SOLUTION OF ASSIGNMENT 2

Q1)

The values of twiddle factor are as under:

$$W_8^0 = e^0 = 1$$

$$W_8^1 = e^{-\frac{j\pi}{4}} = 0.707 - j0.707$$

$$W_8^2 = e^{-\frac{j\pi}{2}} = -j$$

$$W_8^3 = -0.707 - j0.703.$$

Output of Stage 1

$$\boxed{g(0) = x(0) + x(4) = \frac{1}{2} + 0 = 0.5}$$

$$g(1) = x(1) + x(5) = \frac{1}{2} + 0 = 0.5$$

$$g(2) = x(2) + x(6) = \frac{1}{2} + 0 = 0.5$$

$$g(3) = x(3) + x(7) = \frac{1}{2} + 0 = 0.5$$

$$g(4) = [x(0) - x(4)]\ W_8^0 = \left[\frac{1}{2} - 0\right] 1 = 0.5$$

$$g(5) = [x(1) - x(5)]\ W_8^1 = \left[\frac{1}{2} - 0\right] (0.707 - j0.707)$$

or $$g(5) = 0.3535 - j0.3535$$

$$g(6) = [x(2) - x(6)]\ W_8^2 = \left[\frac{1}{2} - 0\right] (-j) = -j0.5$$

$$g(7) = [x(3) - x(7)]\ W_8^3 = \left[\frac{1}{2} - 0\right] (0.707 = j0.707)$$

Output of Stage 2

$$h(0) = g(0) + g(2) = 0.5 + 0.5 = 1$$

$$h(1) = [g(1) + g(3)] = (0.5 + 0.5) = 1$$

$$h(2) = [g(0) - g(2)]\ W_8^0 = (0.5 - 0.5)(+1) = 0$$

$$h(3) = [g(1) - g(3)]\ W_8^2 = (0.5 - 0.5)(-j) = 0$$

$$h(4) = g(4) + g(6) = 0.5 - j\,0.5$$

$$h(5) = g(5) + g(7) = 0.3535 - j0.3535 - j0.3535 - j\,0.3535$$

or $$h(5) = -j0.707$$

$$h(6) = [g(4) - g(6)]\ W_8^0 = [0.5 + j0.5]\ 1 = 0.5 + j\,0.5$$

$$h(7) = [g(5) - g(7)]\ W_8^2$$
$$= [0.3535 - j0.3535 + 0.3535 + j0.3535]\ (-j)$$

