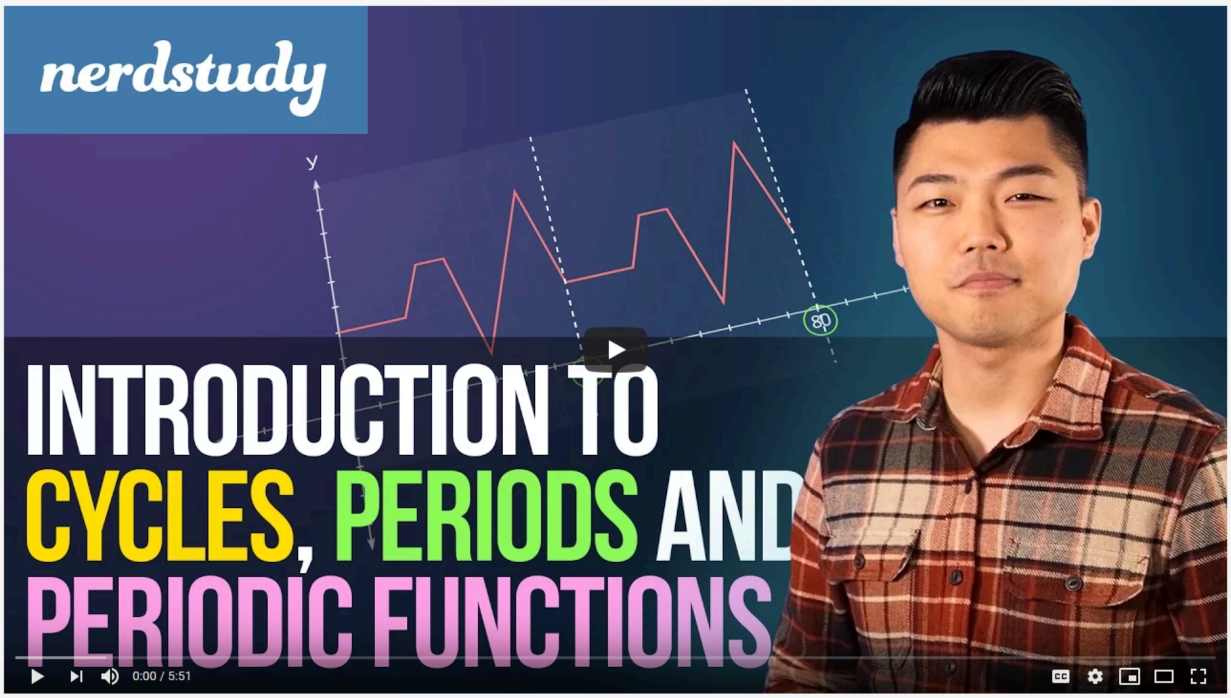
Trigonometric Functions - (**Graph of Sine and Cosine Functions**)

These notes are meant to supplement the material not be a substitute. Please read the book and these notes to help you prepare for doing homework and understand class topics.

1. What are Periodic Functions
    a. Period [**Will be tested on**]
    b. Amplitude [**Will be tested on**]
    c. Phase Shift
    d. Vertical Shift
    e. Frequency
2. Graph of Sine [**Will be tested on**]
3. Graph of Cosine [**Will be tested on**]
4. Examples of Graphing Sine and Cosine Functions [**Will be tested on**]

What are Periodic Functions



Introduction to Cycles, Periods, and Periodic Functions - Nerdstudy

Periodic functions can be explained from two perspectives. From the graph perspective, if its graph repeats after an unchanging period, we call it a periodic function. From the algebraic perspective, if $f(x) = f(x + T)$(T is an unchanging period, x can be any real numbers), then this function is periodic.

The Constant Function(easy to understand)

Now, it's time to talk about the simplest periodic function which is the constant function. It's necessary to prove its periodicity.

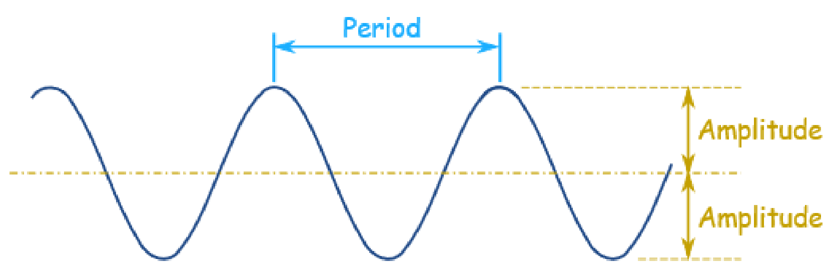
For any constant function, which is defined as having an unchanging y-value, of which the function $y = 1$ is an example, the equation $f(x) = f(x + T)$, where T can be defined as any real number, will always be true.

