ESCUELA POLITÉCNICA NACIONAL

Carrera: Ingeniería Química
Materia: Cálculo Vectorial
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Tema: Valores máximos y mínimos.

Calcule los valores máximos y mínimos relativos, y punto o puntos sillas de la función.

5. $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

$f_x = -2 - 2x$

$f_y = 4 - 8y$

$-2 - 2x = 0 \qquad 4 - 8y = 0$

$x = -1 \qquad x = \frac{1}{2}$

$f_{xx} = -2 \qquad f_{xy} = 0 \qquad f_{yy} = -8$

Punto crítico $\left(-1, \frac{1}{2}\right)$

$A = -2 \quad B = 0 \quad C = -8$

$\Delta = AC - B^2 = (-2)(-8) - 0^2 = 16$

$\Delta\left(-1, \frac{1}{2}\right) = 16 > 0 \therefore \exists\, M_R$

$A = -2 < 0$

$f\left(-1, \frac{1}{2}\right) = 11$

7. $f(x,y) = x^4 + y^4 - 4xy + 2$

$f_x = 4x^3 - 4y \quad ; \quad f_y = 4y^3 - 4x \qquad 4x^3 - 4y = 0 \qquad 4y^3 - 4x = 0$

$x^3 = y \qquad y^3 = x$

$f_{xx} = 12x^2 \qquad f_{xy} = -4 \qquad f_{yy} = 12y^2 \qquad 4y^9 - 4y = 0 \qquad 4(x^9 - x) = 0$

$4(y^9 - y) = 0 \qquad x(x^8 - 1) = 0 \quad x = 0 \quad x = \pm 1$

$y = 0 \wedge y = \pm 1$

Puntos críticos

$(0,0)\ ;\ (1,1)\ ,\ (-1,-1)$

$(0,0) \qquad A = 0 \quad B = -4 \quad C = 0$

$\Delta = AC - B^2 = 0(0) - (-4)^2 = -16$

$\Delta = -16 < 0$ Punto de silla $(0,0)$

$(1,1) \qquad A = 12 \quad B = -4 \quad C = 12$

$\Delta = (12)(12) - (-4)^2 = 128$

$128 > 0$

$12 > 0 \therefore \exists\, m_R$

$f(1,1) = 0$ Mínimo

$(-1,-1) \qquad A = 12 \quad B = -4 \quad C = 12$

$\exists\, m_R \qquad f(-1,-1) = 0$ Mínimo

9. $f(x,y) = (1+xy)(x+y)$

$f(x,y) = x + x^2y + y + xy^2$

$f_x = 1 + 2xy + y^2$, $f_y = 1 + x^2 + 2xy$

① $f_x = 0 = 1 + 2xy + y^2$ ② $0 = 1 + x^2 + 2xy$ ② - ① $1 + x^2 + \cancel{2xy} = 0$

$-1 - y^2 - \cancel{2xy} = 0$

③ $x^2 = y^2$

③ en ①

Si $y = x$ $1 + 2x^2 + x^2 = 0 \rightarrow 1 + 3x^2 = 0$ No hay solución $\mathbb{R}$

Si $y = -x$ $1 - x^2 = 0$ $x = \pm 1$ $1 + 1 + 2(1)y = 0$

$y = -1$

$x = 1$ $y = -1$ $\wedge$ $x = -1$ $y = 1$

Puntos críticos $(1,-1)$ $(-1,1)$

$f_{xx} = 2y$, $f_{xy} = 2x + 2y$, $f_{yy} = 2x$

$(1,-1)$

$A = -2$, $B = 0$ $C = 2$

$\Delta = AC - B^2 = (-2)(2) - 0^2$

$-4 < 0$ Punto de silla

$(-1,1)$ $A = -2$ $B = 0$ $C = 2$

$\Delta = -4 < 0$ Punto de silla

13. $f(x,y) = e^x \cos y$

$f_x = e^x \cos y$ $f_y = e^x(-\operatorname{sen} y)$

① $e^x \cos y = 0$ ② $-e^x \operatorname{sen} y = 0$ ① + ②

① $\cos y = 0$ $y = \frac{\pi}{2} + n\pi$ $e^x \cos y = 0$

$-e^x \operatorname{sen} y = 0$

$\cos y \neq \operatorname{sen} y$ si $y = \frac{\pi}{2} + n\pi$ $e^x(\cos y - \operatorname{sen} y) = 0$

