

Laws of Exponents

Exponents are also called **Powers** or **Indices**

The exponent of a number says how *many times* to use the number in a **multiplication**.

In this example: $8^2 = 8 \times 8 = 64$

In words: 8^2 could be called "8 to the second power", "8 to the power 2" or simply "8 **squared**"

Try it yourself:

So an Exponent saves us writing out lots of multiplies!

Example: a^7

$$a^7 = a \times a \times a \times a \times a \times a \times a = \text{aaaaaaa}$$

Notice how we wrote the letters together to *mean multiply*? We will do that a lot here.

Example: $x^6 = x x x x x x$

The Key to the Laws

Writing all the letters down is the key to understanding the Laws

Example: $x^2 x^3 = (x x)(x x x) = x x x x x = x^5$

Which shows that $x^2 x^3 = x^5$, but *more* on that later!

So, when in doubt, just *remember* to write down all the letters (as *many* as the exponent tells you to) and see if you can *make sense* of it.

All you need to know ...

The "Laws of Exponents" (also called "Rules of Exponents") *come from* **three ideas**:



The exponent says **how *many times*** to use the *number* in a multiplication.



A **negative exponent** *means divide*, because the opposite of *multiplying* is dividing



A [fractional exponent](#) like $\frac{1}{n}$ means to **take the [nth root](#)**: $x^{\frac{1}{n}} = \sqrt[n]{x}$

If you understand those, then you understand exponents!

And all the laws below are based on those ideas.

Laws of Exponents

Here are the Laws (explanations follow):

Law

$$x^1 = x$$

$$x^0 = 1$$

$$x^{-1} = \frac{1}{x}$$

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{-n} = \frac{1}{x^n}$$

Example

$$6^1 = 6$$

$$7^0 = 1$$

$$4^{-1} = \frac{1}{4}$$

$$x^2 x^3 = x^{2+3} = x^5$$

$$\frac{x^6}{x^2} = x^{6-2} = x^4$$

$$(x^2)^3 = x^{2 \times 3} = x^6$$

$$(xy)^3 = x^3 y^3$$

$$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$$

$$x^{-3} = \frac{1}{x^3}$$

And the law about Fractional Exponents:

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

$$x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

Laws Explained

The first three laws above ($x^1 = x$, $x^0 = 1$ and $x^{-1} = \frac{1}{x}$) are just part of the natural sequence of exponents. Have a look at this:

Example: Powers of 5

5²	1 × 5 × 5	25
5¹	1 × 5	5
5⁰	1	1
5⁻¹	1 ÷ 5	0.2
5⁻²	1 ÷ 5 ÷ 5	0.04

.. etc..

Look at that table for a while, notice that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or 5 times smaller) depending on whether the exponent gets larger (or smaller).

The law that $x^m x^n = x^{m+n}$

With $x^m x^n$, how many times do we end up multiplying "x"? *Answer:* first "m" times, then **by another** "n" times, for a total of "m+n" times.

Example: $x^2 x^3 = (x x)(x x x) = x x x x x = x^5$

So, $x^2 x^3 = x^{(2+3)} = x^5$

The law that $\frac{x^m}{x^n} = x^{m-n}$

Like the previous example, how many times do we end up multiplying "x"? Answer: "m" times, then **reduce that** by "n" times (because we are dividing), for a total of "m-n" times.

Example: $x^4 / x^2 = (x x x x) / (x x) = x x = x^2$

So, $\frac{x^4}{x^2} = x^{(4-2)} = x^2$

(Remember that $x/x = 1$, so every time you see an x "above the line" and one "below the line" you can cancel them out.)

This law can also show you why $x^0=1$:

Example: $x^2 / x^2 = x^{2-2} = x^0 = 1$

The law that $(x^m)^n = x^{mn}$

First you multiply "m" times. Then you have **to do that "n" times**, for a total of $m \times n$ times.

Example: $(x^3)^4 = (x x x)^4 = (x x x)(x x x)(x x x)(x x x) = x x x x x x x x x x x x = x^{12}$

So $(x^3)^4 = x^{3 \times 4} = x^{12}$

The law that $(xy)^n = x^n y^n$

To show how this one works, just think of re-arranging all the "x"s and "y"s as in this example:

$$\text{Example: } (xy)^3 = (xy)(xy)(xy) = xy\ xy\ xy = xxxyyy = (xxx)(yyy) = x^3 y^3$$

The law that $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Similar to the previous Example, just re-arrange the "x"s and "y"s

$$\text{Example: } \left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{(xxx)}{(yyy)} = \frac{x^3}{y^3}$$

The law that $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

OK, this one is a little more complicated!

I suggest you read [Fractional Exponents](#) first, so this makes more sense.

Anyway, the important idea is that:

$$x^{\frac{1}{n}} = \text{The } n\text{-th Root of } x$$

And so a fractional exponent like $4^{\frac{3}{2}}$ is really saying to do a **cube** (3) and a **square root** (1/2), in any order.

Just remember from fractions that $\frac{m}{n} = m \times \frac{1}{n}$

$$\text{Example: } x^{\frac{m}{n}} = x^{m \times \frac{1}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

The order does not matter, so it also works for $\frac{m}{n} = \left(\frac{1}{n}\right) \times m$:

$$\text{Example: } x^{\frac{m}{n}} = x^{\frac{1}{n} \times m} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$$

Exponents of Exponents ...

What about this Example?

$$4^{3^2}$$

We do the exponent at the **top first**, so we calculate it this way:

$$\begin{aligned} \text{Start with: } & 4^{3^2} \\ 3^2 = 3 \times 3: & 4^9 \\ 4^9 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4: & 262144 \end{aligned}$$

And That Is It!

If you find it hard to remember all these rules, then remember this:

You can work them out when you understand the [three ideas](#) near the top of this page:

- The exponent says **how many times** to use the number in a multiplication
- A **negative exponent** means **divide**
- A fractional exponent like $1/n$ means to **take the n th root**: $x^{\frac{1}{n}} = \sqrt[n]{x}$

Oh, One More Thing ... What if $x = 0$?

Positive Exponent ($n > 0$)	$0^n = 0$
Negative Exponent ($n < 0$)	0^{-n} is undefined (because dividing by 0 is undefined)
Exponent = 0	0^0 ... <i>ummm</i> ... see below!

The Strange Case of 0^0

There are different arguments for the correct value of 0^0

0^0 could be 1, or possibly 0, so some people say it is really "indeterminate":



$x^0 = 1$, so ...	$0^0 = 1$
$0^n = 0$, so ...	$0^0 = 0$
When in doubt ...	$0^0 = \text{"indeterminate"}$