If the graph of $y = 1$ is drawn(the graph above), then it can be shown that its graph repeats forever, because the y-value never changes.

Frome the evidence above, we are sure that constant functions are periodic.

Period

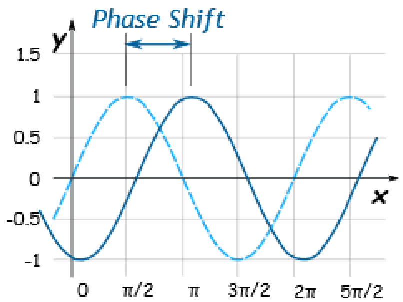
The **Period** goes from one peak to the next (or from any point to the next matching point):



Amplitude

The **Amplitude** is the height from the center line to the peak (or to the trough). Or we can measure the height from highest to lowest points and divide that by 2.

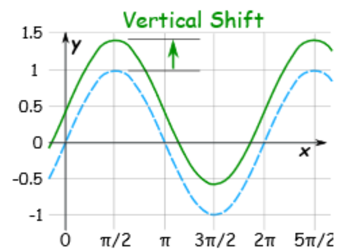
Phase Shift



The **Phase Shift** is how far the function is shifted **horizontally** from the usual position.

Vertical Shift

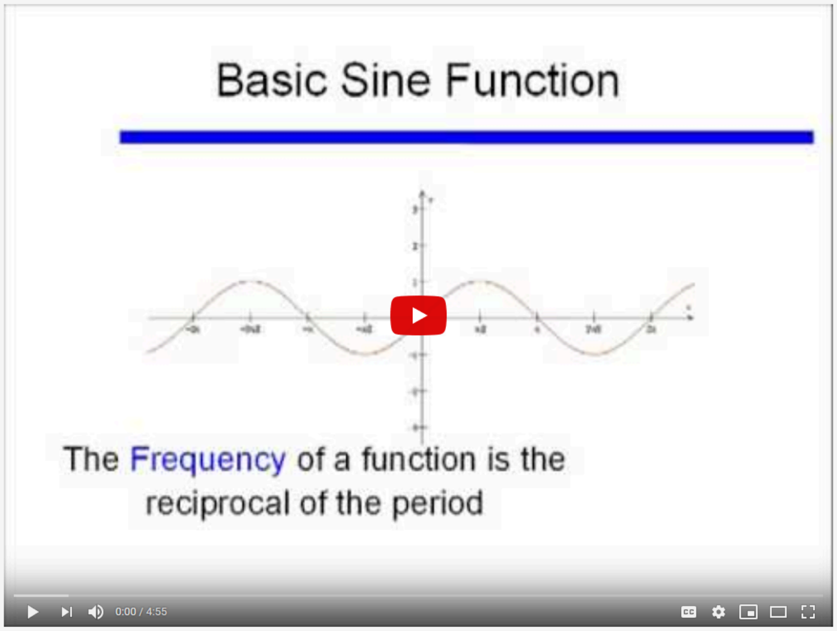
The **Vertical Shift** is how far the function is shifted **vertically** from the usual position.



Frequency

Frequency is how often something happens per unit of time (per "1").

Here is a video which will use a Sine function to define and use the terms which are defined above. We will go more into detail about Sine and Cosine functions below:

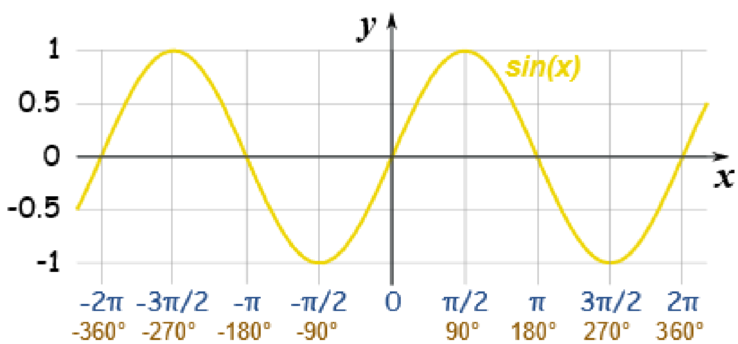


Introduction to Periodic Functions

Graph Of Sine

The Sine Function has this beautiful up-down curve (which repeats every 2π radians, or 360°).

