Building a Mountain Resort Part 1 - The Analysis

The weather is heating up, the flora is in bloom, and everyone you talk to is catching the travel bug. This sparks a brilliant idea which you share to a group of colleagues - why not throw in your hat to this business and build a resort? You're certain that with your expertise, you can design a resort filled with attractions that will bring in thousands of people. But can you be sure there is sufficient demand for it? Will it ultimately be profitable? And if so, where will you build it? These are the questions that you will explore in this analysis.

A Note to the Instructor: Problems 1(c) and 1(d) both involve limits at infinity, which are not taught in the text until Module 4. If you cover them earlier, you may use the problems as they are. If you plan to cover them in sequence with the rest of the material, you can still give the problems, but encourage students to use tables of values rather than trying to perform any algebraic calculations of the limit.

1. **Demand and Profits**

As many of those you talk to discuss plans to retreat to the mountains or relax at the beach, you narrow down your possible resort locations to the Mountains of Calculusia or the Bernoulli Beach. But which has the highest demand? And which looks like it will be the most profitable in the long run?

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(a) First you consider the Mountains of Calculusia. Three nearby mountain resorts kindly offer you their data on the average number of rooms that were occupied per day over the last 10 years. Their data is shown in the table below. What class of function do you think best fits this data? Explain your reasoning. What does that imply about the growth in demand for rooms in a mountain resort? (Bonus: Can you write down a function that fits this data well?) ***(LO 1.2.2, 1.5.1)***

Average Daily Occupied Rooms for Mountain Resorts			
	Mountain 1	Mountain 2	Mountain 3
10 Years Ago	59	100	53
9 Years Ago	111	132	62
8 Years Ago	80	149	90
7 Years Ago	148	154	111
6 Years Ago	175	207	141
5 Years Ago	188	190	212
4 Years Ago	318	264	257
3 Years Ago	325	337	344
2 Years Ago	503	419	429
1 Year Ago	540	558	536

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(b) Next you consider the Bernoulli Beach. Some nearby beach resorts also offer you their data on the average number of rooms that were occupied per year over the last 10 years. What class of function do you think best fits this data? Explain your reasoning. What does that imply about the growth in demand for rooms in a beach resort? (Bonus: Can you write down a function that fits this data well?) ***(LO 1.2.1)***

Average Daily Occupied Rooms for Beach Resorts			
	Beach 1	Beach 2	Beach 3
10 Years Ago	135	143	131
9 Years Ago	155	167	239
8 Years Ago	190	252	222
7 Years Ago	262	248	301
6 Years Ago	290	338	305
5 Years Ago	329	352	385
4 Years Ago	406	423	404

3 Years Ago	429	419	465
2 Years Ago	503	508	535
1 Year Ago	537	550	581

(c) After observing the data in parts (a) and (b), you pass it along to a market specialist who forecasts your profits from building a mountain resort and a beach resort. The function she gives you for your yearly profit if you build in the Mountains of Calculusia is

$$\pi_M(t) = \frac{2t^2+t-5}{t^2+1}, \quad t \geq 0$$

where t is the year after opening ($t = 0$ corresponds to the initial year the resort is open) and $\pi_M(t)$ is the profit that year (in millions). What will your forecasted initial profit be if you build a mountain resort? What will happen to profits in the first 5 years? What will happen to profits in the long run (as t grows really large)? (Hint: Try using a table of values to see what happens when you plug in large values of t) ***(LO 2.2.2, 4.6.1)***

(d) The function she gives you for your yearly profit if you build on Bernoulli Beach is

$$\pi_B(t) = \frac{8ln(t+0.5)}{t+0.5}, \quad t \geq 0$$

where t is the year after opening ($t = 0$ corresponds to the initial year the resort is open) and $\pi_B(t)$ is the profit that year (in millions). What will your initial profit be if you build a beach resort? What will happen to profits in the first 5 years? What will happen to profits in the long run (as t grows really large)? (Hint: You may want to use the fact that $ln(t + 0.5) < \sqrt{t + 0.5}$, or a table of values plugging in large values of t) ***(LO 2.2.2, 2.3.6, 4.6.1)***