or $$h(7) = -j0.707$$

Final Output

$$X(0) = h(0) + h(1) = 1 + 1 = 2$$

$$X(1) = h(4) + h(5) = 0.5 - j0.5 - j.0707 = 0.5 - j1.207$$

$$X(2) = h(2) + h(3) = 0 + 0 = 0$$

$$X(3) = [h(6) + h(7)] = [0.5 + j0.5 - j0.707].1 = 0.5 - j0.207$$

$$X(4) = [h(0) - h(1)]\ W_8^0 = [1 - 1].1 = 0$$

$$X(5) = [h(4) - h(5)]\, W_8^0 = [(0.5 - j\,0.5) + j\,0.707].1$$

or

$$X(5) = 0.5 + j0.207$$

$$X(6) = [h(2) - h(3)]\, W_8^0 = 0$$

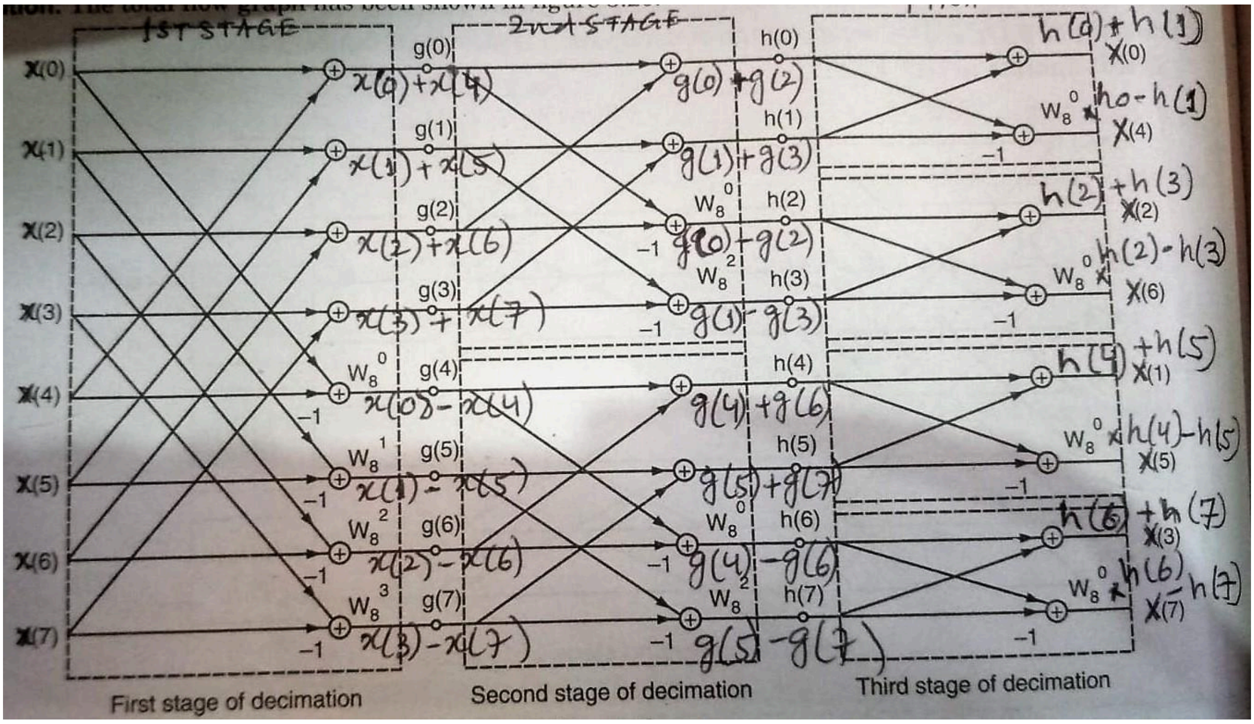
$$X(7) = [h(6) - h(7)]\, W_8^0 = [0.5 + j\,0.5 + j\,0.707] = 0.5 + j\,1.21$$

Therefore, we get

$$X(k) = [X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)]$$

or

$$X(k) = \{2,\ 0.5 - j1.207,\ 0,\ 0.5 - j0.207,\ 0,\ 0.5 + j0.207,\ 0,\ 0.5 + j1.21\}$$



Q2)

Solution: Let us consider a sequence x (n) such that:

$$x(n) = x_1(n) + j\,x_2(n)$$

Therefore, $x(n) = \{1 + j2,\ 1 + j1,\ 1 + j2,\ 1 + j1\}$

Here, $x(0) = 1 + j2,\ x(1) = 1 + j1,\ x(2) = 1 + j2,\ x(3) = 1 + j1.$

We shall obtain DFT using DIT FFT, the flow graph has been shown in figure 3.25.

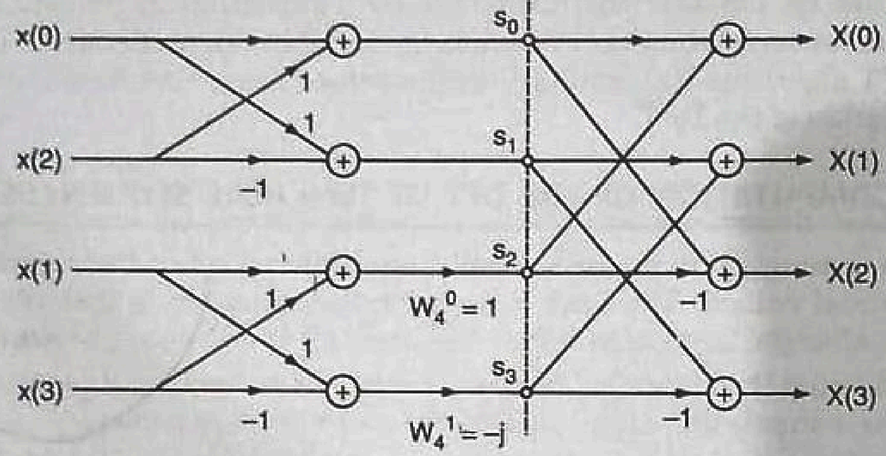


Fig. 3.25.