It starts at $\mathbf{0}$, heads up to $\mathbf{1}$ by $\pi/2$ radians (90°) and then heads down to $-\mathbf{1}$.



Properties of the Sine Function, $y = \sin(x)$

Domain : $(-\infty, \infty)$

Range : $[-1, 1]$ or $-1 \leq y \leq 1$

y -intercept : $(0, 0)$

x -intercept : $n\pi$, where n is an integer.

Period: 2π

Continuity: continuous on $(-\infty, \infty)$

Symmetry: origin (odd function)

The maximum value of $y = \sin(x)$ occurs when $x = \frac{\pi}{2} + 2n\pi$, where n is an integer.

The minimum value of $y = \sin(x)$ occurs when $x = \frac{3\pi}{2} + 2n\pi$, where n is an integer.

Amplitude and Period of a Since Function

The amplitude of the graph of $y = a\sin(bx)$ is the amount by which it varies above and below the x -axis.

Amplitude = | a |

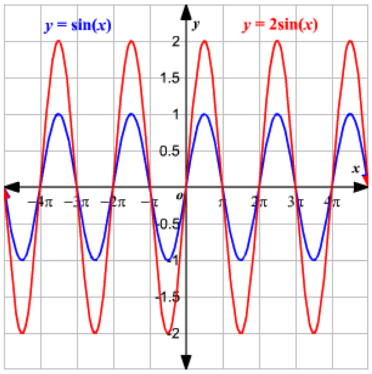
The period of a sine function is the length of the shortest interval on the x -axis over which the graph repeats.

Period = $\frac{2\pi}{|b|}$

Example:

Sketch the graphs of $y = \sin(x)$ and $y = 2\sin(x)$. Compare the graphs.

For the function $y = 2\sin(x)$, the graph has an amplitude 2 . Since $b = 1$, the graph has a period of 2π . Thus, it cycles once from 0 to 2π with one maximum of 2 , and one minimum of -2 .

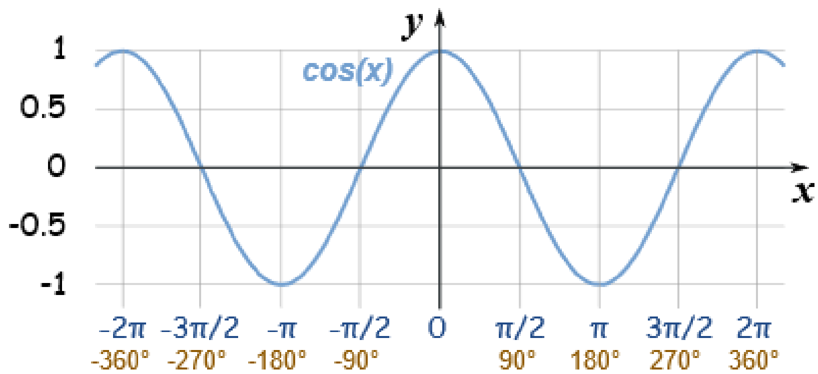


Observe the graphs of $y = \sin(x)$ and $y = 2\sin(x)$. Each has the same x -intercepts, but $y = 2\sin(x)$ has an amplitude that is twice the amplitude of $y = \sin(x)$.

Graph of Cosine

Plot of Cosine

Cosine is just like Sine, but it starts at 1 and heads down until π radians (180°) and then heads up again.



Properties of the Cosine Function, $y = \cos(x)$ **.**

Domain : $(-\infty, \infty)$

Range : $[-1, 1]$ or $-1 \leq y \leq 1$

y -intercept : $(0, 1)$

x -intercept : $\left(\frac{n\pi}{2}, 0\right)$, where n is an integer.

Period: 2π

Continuity: continuous on $(-\infty, \infty)$

Symmetry: y -axis (even function)

The maximum value of $y = \cos(x)$ occurs when $x = 2n\pi$, where n is an integer.

