

Game Theory Worksheet

Note: Answer key is at the end of this document

Video: "The Smartest Soccer Player"

Link:

https://www.youtube.com/watch?v=ZC_Hul1jUQM&t

Description: Video examines which soccer player played the correct mixed equilibrium in penalty kicks.

Resource Type: Problems, Students will practice calculating pure and mixed equilibria.

Assumptions: Students will need to know how to calculate pure and mixed equilibria.

Learning Objectives:

1. Practice finding equilibria in both a pure and mixed setting.

Economics Vocabulary Used:

Nash Equilibrium, Mixed Strategy

You can choose to do the worksheet before or after the video. Before allows the students to check their results when they watch the video. After allows the students to make sure they understand the concepts.

1. COOKIE GAME

Solve for the Nash equilibrium for the following payoff matrix (this represents the cookie game in the video)--

		Player 2	
		Split	Steal
Player 1	Split	1, 1	0, 2
	Steal	2, 0	0.1, 0.1

2. BASIC PENALTY KICKS

Solve for the mixed equilibrium for the kicker should follow for the payoff matrix below--

		Goalie	
		Right	Left
Kicker	Right	0, 0	1, -1
	Left	1, -1	0, 0

3. ADVANCED PENALTY KICKS

Solve for the mixed equilibrium for the kicker should follow for the payoff matrix below–

		Goalie	
		Right	Left
Kicker	Right	0.69, -0.69	0.96, -0.96
	Left	0.91, -0.91	0.71, -0.71

ANSWER KEY

1. COOKIE GAME

Solve for the Nash equilibrium for the following payoff matrix (this represents the cookie game in the video)--

		Player 2	
		Split	Steal
Player 1	Split	1, 1	0, 2
	Steal	2, 0	0.1, 0.1

2. BASIC PENALTY KICKS

Solve for the mixed equilibrium for the kicker should follow for the payoff matrix below–

		Goalie	
		Right	Left
Kicker	Right	0, 0	1, -1
	Left	1, -1	0, 0

The kicker must randomize such that the goalie does not have an advantage going right or left, i.e. the kicker needs to make sure the goalies' expected payoffs from going right and left are equal. Let's label the kicker's probability of going right as x and left as $1-x$.

Then the goalie's expected payoff from going right is:

$$0x + -1(1-x)$$

And the expected payoff from going left is:

$$-1x + 0(1-x)$$

The kicker should make sure these two are equal, therefor

$$-1*(1-x)=-1x$$

$$-1+x=-1x$$

$$-1=-2x$$

$$x=\frac{1}{2}$$

The kicker should go left half the time and right half the time.

3. ADVANCED PENALTY KICKS

Solve for the mixed equilibrium for the kicker should follow for the payoff matrix below–

		Goalie	
		Right	Left
Kicker	Right	0.69, -0.69	0.96, -0.96
	Left	0.91, -0.91	0.71, -0.71

The kicker must randomize such that the goalie does not have an advantage going right or left, i.e. the kicker needs to make sure the goalies' expected payoffs from going right and left are equal. Let's label the kicker's probability of going right as x and left as $1-x$.

Then the goalie's expected payoff from going right is:

$$-.69x+-.91*(1-x)$$

And the expected payoff from going left is:

$$-.96x+-.71*(1-x)$$

The kicker should make sure these two are equal, therefor

$$-.69x+-.91*(1-x)=-.96x+-.71*(1-x)$$

$$-.69x-.91+.91x=-.96x-.71+.71x$$

$$.22x-.91=-.25x-.71$$

$$.47x=.2$$

$$x=.43$$

$$1-x=.57$$

The kicker should go left roughly 57% of the time and right 43% .