Here,

$$s_0 = x(0) + x(2) = 1 + j2 + 1 + j2 = 2 + j4$$

$$s_1 = x(0) - x(2) = 1 + j2 - 1 - j2 = 0$$

$$s_2 = [x(1) + x(3)] = [1 + j1 + 1 + j1] = 2 + j2$$

$$s_3 = [x(1) - x(3)] = [(1 + j1 - 1 - j1] = 0$$

The final output will be

$$X(0) = s_0 + s_2 = 2 + j4 + 2 + j2 = 4 + j6$$

$$X(1) = s_1 + s_3 = 0$$

$$X(2) = (s_0 - s_2)\ W_4^0 = 2 + j4 - 2 - j2 = j2$$

$$X(3) = (s_1 - s_3)\ W_4^1 = (0 - 0)(-j) = 0$$

Hence, $X(k) = \{4 + j6, 0, j2, 0\}$

Calculation of $X_1(k)$

Now, we have

$$X_1(k) = \frac{1}{2}[X(k) + X^*(N-k)] \quad ...(i)$$

Here, $N = 4$ and $X^*(k) = \{4 - j6, 0, -j2, 0\}$ ...(ii)

In equation (ii), $X^*(N-k) = X^*((-k))$ which represents circular folding of $X^*(k)$. It has been shown in figure 3.26. ...(iii)

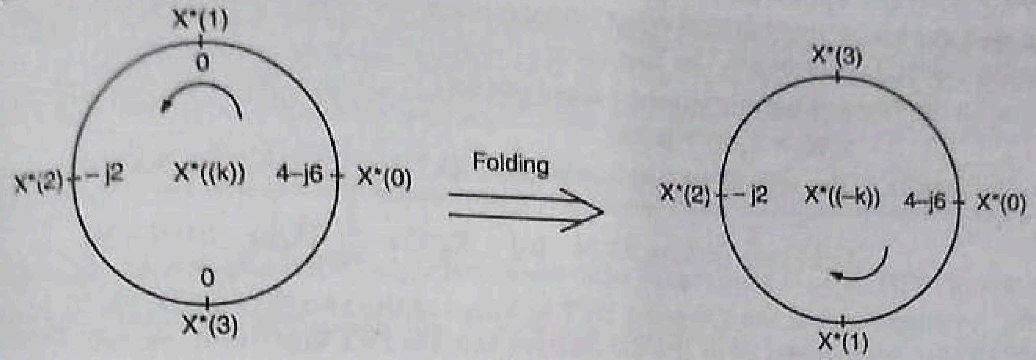


Fig. 3.26.

Hence, $X^*(-k) = \{4 - j6, 0, -j2, 0\}$

Using equation (ii), we can obtain DFT $X_1(k)$ as under:

We have, $X_1(k) = \frac{1}{2}[X(k) + X^*((-k))]$

For $k = 0$, we have $X_1(0) = \frac{1}{2}[X(0) + X^*(0)] = \frac{1}{2}[4 + j6 + 4 - j6] = 4$

For $k = 1$, we have $X_1(1) = \frac{1}{2}[X(1) + X^*(1)] = \frac{1}{2}[0 + 0] = 0$

For $k = 2$, we have $X_1(2) = \frac{1}{2}[X(2) + X^*((-2))] = \frac{1}{2}[+j2 - j2] = 0$

For $k = 3$, we have $X_1(3) = \frac{1}{2}[X(3) + X^*((-3))] = \frac{1}{2}[0 + 0] = 0$

Therefore, we get

$X_1(k) = \{4, 0, 0, 0\}$

Calculation of $X_2(k)$

We have,

$$X_2(k) = \frac{1}{2j}[X(k) - X^*(N-k)] = \frac{1}{2j}[X(k) - X^*((-k))]$$

For $k = 0$, we have $X_2(0) = \frac{1}{2j}[X(0) - X^*(0)] = \frac{1}{2j}[4 + j6 - 4 + j6] = 6$

For $k = 1$, we have $X_2(1) = \frac{1}{2j}[X(1) - X^*((-1))] = \frac{1}{2j}[0 + 0] = 0$

For $k = 2$, we have $X_2(2) = \frac{1}{2j}[X(2) - X^*((-2))] = \frac{1}{2j}[j2 + j2] = 2$

For $k = 3$, we have $X_2(3) = \frac{1}{2j}[X(3) - X^*((-3))] = \frac{1}{2j}[0 - 0] = 0$

Hence, $X_2(k) = \{6, 0, 2, 0\}$ **Ans.**

— J.B. Massie