The minimum value of $y = \cos(x)$ occurs when $x = \pi + 2n\pi$, where n is an integer.

Amplitude and Period a Cosine Function

The amplitude of the graph of $y = a\cos(bx)$ is the amount by which it varies above and below the x -axis.

Amplitude = | a |

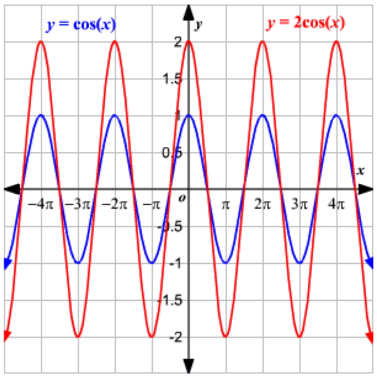
The period of a cosine function is the length of the shortest interval on the x -axis over which the graph repeats.

Period = $\frac{2\pi}{|b|}$

Example:

Sketch the graphs of $y = \cos(x)$ and $y = 2\cos(x)$. Compare the graphs.

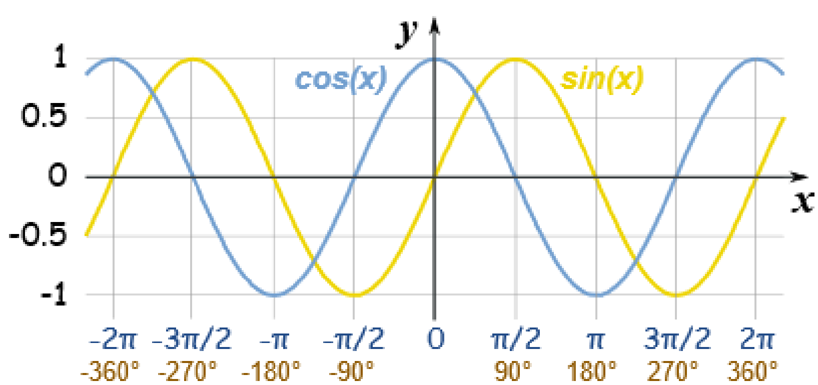
For the function $y = 2\cos(x)$, the graph has an amplitude 2 . Since $b = 1$, the graph has a period of 2π . Thus, it cycles once from 0 to 2π with one maximum of 2 , and one minimum of -2 .



Observe the graphs of $y = \cos(x)$ and $y = 2\cos(x)$. Each has the same x -intercepts, but $y = 2\cos(x)$ has an amplitude that is twice the amplitude of $y = \cos(x)$.

