7.7 APPLICATIONS OF DOT AND CROSS PRODUCT

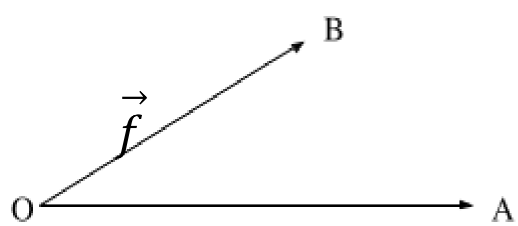
Formula for the Calculation of Work

$W = \vec{f} \cdot \vec{s}$, where $\vec{f}$ is the force acting on an object, measured in newtons (N); $\vec{s}$is the displacement of the object, measured in meter (m); and W is the work done, measured in joules (J).

Note: when a 1 N force moves an object 1 m, the amount of work done is 1 J.

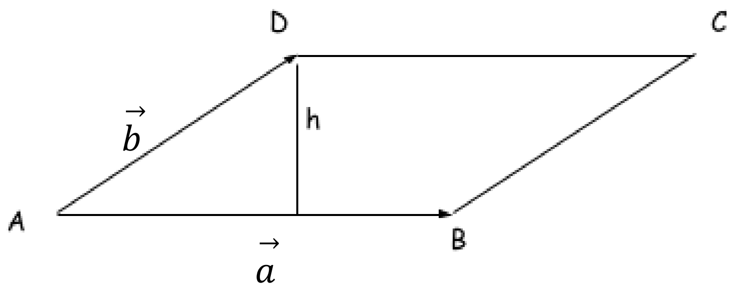
When a force is acting on an abject so that the object is moved from one point to another, we say that the force has done __________. Work is defined as the ______________ of the ______________ an object has been displaced and the component of the ___________ along the line of displacement.

In the following diagram, $\vec{OB}$ represents a constant force, $\vec{f}$, acting on an object on an object at O so that this force moves the object from ____ to _____. We will call the distance that the object is displaced s, which is a ___________, where we are assuming that $\vec{s} = \vec{OA}$ and $S =$ _______. The scalar projection of $\vec{f}$ on $\vec{OA}$ equals ON, or _________________, which is the same calculation for the scalar projection that was done earlier. The work, W, done by $\vec{f}$ in moving the object is calculated by W =



Ex. 1 Kennedy is pulling her daughter in a toboggan and is exerting a force of 40 N, acting at 24º to the ground. If Kennedy pulls the child a distance of 100 m, how much work was done?

Determining the area of a parallelogram.



It can be proven that this formula for the area is equal to ________________.

That is, ______________________________________ which can be found on page 411.

Ex. 2 Determine the area of the parallelogram determined by the vectors $\vec{p} = (-1, 5, 6)$ and $\vec{q} = (2, 3, -1)$.

Ex. 3 Determine the area of the of the triangle formed by the points A(-1,2,1) B(-1,0,0), and C(3,-1,4).

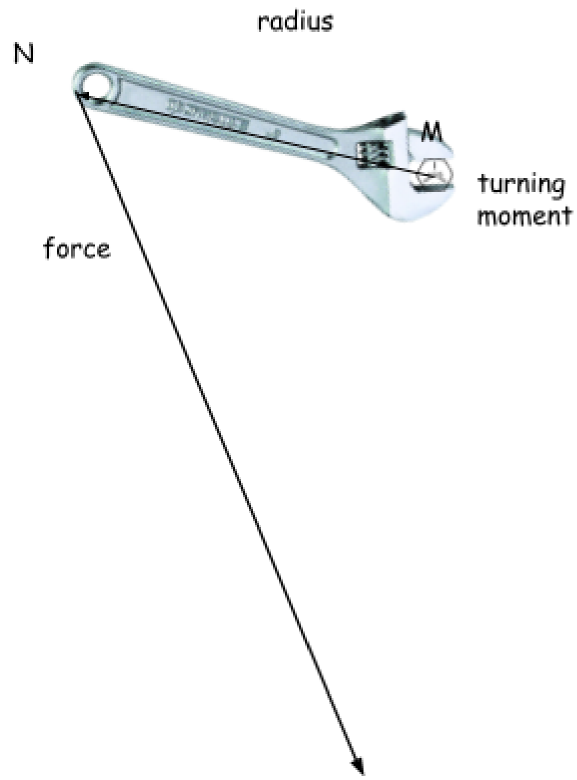
Physical Applications of the Cross Product

The cross product can also be used in the consideration of forces that involve rotation, or turning about a point or an axis. Such examples are:

- Tightening or loosening of a nut using a wrench
- Application of force to a bicycle pedal to make the crank arm rotate
- The act of opening a door by pushing or pulling on it

In the following situation, a bolt with a right-hand thread is being screwed into a piece of wood by a wrench, as shown. A force $\vec{f}$ is applied to the wrench at point N and is rotating about point M. The vector $\vec{r} = \overrightarrow{MN}$ is the position vector of N with respect to M.

The torque, or the turning effect, of the force ____ about the point _____ is defined to be the vector __________. This vector is ___________________ to the plane formed by the vectors $\vec{r}$ and $\vec{f}$, and gives the direction of the axis through M about which the force tends to twist. In this situation, the vector representing the ___________ _____________ is directed down as the bolt tightens into the wood and would normally be directed along the axis of the bolt.



The magnitude of the force depends on two factors: the exerted force, $\vec{f}$, and the distance between the line of the exerted force, d, and the point of rotation, M.

The magnitude of the force = (magnitude of the force) (the perpendicular distance from M to the force)

=

=

=

Note: Torque is measured in joules (J), which is the same unit that work is measured in.

Ex. 4 A 20 N force is applied at the end of a wrench that is 40 cm in length. The force is applied at an angled of 60º to the wrench. Calculate the magnitude of the torque about the point of rotation M. Include a diagram.

Note: the magnitude of the torque is $\left|\vec{r} \times \vec{f}\right| =$__________________, which is at its maximum when

______________ and when the ___________ is applied as far possible form the turning point. That is, ______ is

as large as possible. Thus to get the best effect when tightening a bolt, the force should be applied at

______________ angles to the wrench and as _______ _____________ the handle of the wrench as possible from

the _________________ _____________.