15. $f(x,y) = (x^2 + y^2)e^{y^2 - x^2}$

$f_x = (x^2+y^2)e^{y^2-x^2}(-2x) + 2x\,e^{y^2-x^2} = 2xe^{y^2-x^2}(1 - x^2 - y^2)$

$f_y = (x^2+y^2)e^{y^2-x^2}(2y) + 2y\,e^{y^2-x^2} = 2y\,e^{y^2-x^2}(1 + x^2 + y^2)$

① $2xe^{y^2-x^2}(1 - x^2 - y^2) = 0$ ② $2y\,e^{y^2-x^2}(1 + x^2 + y^2) = 0$

③ $y = 0$ $2ye^{y^2}(1 + y^2) = 0$

$y = 0$

③ en ①

$2x\,e^{-x^2}(1 - x^2) = 0$ $x = 0$ $x = \pm 1$

$y = 0$

Puntos críticos $(0,0)$; $(+1,0)$, $(-1,0)$

$f_{xx} = 2e^{y^2-x^2}((1 - x^2 - y^2)(1 - 2x^2) - 2x^2)$ $f_{xy} = -4xye^{y^2-x^2}(x^2+y^2)$ $f_{yy} = 2e^{y^2-x^2}((1+x^2+y^2)(1+2y^2) + 2y^2)$

$(0,0)$ $A = 2$ $B = 0$ $C = 2$

$\Delta = 2(2) - 0^2 = 4 > 0$ $A > 0$ $\therefore \exists$ mA $f(0,0) = 0$

$(1,0)$ $\wedge$ $(-1,0)$ $A = -4e^{-1}$ $B = 0$ $C = +4e^{-1}$

$\Delta = (-4e^{-1})(4e^{-1}) - 0$ $\Delta < 0$ $\therefore$ Puntos de silla

17. $f(x,y) = y^2 - 2y\cos x \quad 1 \leq x \leq 7$

$f_x = 2y\,\text{sen}\,x \qquad f_y = 2y - 2\cos x$

① $f_x = 0 = 2y\,\text{sen}\,x \qquad$ ② $f_y = 0 = 2y - 2\cos x$

③ $\cancel{2}y = \cancel{2}\cos x$

③ en ①

$2\cos x\,\text{sen}\,x = 0$

$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = 0 \quad x = \pi \quad x = 2\pi \quad -1 \leq x \leq 7$

$y = 0 \quad y = 0 \quad y = 1 \quad y = -1 \quad y = 1$

Puntos críticos $(0,1), (\pi,-1), (2\pi,1), (\frac{\pi}{2},0), (\frac{3\pi}{2},0)$

$f_{xx} = 2y\cos x \qquad f_{xy} = 2\,\text{sen}\,x \qquad f_{yy} = 2$

$\Delta(\frac{\pi}{2},0) = \Delta(\frac{3\pi}{2},0) = -4 < 0$ ∴ Puntos de silla

$(\pi,-1)(2\pi,1)(0,1) \quad A = 2 \quad B = 0 \quad C = 2$

$\Delta = AC - B^2 = 2(2) - 0 = 4 \quad A > 0$

$4 > 0$ ∴ $\exists$ mR

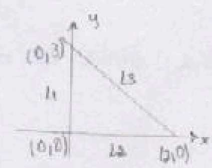
$f(\pi,-1) = f(2\pi,1) = f(0,1) = 1 - 2(1) = -1$ Punto mínimo

. Determine los valores máximos y mínimos absolutos de f en el conjunto D.