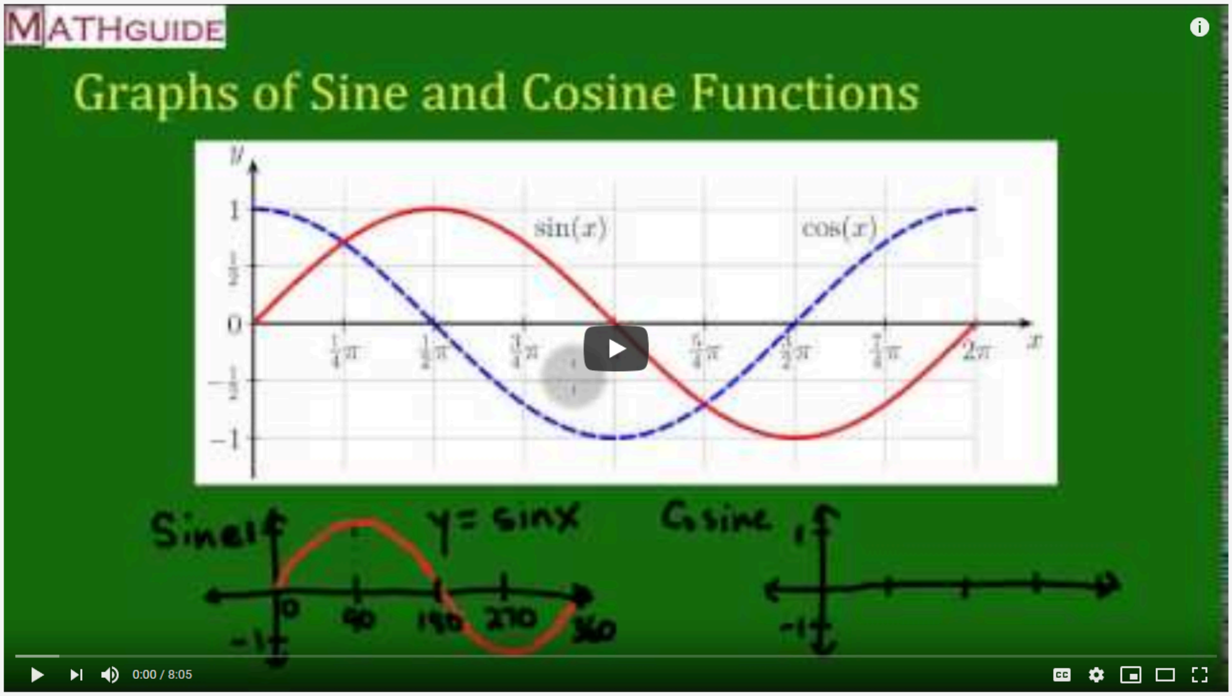
Plot of Sine and Cosine

In fact Sine and Cosine are like **good friends**: they follow each other, exactly $\pi/2$ radians (90°) apart.



Trigonometric Functions - (**Graph of Sine and Cosine Functions**)

The graphs of trigonometric functions have several properties to elicit. To be able to graph these functions by hand, we have to understand them. Before we progress, take a look at this video that describes some of the basics of sine and cosine curves.



Graphing Basics: Sine and Cosine Functions

Examples of Graphing Sine and Cosine Functions

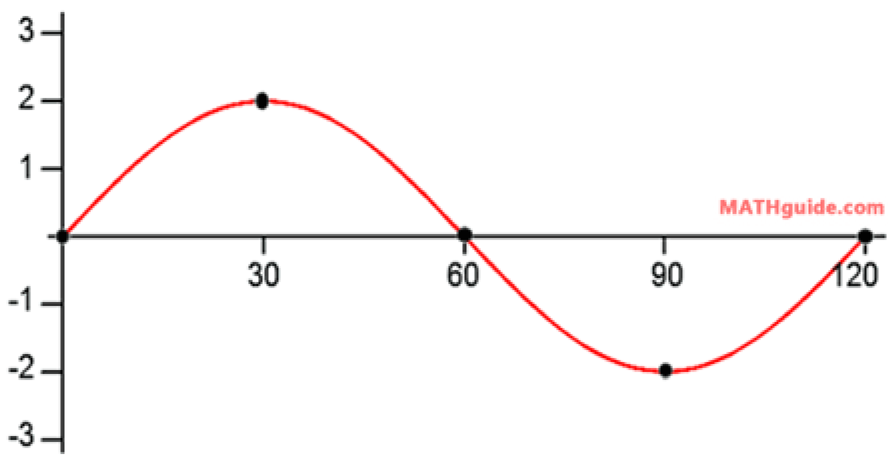
Here is a sine function we will graph.

$$y = 2\sin 3x$$

The a-value is the number in front of the sine function, which is 2. This makes the amplitude equal to |2| or simply 2. The graph of the function has a maximum y-value of 2 and a minimum y-value of -2.

The b-value is the number next to the x-term, which is 3. This means the period is 360 degrees divided by 3 or 120. So, the curve has a y-intercept of zero (because it is a sine curve it passes through the origin) and it completes one cycle in 120 degrees.

