# Banach Algebras and Spectral Theory (6 weeks)

**Lecturers:**

Prof. Rudi Brits (University of Johannesburg)
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**Study material:**

Comprehensive lecture notes taken from a variety of sources; most notably:

- J. B. Conway, A course in Functional Analysis.
- W. Rudin, Functional Analysis.
- B. Aupetit, A primer on Spectral Theory.
- F. F. Bonsall and J. Duncan, Complete Normed Algebras.

**Topics:**

(week 1-3)

Introduction to Banach Algebras:

Normed algebras
Completeness
Ideals and subalgebras
Unitization

Some important examples:

C(X) – continuous complex-valued functions on a topological space X
(compact metric space; compact Hausdorff space)
A(\Delta) – disk algebra
B(X) – bounded linear operators on a Banach space X
\ell^p, \ell^{\infty}, c_0 – sequence algebras

Invertibility
Jacobson's Lemma
Jacobson radical and semisimplicity
Banach algebra quotients
Some examples of semisimple algebras:
A(\Delta), B(X), A/Rad(A)

Spectral Theory:

Spectrum, resolvent, spectral radius
Basic properties of the spectrum
Compactness of the spectrum
Non-emptiness of the spectrum
Gelfand-Beurling Formula
Gelfand-Mazur Theorem

(week 3-5):

Holomorphic Functional Calculus:

Vector valued integration via bounded linear functionals
Main definition
Rational functions
Existence and uniqueness
Holomorphic Functional Calculus
Spectral Mapping Theorem
Principal component of G(A)
Upper semicontinuity of the spectrum

Gelfand Theory for Commutative Banach algebras:

Gleason-Kahane-Żelazko spectral characterization of characters
Identification of characters with maximal ideals
Gelfand transform
Gelfand topology
Some applications of Gelfand theory

(week 6):

Assessment