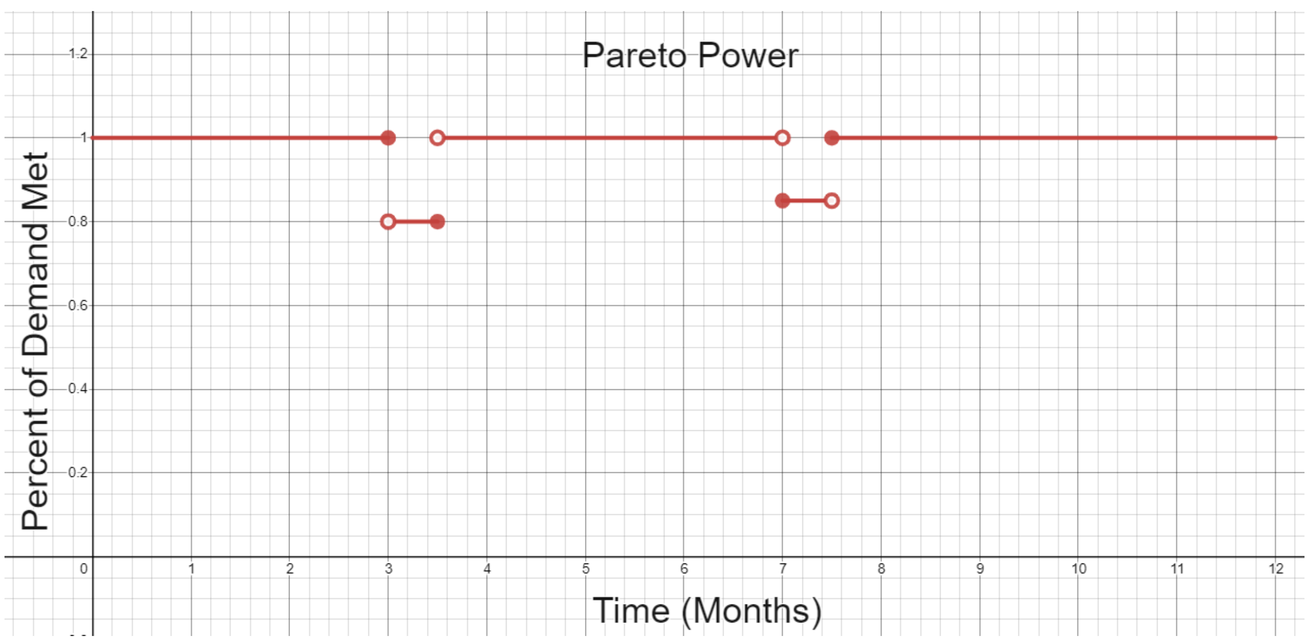
(e) Based on parts (a) through (d), do you think it is a better decision to build a mountain resort in the Mountains of Calculusia or a beach resort on Bernoulli Beach? Is your answer the same if your time horizon is the short run (under 5 years) or the long run (over 5 years)? How does what you know about both growth in demand and forecasted profits inform your decision? ***(No particular LO - tests general understanding of previous parts of the problem)***

2. Power Supply

Beyond the financial factors you've explored, there are many variables which are important to consider when choosing where to build a resort. For example, wherever you build, you will need to make sure that you can get power there - both for constructing and running the facility.

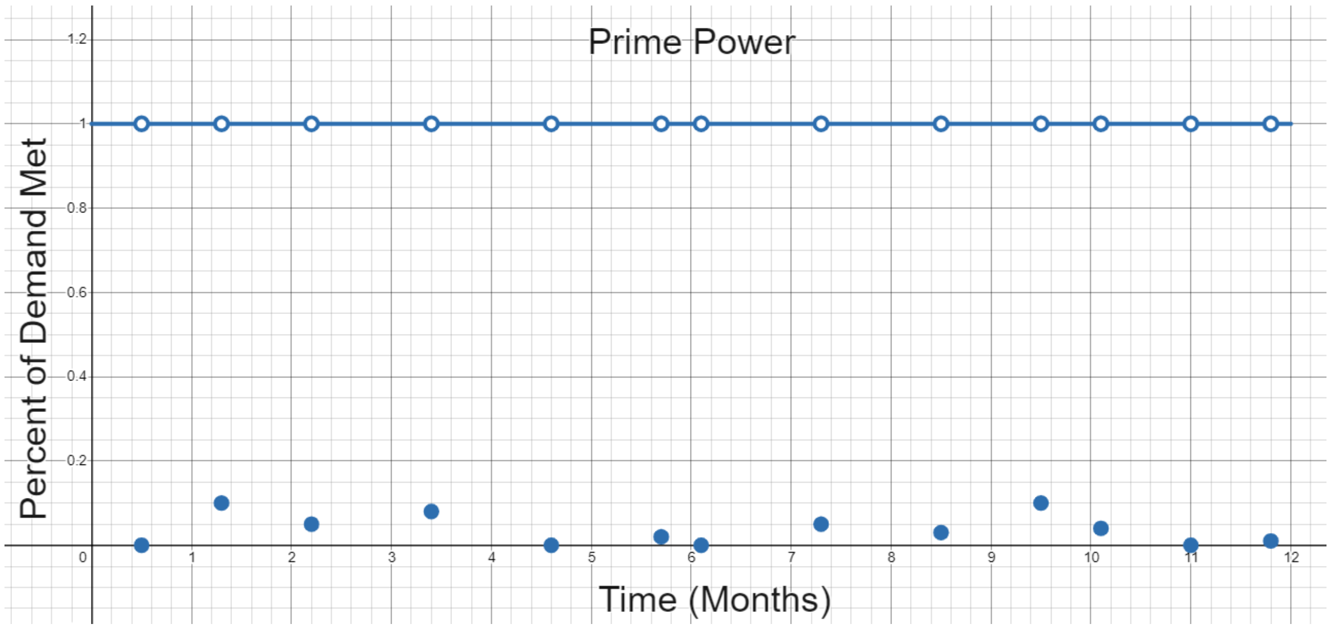
Upon doing some research, you discover that two different companies supply power to Bernoulli Beach and the Mountains of Calculusia. You want to make sure that the power supply is steady and reliable, so you do some digging and find information on historical power output from the two companies over the last year. Comparing the two will likely help you to come closer to finalizing your decision on where to build.