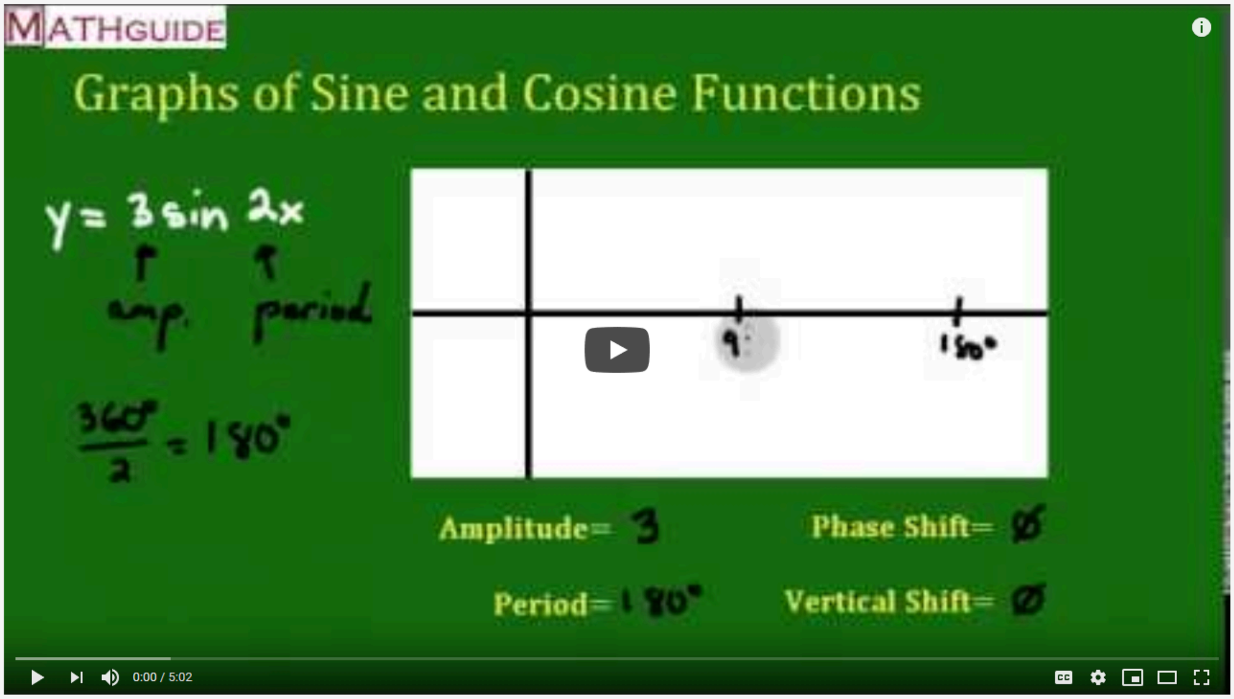
This is the graph of the sine curve.



This particular interval of the curve is obtained by looking at the starting point (0,0) and the end point (120,0). The domain (the x-values) of this cycle go from 0 to 120. So, we write this interval as [0,120].

Trigonometric Functions - (**Graph of Sine and Cosine Functions**)

This video will demonstrate how to graph a different sine function with two parameters: amplitude and period.



Graphing a Sine Function: Amplitude and Period

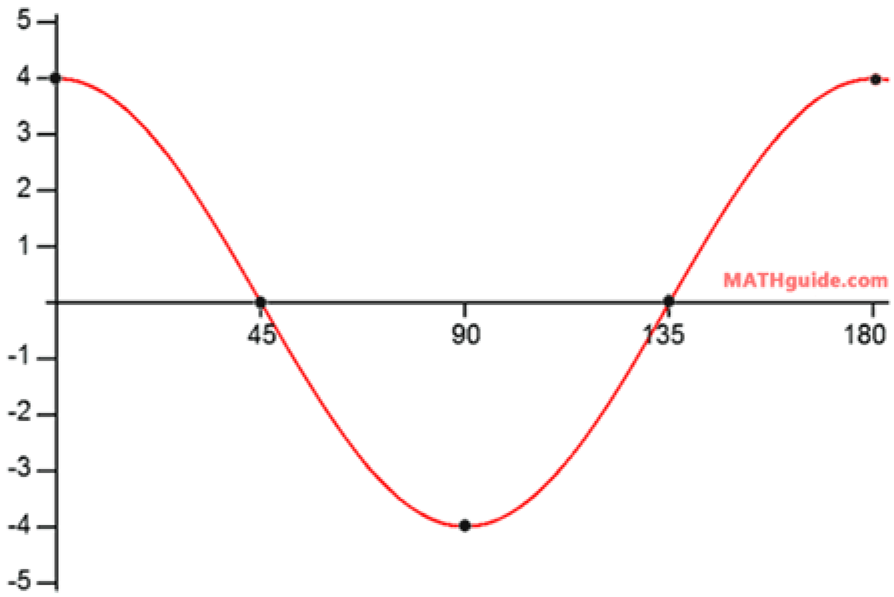
Here is a cosine function we will graph.

$$y = 4\cos 2x$$

The a-value is the number in front of the sine function, which is 4. This makes the amplitude equal to |4| or 4. The graph of the function has a maximum y-value of 4 and a minimum y-value of -4.