29. $f(x,y) = 1 + 4x - 5y$, D es la región cerrada con vértices $(0,0)$, $(2,0)$ y $(0,3)$

$f_x = 4 \qquad f_y = -5$

$L_1: x = 0 \quad f(0,y) = 1 - 5y$ para $0 \leq y \leq 3$



$f(0,0) = 1$ P. máximo

$f(0,3) = -14$ P mínimo

$L_2: y = 0 \quad f(x,0) = 1 + 4x$ para $0 \leq x \leq 2$

$f(0,0) = 1$ Pmáximo

$f(2,0) = 9$ mínimo

$L_3: y = -\frac{3}{2}x + 3 \quad f(x, -\frac{3}{2}x + 3) = \frac{23}{2}x - 14$ para $0 \leq x \leq 2$

$f(0,3) = -14$ máximo

$f(2,0) = 9$ mínimo

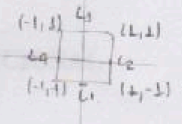
Máximo absoluto $f(2,0) = 9$

Mínimo absoluto $f(0,3) = -14$.

31. $f(x,y) = x^2+y^2+x^2y+4$, $D=\{(x,y) / |x|\leq 1, |y|\leq 1\}$

$f_x = 2x+2xy$

$f_y = 2y+x^2$ $\quad f_x = f_y = 0$



Punto crítico $(0,0)$ $f(0,0)=0$

L_1 $\quad y=-1$, $f(x,-1)=5$

L_2 $\quad x=+1$, $f(1,y)=y^2+y+5$ $\quad$ max $(1,1)$ $f(1,1)=7$ $\quad$ min $(1,-\frac{1}{2})$, $f(1,-\frac{1}{2})=\frac{19}{4}$

L_3 $\quad f(x,1)=2x^2+5$ $\quad$ max $(-1,1)$ $(1,1)$ $\quad f(\pm 1,1)=7$ $\quad$ min $(0,1)$ $f(0,1)=5$

L_4 $\quad f(-1,y)=y^2+y+5$ $\quad$ max $(-1,1)$, $f(-1,1)=7$ $\quad$ min $(-1,-\frac{1}{2})$ $f(-1,-\frac{1}{2})=\frac{19}{4}$

Máximo absoluto $\quad (\pm 1,1)$ $\quad f(\pm 1,1)=7$

Mínimo absoluto $\quad (0,0)$ $\quad f(0,0)=4$

35. $f(x,y) = 2x^3+y^4$, $D=\{(x,y) \mid x^2+y^2\leq 1\}$

$f_x = 6x$ $\quad f_y = 4y^3$ $\quad f_x = f_y = 0$ $\quad x=y=0$

Punto crítico $f(0,0)=0$

Sobre el círculo $x^2+y^2=1$ $\quad y^2=1-x^2$

$g(x)=f(x,y)=2x^3+(1-x^2)^2 = x^4-2x^3-2x^2+1$ $\quad -1\leq x\leq 1$

$g'(x)=4x^3+6x^2-4x \Rightarrow x=0, -2$ o $\frac{1}{2}$ $\quad f(0,\pm 1)=g(0)=1$

$f(\frac{1}{2},\pm\frac{\sqrt{3}}{2}) = g(\frac{1}{2}) = \frac{13}{16}$ $\quad (-2,-3)$ no está en D

$f(-1,0)=g(-1)=-2$ $\quad f(1,0)=g(1)=2$

Max y min absoluto de f en D son $f(1,0)=2$ y $f(-1,0)=-2$

40. Determine el punto sobre el plano $x-y+z=4$ que está más cerca al punto $(1,2,3)$

$d=\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

$z=4-x+y$

$d^2=f(x,y) = (x-1)^2+(y-2)^2+(1-x+y)^2$

$f_x = 2(x-1)+2(1-x+y)(-1) = 4x-2y-4$

$f_y = 2(y-2)+2(1-x+y) = 4y-2x-2$

① $4x-2y-4=0$ $\quad$ ② $4y-2x-2=0$

③ $x=\frac{4+2y}{4}$ $\quad$ ③ en ② $\quad 4y-2\left(\frac{4+2y}{4}\right)-2=0$

$16y-8-4y-8=0$ $\quad 12y=16$

$y=\frac{4}{3}$ $\quad x=\frac{5}{3}$

mínima distancia, el punto en el plano para $(1,2,3)$ es $(\frac{5}{3},\frac{4}{3},\frac{11}{3})$