Class 11 Physics

Chapter 13 - Kinetic Theory

Some solved examples related to:

- Size of Molecules
- Distance between molecules
- Ideal Gas equation

Detailed notes can be seen here

Solved example 13.1

The density of water is 1000 kg m^{-3} . The density of water vapour at 100 °C and 1 atm pressure is 0.6 kg m^{-3} . The volume of a molecule multiplied by the total number gives, what is called, molecular volume. Estimate the ratio (or fraction) of the molecular volume to the total volume occupied by the water vapour under the above conditions of temperature and pressure.

Solution:

1. Initial and final states:

• The initial state is when water is in the liquid state

♦ Let V_i be the volume in the initial state

• Final state is when water is in the gaseous state

♦ Let V_f be the volume in the final state

• Mass m will be the same in both initial state and final state

2. Given that:

• Density in the initial state = 1000 kg m^{-3}

♦ But density = $\frac{Mass}{Volume}$

♦ So we can write: $\frac{m}{V_i}$= 1000 kg m^{-3}

3. Given that:

• Density in the final state = 0.6 kg m^{-3}

♦ But density = $\frac{Mass}{Volume}$

♦ So we can write: $\frac{m}{V_f}$= 0.6 kg m^{-3}

4. In the initial state, the molecules are closely packed. So the volume occupied by the mass of the water is same as the total volume of the molecules

• We can write: Molecular volume = V_i

5. So the ratio $\frac{Molecular\ Volume}{Volume\ of\ water\ vapour}$ is: $\frac{V_i}{V_f}$

• From (2) and (3), we get:

$$\left[\frac{m}{V_f} \div \frac{m}{V_i}\right] = \left[\frac{m}{V_f} \times \frac{V_i}{m}\right] = \left[\frac{V_i}{V_f}\right] = [0.6 \div 1000] = 6 \times 10^{-4}$$

Solved example 13.2

Estimate the volume of a water molecule using the data in Example 13.1.

Solution:

1. Density of liquid water = 1000 kg m^{-3}

♦ So 1000 kg of water will have a volume of one m^3

2. Molar mass of water = 18 grams

♦ So number of moles in 1000 kg of water = $\frac{1000 \times 10^3}{18}$

♦ So number of molecules in 1000 kg of water =

$\frac{1000 \times 10^3}{18} \times 6.023 \times 10^{23}$

3. In liquid water, molecules are closely packed

• So we can write:

Volume of one molecule × Number of molecules = Volume of sample

⇒ Volume of one molecule × $\frac{1000 \times 10^3}{18} \times 6.023 \times 10^{23}$ = 1 m^3

⇒ Volume of one molecule = 2.99 × 10^{-29}

4. We have: Volume of a sphere = $\frac{4}{3}\pi r^3$

♦ Where r is the radius of the sphere

• A molecule can be considered to be spherical

♦ Let r_m be the radius of one spherical molecule

• Then we can write: $\frac{4}{3}\pi r_m^3 = 2.99 \times 10^{-29}$

$\Rightarrow r_m = 1.93 \times 10^{-10}$ m $\simeq 2 \times 10^{-10}$ m = 2 Å

Solved example 13.3

What is the average distance between atoms (interatomic distance) in water? Use the data given in Examples 13.1 and 13.2.

Solution:

1. The initial state:

• The Let n be the number of molecules in V_i

♦ V_i is one m^3. It is a cube of edge 1 m

♦ So $\sqrt[3]{n}$molecules will be packed along each edge of that cube

• Those molecules will be closely packed

♦ So distance between the centers of the molecules = $(2r_m)$

$= 4 \times 10^{-10}$ m

• Length of one edge of the cube

= No. of molecules in one edge × Distance between molecules

$=\sqrt[3]{n} \times 4 \times 10^{-10}$

$\Rightarrow V_i = (\sqrt[3]{n} \times 4 \times 10^{-10})^3$

2. The final state

• In the water vapour state, V_i is enlarged into V_f

♦ The number of molecules is the same n

♦ V_f is a cube

♦ So $\sqrt[3]{n}$molecules will be arranged along each edge of that cube

• The distance between those molecules will be greater than 4×10^{-10} m

♦ Let the new distance be D

• Length of one edge of the cube

= No. of molecules in one edge × Distance between molecules

$=\sqrt[3]{n} \times D$

$\Rightarrow V_f = (\sqrt[3]{n} \times D)^3$

3. Taking ratios

$$\frac{V_f}{V_i} = \frac{(\sqrt[3]{n}\, D)^3}{(\sqrt[3]{n} \times 4 \times 10^{-10})^3} = \frac{D^3}{(4 \times 10^{-10})^3}$$

• But from example 13.1, we have: $\frac{V_f}{V_i} = \frac{1}{6 \times 10^{-4}}$

• Equating the two, we get: $\frac{D^3}{(4 \times 10^{-10})^3} = \frac{1}{6 \times 10^{-4}}$

$\Rightarrow D = 47.42 \times 10^{-10}$ m $\simeq 40$ Å

Solved example 13.4

A vessel contains two non-reactive gases: neon (monatomic) and oxygen (diatomic). The ratio of their partial pressures is 3:2. Estimate the ratio of (i) number of molecules and (ii) mass density of neon and oxygen in the vessel. Atomic mass of Ne = 20.2 u, molecular mass of O_2 = 32.0 u.

Solution:

Part (i):

1. Let V and T be the volume of the vessel and temperature of the mixture

♦ Let P_{Ne} and μ_{Ne} be the partial pressure and number of moles of Ne

♦ Let P_O and μ_O be the partial pressure and number of moles of O

2. The partial pressures:

♦ $P_{Ne} = \frac{\mu_{Ne} RT}{V}$

♦ $P_O = \frac{\mu_O RT}{V}$

3. Taking ratios

$$\frac{P_{Ne}}{P_O} = \frac{\mu_{Ne}}{\mu_O} = \frac{3}{2}$$

4. We have:

Number of molecules = Number of moles (μ) × N_A

• So the result in (3) can be modified as:

$$\frac{Number\ of\ Ne\ molecules}{Number\ of\ O\ molecules} = \frac{\mu_{Ne} \times N_A}{\mu_O \times N_A} = \frac{3}{2}$$

Part (ii):

- We have: Mass = Number of moles × Molar mass
- So the result in part (i) step (3) can be modified as:

$$\frac{Mass\ of\ Ne}{Mass\ of\ O} = \frac{\mu_{Ne} \times 20.2}{\mu_O \times 32} = \frac{3 \times 20.2}{2 \times 32} = 0.947$$

Solved example 13.5

Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3 Å.

Solution:

1. To find the volume of one O_2 molecule:

- Diameter of one O_2 molecule = 3×10^{-10} m

⇒ Radius of one O_2 molecule = 1.5×10^{-10} m

⇒ Volume of one O_2 molecule

$= \frac{4}{3}\pi r_{O2}^{3} = \frac{4}{3}\pi(1.5 \times 10^{-10})^3 = 14.13 \times 10^{-30}$

2. Let the gas be at STP

- Consider one mole of the gas
- Total volume of all molecules in that one mole

= Volume of one molecule × N_A

$= 14.13 \times 10^{-30} \times 6.023 \times 10^{23} = 85.10 \times 10^{-7}\ m^3$

3. Volume of that one mole at STP = 22.4 liters = $22.4 \times 10^{-3}\ m^3$

- So the required fraction = $\frac{85.10 \times 10^{-7}}{22.4 \times 10^{-3}} = 3.8 \times 10^{-4}$

Solved example 13.6

The Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.