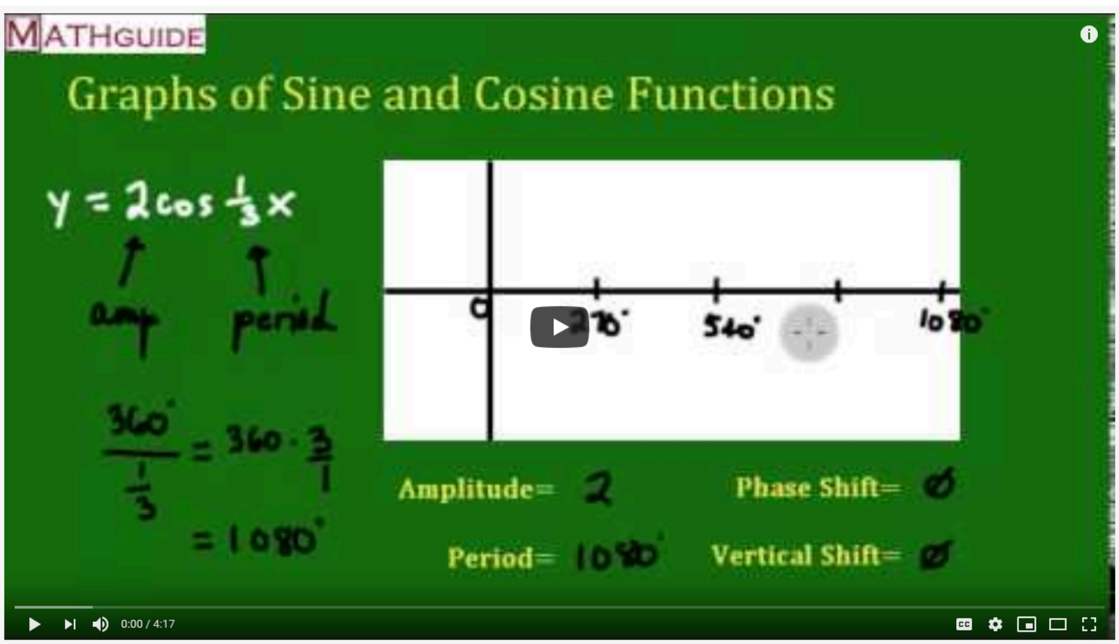
The b-value is the number next to the x-term, which is 2. This means the period is 360 degrees divided by 2 or 180. So, the curve has a y-intercept at its maximum (0,4) (because it is a cosine curve) and it completes one cycle in 180 degrees.

This is the graph of the cosine curve.



This particular interval of the curve is obtained by looking at the starting point (0,4) and the end point (180,4). The domain (the x-values) of this cycle go from 0 to 180. So, we write this interval as [0,180].

This video will demonstrate how to graph a different cosine function with two parameters: amplitude and period.



Graphing a Cosine Function: Amplitude and Period

To calculate phase shift and vertical shift, the equation of our sine and cosine curves have to be in a specific form. The equations have to look like this.

$$y = d + a\sin b(x - c)$$
$$y = d + a\cos b(x - c)$$

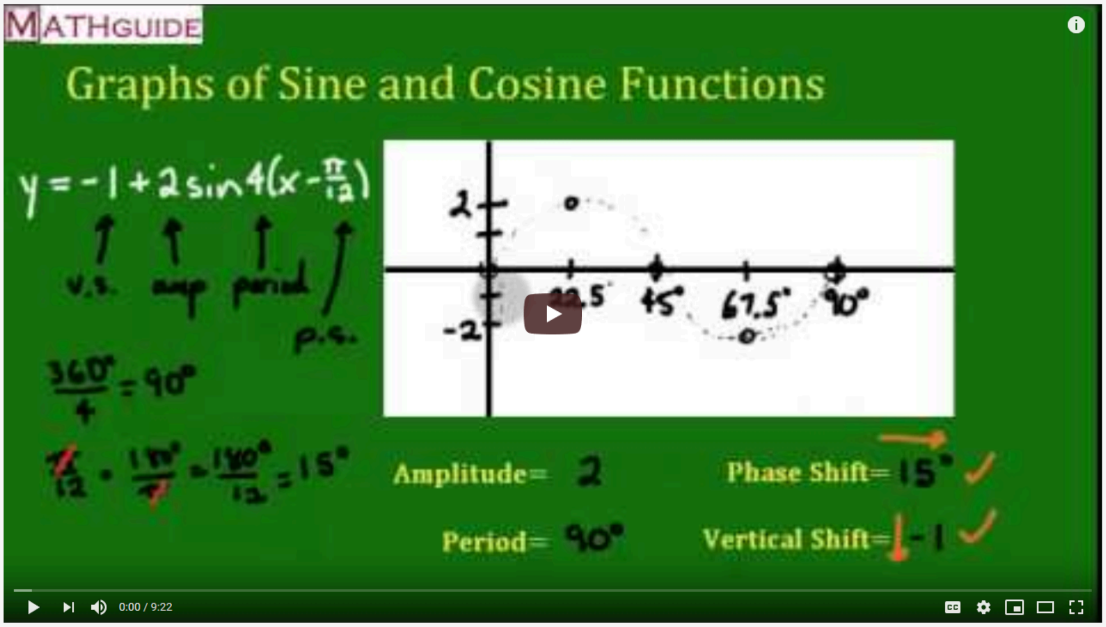
Once in that form, all the parameters can be calculated as follows.

