review

2.5: 26 . Find the derivative

$$s(t) = \sqrt{\frac{1+\sin sin\, t}{1+\cos cos\, t}}$$

2.5: 40. Find the derivative

$$g(u) = \left[\left(u^2 - 1\right)^6 - 3u\right]^4$$

2.5: 60. The function $f(x) = \sin sin\,(x + sin2x)$, $0 \le x \le \pi$, arises in application to frequency modulation (FM) synthesis.

a. Use a graph of f produced by a calculator or computer to make a rough sketch of the graph of f'.

b. Calculate $f'(x)$ and use this expression, with a calculator or computer, to graph f'. Compare with your sketch in part a.

2.5: 62. At what point on the curve $y = \sqrt{1 + 2x}$ is the tangent line perpendicular to the line $6x + 2y = 1$

2.6: 4 $\frac{2}{x} - \frac{1}{y} = 4$

a) Find y' by implicit differentiation

b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

c) Check that your solutions are consistent by substituting the expression for y into your solution for part a

2.6: 24: Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find dx/dy

$$y \sec x = x \tan y$$

2.6: 30. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

a) $x^2 + 2xy + 4y^2 = 12,\ (2, 1)$ (ellipse)

b) $x^2 + 3xy + 9y^2 = 19,\ (2, 1)$ (ellipse)

2.6: 36.

a) The curve with equation $y^2 = x^3 + 3x^2$ is called the Tschirnhausen cubic. Find an equation of the tangent line to this curve at the point $(1, \ -2)$.

b) At what points does this curve have horizontal tangents?

c) Illustrate parts (a) and (b) by graphing the curve and the tangent lines on a common screen.

2.7: 18

a. The volume of a growing spherical cell is $V = \frac{4}{3}\pi r^3$, where the radius r is measured in micrometers ($1\ \mu m = 10^{-6} m$). Find the average rate of change of V with respect to r when r changes from

i. $5\ to\ 8\ \mu m$

ii. $5\ to\ 6\ \mu m$

iii. $5\ to\ 5.1\ \pi m$

b. Find the instantaneous rate of change of V with respect to r when $r = 5\ \mu m$.

c. Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area. Explain geometrically why this result is true.

2.7: 30 The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density.

a) Find the rate of change of the frequency with respect to

   i. the length (when T and ρ are constant),

   ii. the tension (when L and ρ are constant), and

   iii. the linear density (when L and T are constant).

b) The pitch of a note (how high or low the note sounds) is determined by the frequency f. (The higher the frequency, the higher the pitch.) Use the signs of the derivatives in part a to determine what happens to the pitch of a note

   i. when the effective length of a string is decreased by placing a finger on the string so a shorter portion of the string vibrates,.

   ii. when the tension is increased by turning a tuning peg,

iii. when the linear density is increased by switching to another string,

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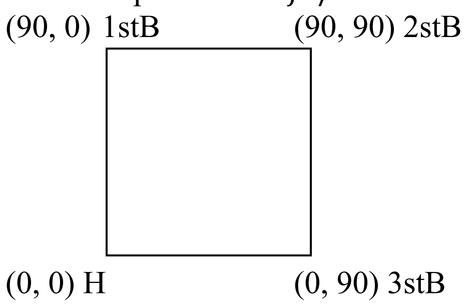
2.8: 6. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing (in square cm/s)?

2.8: 20. A baseball diamond is a square with side $90\ ft$. A batter hits the ball and runs toward first base with a speed of $24\ ft/s$.



a) At what rate is his distance from second base decreasing when he is halfway to first base?

b) At what rate is his distance from third base increasing at the same moment?

2.8: 28. A trough is $10\ ft$ long and its ends have the shape of isosceles triangles that are $3\ ft$ across at the top and have a height of $1\ ft$. If the trough is being filled with water at a rate of $12\ ft^3/min$, how fast is the water level rising when the water is $6\ inches$ deep?

2.9: 2. Find the linearization $L(x)$ of the function at a

$f(x) = \cos 2x\,, a = \pi/6$

2.9: 4. Find the linearization $L(x)$ of the function at a

$$f(x) = 2/\sqrt{x^2 - 5},\ a = 3$$

2.9: 26. Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram showing the line segments with lengths dx, dy, and Δy.

$$y = x^3,\ x = 0,\ \Delta x = 0.5$$

3.1: 12

a) Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no local maximum.

b) Sketch the graph of a function on $[-1, 2]$ that has a local maximum but no absolute maximum.

3.1: 28. Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f.

$$f(x) = \{ \ 2x + 1 \quad if\ 0 \leq x < 1 \quad and\ 4 - 2x \quad if\ 1 \leq x \leq 3\}$$

3.1: 42. Find the critical numbers of the function.

$$h(x) = x^{-1/3}(x - 2)$$

3.1: 46. Find the critical numbers of the function.

$$g(x) = \sqrt{1 - x^2}$$

3.1: 54. Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = (t^2 - 4)^3,\ [-2,\ 3]$$

3.2: 6. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing (in square cm/s)?

3.2: 16. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 - 3x + 2,\ [-2,\ 2]$$

3.2: 20. Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. Graph the function, the secant line through the endpoints, and the tangent line at $(c,\ f(c))$. Are the secant line and the tangent line parallel?

$$f(x) = x^3 - 2x,\ [2,\ \ -2]$$

3.2: 23. Show that the equation has exactly one real solution.

$$2x + cos\, x = 0$$

3.3: 16. Find the intervals on which f is concave upward or concave downward, and find the infection points of f.

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

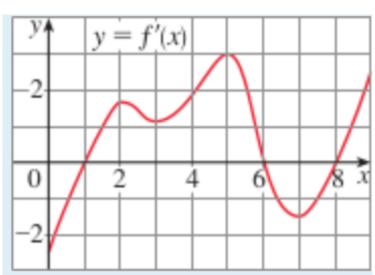
3.3: 22. Find the intervals on which f is concave upward or concave downward, and find the infection points of f.

$$f(x) = cos^2 x - 2sin\, x,\ 0 \leq x \leq 2\pi$$

3.3: 24. Find the local maximum and minimum values of using both the First and Second Derivative Tests. Which method do you prefer?

$$f(x) = \frac{x^2}{x-1}$$

3.3: 38. The graph of the derivative f' of a continuous function f is shown.



a) On what intervals is f increasing? Decreasing?

b) At what values of x does f have a local maximum? Local minimum?

c) On what intervals is f concave upward? Concave downward?

d) State the x-coordinate (s) of the point(s) of inflection.

e) Assuming that $f(0) = 0$, sketch a graph of f.