Solution:

1. We have: The ideal gas equation: PV = μRT

- This can be rearranged as: $V = \frac{\mu RT}{P}$
- Where
  - ♦ μ= number of moles
  - ♦ R = 8.314 J^{-1} $mole^{-1}$ K^{-1}
  - ♦ T = Temperature in kelvin
  - ♦ P = Pressure

2. In our present case,
  - ♦ μ= 1
  - ♦ T = 0 C = 273 K
  - ♦ P = 1 atm = 101325 N m^{-2}

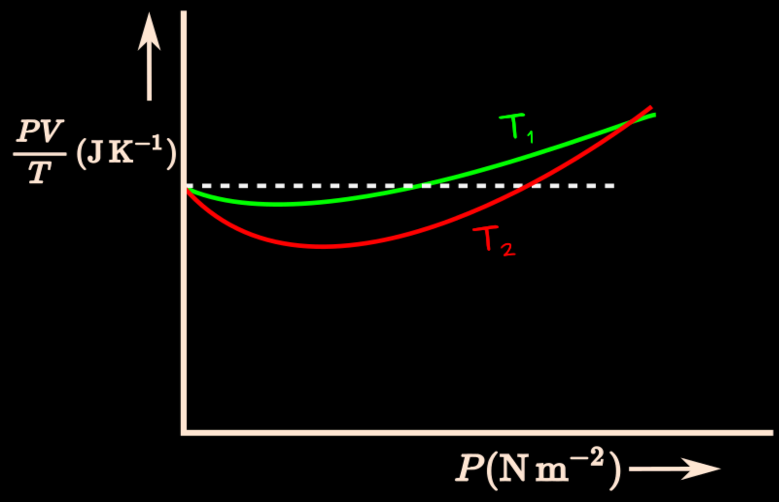
3. Substituting the known values in (1), we get:

$$V = \frac{1(mole)\times 8.314(J\ K^{-1}\ mole^{-1})\times 273(K)}{101325(Nm^{-2})} = 2.24 \times 10^{-2}\ m^3$$

- One m^3 = 1000 litres
  - ♦ So 2.24×10^{-2} m^3 = 22.4 litres

Solved example 13.7

Figure below shows a plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.



(a) What does the dotted plot signify?

(b) Which is true: $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y- axis?

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mo1^{-1} K^{-1}.)

Solution:

Part a:

1. We have: $\frac{PV}{T} = \mu R$

• While keeping the temperature at T1, the given sample is subjected to various pressures: P1, P2, P3, . . .

• For each of those pressure values, the corresponding volumes V1, V2, V3, . . . are measured

• Thus we get a number of pairs: (P_1, V_1), (P_2, V_2), (P_3, V_3), . . .

2. For each of those pairs, calculate the ratio $\frac{PV}{T}$

• Thus we get some new pairs:

$\left(P_1, \frac{P_1V_1}{T_1}\right), \left(P_2, \frac{P_2V_2}{T_1}\right), \left(P_3, \frac{P_3V_3}{T_1}\right), . .$

• In the above pairs, note that, P and V change but T is constant at T_1

3. Now a graph is plotted with

♦ P along the x-axis

♦ $\frac{PV}{T}$ along the y-axis

• Theoretically, $\frac{PV}{T}$ must be a constant. It's value does not change. So we expect a horizontal line

• But in practice, $\frac{PV}{T}$ is not a constant. We will not get a perfect horizontal line. That is why, it is shown as a dotted line

Part b:

1. In the given graph, we see that:

- ♦ When the temperature is T_1, the deviation from ideal behaviour is small
- ♦ When the temperature is T_2, the deviation from ideal behaviour is large

2. We know that, the deviation is small when temperature is high

• So we can conclude that $T_1 > T_2$ is true

Part c:

1. In the given graph, we see that,
   - ♦ the curves
   - ♦ and the horizontal dotted line
   - ♦ meet the y-axis at the same point

• So the 'required $\frac{PV}{T}$ value' is the theoretical constant value of $\frac{PV}{T}$

2. But the theoretical constant value of $\frac{PV}{T}$ is μR

• So our next aim is to find μ

3. The given sample contains 1 gram of O_2

• The molar mass of O_2 is 32 grams

• So $\mu = \frac{1}{32}$

4. Thus we get:

• Required value of $\frac{PV}{T}$

= μR = $\frac{1}{32}\,(mole)\ \times\ 8.314(J\ \ K^{-1}mole^{-1})$= 0.2598 J K^{-1}

