Discussion 10:

Theorems About Definite Integrals

In this module, you learned several theorems about definite integrals and how to evaluate them.

With your assigned partner or group, discuss the following questions:

1. You were presented with a theorem that states if $f(x)$ is continuous on $[a, b]$, then $f(x)$ is integrable on$[a, b]$. Suppose $f(x)$ is not continuous on $[a, b]$. Does this theorem imply that it is definitely not integrable? Why or why not?

2. Is it possible to find a piecewise-defined function that has a jump discontinuity at only one point in $[a, b]$ and that has finite area between the curve and the x -axis?If so, find one. If not, why not?

3. One of the properties of an definite integral is that $\int_a^a f(x)dx = 0$. Why is this always true, no matter the function $f(x)$ and no matter the value of a?

4. Is it possible to find a function such that $0 < a < b$ and $\int_a^b f(x)dx = 0$? If so, find one. If not, why not?

5. The Fundamental Theorem of Calculus, Part 2, states that if $f(x)$ is continuous on $[a, b]$, and $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$. Why is it important that $f(x)$ is continuous?

6. The general form of an antiderivative theorem showed that if $F(x)$ is an antiderivative of $f(x)$, then so is $G(x) = F(x) + C$ for any constant C. Why are you allowed to use *any* antiderivative when applying the Fundamental Theorem of Calculus, Part 2?

7. The derivative of the natural logarithmic function theorem showed that if $x > 0$ and $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$, but one of the formulas that can be used to evaluate integrals involving logarithmic functions states says that

$\int \frac{1}{x} dx = \ln|x| + C$. Is this a contradiction? Why is the absolute value needed in the second formula? What does this imply about the derivative of $f(x) = \ln|x|$?