Laws of Exponents

Exponents are also called **Powers** or **Indices**

The exponent of a number says how *m*any ti*m*es to use the number in a **multiplication.**

In this example: $8^2 = 8 \times 8 = 64$

In words: 8^2 could be called "8 to the second power", "8 to the power 2" or simply "8 **squared**"

Try it yourself:

So an Exponent saves us writing out lots of multiplies!

Example: a^7

$$a^7 = a \times a \times a \times a \times a \times a \times a = aaaaaaa$$

Notice how we wrote the letters together to *m*ean *m*ultiply? We will do that a lot here.

Example: $x^6 = x\,x\,x\,x\,x\,x$

The Key to the Laws

Writing all the letters down is the key to understanding the Laws

Example: $x^2\,x^3 = (x\,x)(x\,x\,x) = x\,x\,x\,x\,x = x^5$

Which shows that $\boldsymbol{x^2\,x^3 = x^5}$, but *m*ore on that later!

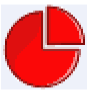
So, when in doubt, just re*mem*ber to write down all the letters (as *m*any as the e*x*ponent tells you to) and see if you can *m*ake sense of it.

All you need to know ...

The "Laws of Exponents" (also called "Rules of Exponents") come from **three ideas**:

 The exponent says **how many times** to use the number in a multiplication**.**

 A **negative exponent** means **divide**, because the opposite of multiplying is dividing

 A fractional exponent like $\frac{1}{n}$ means to **take the nth root**: $x^{\frac{1}{n}} = \sqrt[n]{x}$

If you understand those, then you understand exponents!

And all the laws below are based on those ideas.

Laws of Exponents

Here are the Laws (explanations follow):

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = \frac{1}{x}$	$4^{-1} = \frac{1}{4}$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^6}{x^2} = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3y^3$
$(\frac{x}{y})^n = \frac{x^n}{y^n}$	$(\frac{x}{y})^2 = \frac{x^2}{y^2}$
$x^{-n} = \frac{1}{x^n}$	$x^{-3} = \frac{1}{x^3}$
And the law about Fractional Exponents:	
$x^{\frac{m}{n}} = \sqrt[n]{x}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Laws Explained

The first three laws above ($x^1 = x$, $x^0 = 1$ and $x^{-1} = \frac{1}{x}$) are just part of the natural sequence of exponents. Have a look at this:

Example: Powers of 5

5^2	**1 × 5 × 5**	25
5^1	**1 × 5**	5
5^0	**1**	1
5^{-1}	**1 ÷ 5**	0.2
5^{-2}	**1 ÷ 5 ÷ 5**	0.04

.. etc..

Look at that table for a while, notice that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or 5 times smaller) depending on whether the exponent gets larger (or smaller).

The law that $x^m x^n = x^{m+n}$

With $x^m x^n$, how many times do we end up multiplying "x"? *Answer:* first "*m*" times, then **by another** "*n*" times, for a total of "*m*+*n*" times.

Example: $x^2 x^3 = (x\,x)(x\,x\,x) = x\,x\,x\,x\,x = x^5$

So, $x^2 x^3 = x^{(2+3)} = x^5$

The law that $\frac{x^m}{x^n} = x^{m-n}$

Like the previous example, how many times do we end up multiplying " x"? Answer: "m" times, then **reduce that** by "n" times (because we are dividing), for a total of "m-n" times.

Example: $x^4 / x^2 = (x\,x\,x\,x) / (x\,x) = x\,x = x^2$

So, $\frac{x^4}{x^2} = x^{(4-2)} = x^2$

(Remember that $\boldsymbol{x}/\boldsymbol{x} = 1$, so every time you see an $\boldsymbol{x}$ "above the line" and one "below the line" you can cancel them out.)

This law can also show you why $\boldsymbol{x^0=1}$:

Example: $x^2 / x^2 = \boldsymbol{x^{2-2} = x^0 = 1}$

The law that $(x^m)^n = x^{mn}$

First you multiply "m" times. Then you have **to do that "n" times**, for a total of m×n times.

Example: $(x^3)^4 = (x\,x\,x)^4 = (x\,x\,x)(x\,x\,x)(x\,x\,x)(x\,x\,x) = x\,x\,x\,x\,x\,x\,x\,x\,x\,x\,x\,x = x^{12}$

So $(x^3)^4 = x^{3\times4} = x^{12}$

The law that $(xy)^n = x^n y^n$

To show how this one works, just think of re-arranging all the "x"s and "y"s as in this example:

Example: $(xy)^3 = (xy)(xy)(xy) = xy\,xy\,xy = xxxyyy = (xxx)(yyy) = x^3y^3$

The law that $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Similar to the previous Example, just re-arrange the "x"s and "y"s

Example: $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{(xxx)}{(yyy)} = \frac{x^3}{y^3}$

The law that $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

OK, this one is a little more complicated!

I suggest you read Fractional Exponents first, so this makes more sense.

Anyway, the important idea is that:

$$x^{\frac{1}{n}} = \text{The } \boldsymbol{n}\text{-th Root of } x$$

And so a fractional exponent like $4^{\frac{3}{2}}$ is really saying to do a **cube** (3) and a **square root** (1/2), in any order.

Just remember from fractions that $\frac{m}{n} = \boldsymbol{m} \times \frac{1}{n}$

Example: $x^{\frac{m}{n}} = x^{m\times\frac{1}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$

The order does not matter, so it also works for $\frac{m}{n} = \left(\frac{1}{n}\right)\times \boldsymbol{m}$:

Example: $x^{\frac{m}{n}} = x^{\frac{1}{n}\times m} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$

Exponents of Exponents ...

What about this Example?

$$4^{3^2}$$

We do the exponent at the **top first**, so we calculate it this way:

Start with:	4^{3^2}
$3^2 = 3\times3$:	4^9
$4^9 = 4\times4\times4\times4\times4\times4\times4\times4\times4$:	262144

And That Is It!

If you find it hard to remember all these rules, then remember this:

You can work them out when you understand the three ideas near the top of this page:

- The exponent says **how many times** to use the number in a multiplication
- A **negative exponent** means **divide**
- A fractional exponent like **1/n** *m*eans to **take the *n*th root**: $x^{\frac{1}{n}} = \sqrt[n]{x}$

Oh, One *M*ore Thing ... What if $x = 0$?

Positive Exponent (n>0)	$0^n = 0$
Negative Exponent (n<0)	0^{-n} is **undefined** (because dividing by 0 is undefined)
Exponent = 0	0^0 ... *ummm* ... see below!

The Strange Case of 0^0

There are different arguments for the correct value of 0^0

0^0 could be 1, or possibly 0, so so*m*e people say it is really "indeterminate":

$x^0 = 1$, so ...	$0^0 = 1$
$0^n = 0$, so ...	$0^0 = 0$
When in doubt ...	0^0 = *"indeterminate"*