Part d:

1. The horizontal dotted line in the given graph corresponds to μR where μ is the number of moles in one gram O_2

2. If we use one gram H_2, the μwill be different

• So our next aim is to find the mass of H_2 that will give the same $\frac{1}{32}$ moles of H_2

3. We know that, one mole H_2 is 2.02 grams

• So $\frac{1}{32}$ mole is $\left(\frac{1}{32}\ \times\ 2.02\right)$= 6.31×10^{-2} grams

Solved example 13.8

An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder ($R = 8.31$ J mol^{-1} K^{-1} , molecular mass of O_2 = 32 u).

Solution:

1. Given that:

- $P_1 = 15$ atm, $V = 30$ L, $T_1 = 27$ °C
    - ♦ $P_1 = (15 \times 101325) = 1.52 \times 10^6$ N m^{-2}
    - ♦ $V = (30 \times 10^{-3})$ m^3
    - ♦ $T_1 = (273 + 27) = 300$ K
- $P_2 = 11$ atm, $V = 30$ L, $T_2 = 290$ K
    - ♦ $P_2 = (11 \times 101325) = 1.11 \times 10^6$ N m^{-2}
    - ♦ $V = (30 \times 10^{-3})$ m^3
    - ♦ $T_1 = (273 + 17) = 290$ K

2. We have: $PV = \mu RT$

- Substituting the initial values, we get:

1.52×10^6 (N m^{-2}) $\times 30 \times 10^{-3}$(m^3) $= \mu_1 \times 8.31$(J mol^{-1} K^{-1}) $\times 300$ (K)

$\Rightarrow \mu_1 = 18.3$ moles

- Substituting the final values, we get:

1.11×10^6 (N m^{-2}) $\times 30 \times 10^{-3}$(m^3) $= \mu_2 \times 8.31$(J mol^{-1} K^{-1}) $\times 290$ (K)

$\Rightarrow \mu_2 = 13.87$ moles

3. So the decrease in the number of moles

$= (\mu_1 - \mu_2) = (18.3 - 13.87) = 4.41$

- That means, 4.41 moles of oxygen is taken out from the cylinder

4. One mole of oxygen is 32 grams

- So 4.41 moles is $(4.41 \times 32) = 141.3$ grams

Solved example 13.9

An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C ?

Solution:

1. We have: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

• The subscript '1' indicates the conditions when the bubble is at 40 m depth

• The subscript '2' indicates the conditions when the bubble is at the surface

2. P_1 = Atmospheric pressure + Pressure due to 40 m depth of water

• Atmospheric pressure = 1.013×10^5 N m^{-2}

• Pressure due to 40 m depth of water

= ρgh = (1000 × 9.81× 40) = 3.92×10^5

• So $P_1 = 4.94 \times 10^5$

3. P_2 = Atmospheric pressure = 1.013×10^5 N m^{-2}

4. Substituting the known values, we get:

$$\frac{4.94 \times 10^5 \times 1 \times 10^{-6}}{285} = \frac{1.013 \times 10^5 \times V_2}{308}$$

• Thus we get: $V_2 = 5.27 \times 10^{-6}$ m^3 = 5.27 cm^3

Solved example 13.10

Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure

Solution:

1. Given that, pressure P in the room is 1 atm

• This pressure is the sum of the partial pressures exerted by the various gases

• So we can write: $P = (\mu_1 + \mu_2 + \mu_3 ...)\frac{RT}{V}$

2. We are asked to find the number of molecules

- The number of molecules = $(\mu_1 + \mu_2 + \mu_3 ...)\times N_A$

3. We will denote $(\mu_1 + \mu_2 + \mu_3 ...)$ as μ

- So we can write: $P = \frac{\mu RT}{V}$
- Substituting the values, we get:

$1.013 \times 10^5 = \frac{\mu \times 8.31 \times 300}{25}$

$\Rightarrow \mu = 1015.84$ moles

4. So from (2), we get:

Number of molecules = $1015.84 \times 6.023 \times 10^{23} = 6.11 \times 10^{26}$