(a) Pareto Power is the company that supplies power to the Mountains of Calculusia. Below is a graph of their power output from the past year. The y-axis is the percent of the demand that Pareto Power was able to supply, and the x-axis is the time in months. Describe the graph of their power output in terms of continuity. How many points of discontinuity are there? What type of discontinuities are they? What might explain the discontinuities you identify, and what does it imply about the reliability of Pareto Power? ***(LO 2.4.2)***



(b) Prime Power is the company that supplies power to Bernoulli Beach. Below is a graph of their power output from the past year. The y-axis is the percent of the demand that Prime Power was able to supply, and the x-axis is the time in months. Describe the graph of their power output in terms of continuity. How many points of discontinuity are there? What type of discontinuities are they?

What might explain the discontinuities you identify, and what does it imply about the reliability of Prime Power? ***(LO 2.4.2)***



(c) Based on your analyses in (a) and (b), which power company would you consider the most reliable? How would that affect your decision of the location to build a resort? ***(No particular LO - synthesizing understanding of previous parts)***

3. Risk

Another important factor to consider when choosing where to build your resort is risk. Near Bernoulli Beach, the seas are known to become temperamental especially during the winter months of the year, resulting in a higher chance of hurricanes and floods. In the Mountains of Calculusia, on the other hand, the summer months often see drier conditions, resulting in a higher chance of forest fires.

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In order to build a model to make your final projections, you need to take the risk information that you have on hurricanes and fires and translate them to functions. This will provide you with one last piece of information to help you decide where it will be safest to build your resort.

(a) In the Mountains of Calculusia, the probability of a forest fire during the winter months of the year (corresponding to $0 \leq t < 2$ and $10 < t \leq 12$) is 0. From the beginning of March ($t = 2$) to end of September ($t = 9$), the probability of a forest fire increases at a constant rate from 0 to 0.25, except for the 4th of July ($t = 6.13$) when the risk jumps to 0.4 for that single day due to the fireworks. The risk quickly decreases through October ($9 < t \leq 10$)until it reaches 0 at the beginning of November ($t = 10$).

Write a function to represent the probability of a forest fire in the Mountains of Calculusia at a given time t in the year, based on the information above (note: the domain of your function should be $[0, 12]$). Is the function you write down continuous? If not, where are the discontinuities, and what type of discontinuities are they? ***(LO 1.2.1, 1.2.8, 2.4.3)***

(b) On Bernoulli Beach, the probability of a hurricane during the summer months of the year (corresponding to $5 < t < 8$) is 0.1. From the beginning of the year ($t = 0$) to end of May ($t = 5$), the probability of a hurricane is a constant 0.2. From the beginning of September ($t = 8$) to the end of the year, the probability of a hurricane returns to a constant 0.2.

Write a function to represent the probability of a hurricane on Bernoulli Beach at a given time t in the year, based on the information above (note: the domain of your function should be $[0, 12]$). Is the function you write down continuous? If not, where are the discontinuities, and what type of discontinuities are they? ***(LO 1.2.8, 2.4.3)***

(c) Compare the two functions you have written down in parts (a) and (b) - you may find it useful to graph them. Which location seems to have the lowest yearly risk of an adverse event? ***(No particular LO - synthesizing knowledge from previous parts)***

4. The Decision

After going through the analysis above, it's time for you to decide where to build your resort. Based on what you found in questions 1-3, do you think it will be best to build in the Mountains of Calculusia or on Bernoulli Beach? Explain your answer with a brief summary of your findings. ***(No particular LO - summary question from previous parts in assignment)***