Parameter	Calculation
Amplitude	$\lvert a \rvert$
Period	$\frac{360°}{b}$ or $\frac{2\pi}{b}$
Phase Shift	c
Vertical Shift	d

Notice that the equations have subtraction signs inside the parentheses. The c-values have subtraction signs in front of them. However, the phase shift is the opposite. This will be demonstrated in the next two sections.

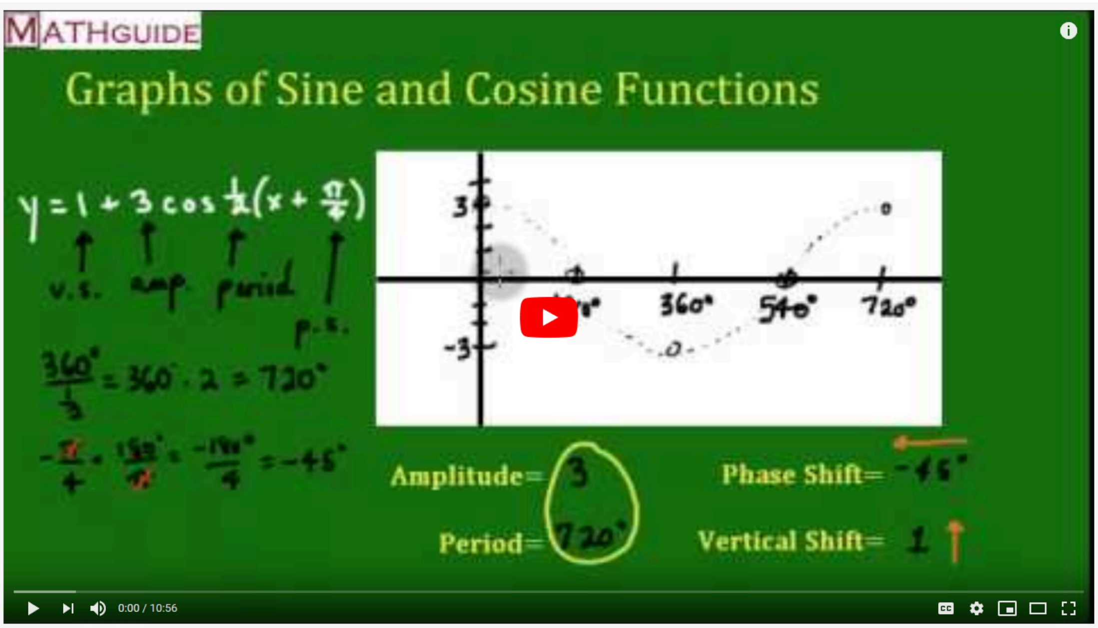
Trigonometric Functions - (**Graph of Sine and Cosine Functions**)

This video will demonstrate how to graph a sine function with four parameters: amplitude, period, phase shift, and vertical shift.



Graphing a Sine Function: Amplitude, Period, Phase Shift, and Vertical Shift

This video will demonstrate how to graph a cosine function with four parameters: amplitude, period, phase shift, and vertical shift.



Graphing a Cosine Function: Amplitude, Period, Phase Shift, Vertical Shift