Solved example 13.11

The Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms :

Substance	Atomic mass (u)	Density (10^3 Kg m^{-3})
Carbon	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (Liquid)	19.00	1.14

[Hint : Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

Solution:

(a) Carbon:

1. Consider a cube of side 1 m, made entirely of carbon

- It will have a mass of 2.22×10^3 kg

2. One mole of carbon will have a mass of 12.01 grams

⇒ One gram of carbon will be $\frac{1}{12.01}$ moles

⇒ One kg of carbon will be $\frac{1000}{12.01}$ moles

⇒ 2.22×10^3 kg will have $\left(2.22 \times 10^3 \times \frac{1000}{12.01}\right)$

3. One mole carbon will have N_A molecules

⇒ $\left(2.22 \times 10^3 \times \frac{1000}{12.01}\right)$ moles will have $\left[\left(2.22 \times 10^3 \times \frac{1000}{12.01}\right) \times N_A\right]$ molecules

$= 1.85 \times 10^5\ N_A$ molecules

4. This much molecules occupy 1 m^3

- Based on the above steps, we can write a formula for the ‘number of molecules (N) in 1 m^3:

$$N = \frac{1000\,D\,N_A}{m_0}$$

  - ♦ Where
    - ☆ D = Density in Kg m^{-3}
    - ☆ m_0 = Molar mass in grams

5. We obtained the number of molecules in 1 m^3

- Since the molecules are tightly packed, we can write:

Volume of one molecule = $\left[1 \div \left(1.85 \times 10^5 \times N_A\right)\right]$

- Substituting the value of N_A we get:

Volume of one molecule = 8.98×10^{-30} m^3

6. We can write the above result also into a formula:

Volume (V) of one molecule = $[1 \div N] = \left[1 \div \frac{1000\,D\,N_A}{m_0}\right]$ $V = \frac{m_0}{1000\,D\,N_A}$

7. We obtained the volume of one carbon molecule

- A molecule can be considered to be a sphere
  - ♦ Let r be the radius of the sphere

• Then volume of the sphere = $\frac{4}{3}\pi r^3$ = volume of atom = 8.98×10^{-30} m^3

$\Rightarrow$ r = 1.29×10^{-10} m = 1.29 Å

8. We can write a formula for radius also:

We have: $V = \frac{m_0}{1000\, D\, N_A} = \frac{4}{3}\pi r^3$

$$\Rightarrow r = \left(\frac{3\, m_0}{4000\, \pi\, D\, N_A}\right)^{\frac{1}{3}}$$

- ♦ Where
  - ☆ D = Density in Kg m^{-3}
  - ☆ m_0 = Molar mass in grams
  - ☆ N_A = Avogadro number

• Check:

Radius (r) of carbon atom = $\left(\frac{3 \times 12.01}{4000 \times \pi \times 2.22 \times 10^3 \times N_A}\right)^{\frac{1}{3}}$

= 1.29×10^{-10} m = 1.29 Å

■ For the rest of the problem, we can use the formula:

$$\Rightarrow r = \left(\frac{3\, m_0}{4000\, \pi\, D\, N_A}\right)^{\frac{1}{3}}$$

(b) Gold:

• Given:
- ♦ m_0 = 197.00 grams
- ♦ D = 19.32 kg m^{-3}

• So r = 1.59×10^{-10} m = 1.59 Å

(c) Nitrogen:

• Given:
- ♦ m_0 = 14.01 grams
- ♦ D = 1.00 kg m^{-3}

• So r = 1.77×10^{-10} m = 1.77 Å

(d) Lithium:

- Given:
  - ♦ m_0 = 6.94 grams
  - ♦ D = 0.53 kg m^{-3}
- So r = 1.73 × 10^{-10} m = 1.73 Å

(e) Fluorine:

- Given:
  - ♦ m_0 = 19.00 grams
  - ♦ D = 1.14 kg m^{-3}
- So r = 1.59 × 10^{-10} m = 1.88 Å
