

Post Correspondence Problem

In this section, we will discuss the undecidability of string and not of Turing machines. The undecidability of the string is determined with the help of Post's Correspondence Problem (PCP). Let us define the PCP.

"The Post's correspondence problem consists of two lists of string that are of equal length over the input. The two lists are $A = w_1, w_2, w_3, \dots, w_n$ and $B = x_1, x_2, x_3, \dots, x_n$ then there exists a non empty set of integers $i_1, i_2, i_3, \dots, i_n$ in such that,
 $w_1, w_2, w_3, \dots, w_n = x_1, x_2, x_3, \dots, x_n$ "

To solve the post correspondence problem we try all the combinations of $i_1, i_2, i_3, \dots, i_n$ to find the $w_1 = x_1$ then we say that PCP has a solution.

Example 1:

Consider the correspondence system as given below

$A = (b, bab^3, ba)$ and $B = (b^3, ba, a)$. The input set is $\Sigma = \{0, 1\}$. Find the solution.

Solution:

A solution is 2, 1, 1, 3. That means $w_2w_1w_1w_3 = x_2x_1x_1x_3$

The constructed string from both lists is bab^3b^3a .

w_2	w_1	w_1	w_3
bab^3	b	b	ba
x_2	x_1	x_1	x_3
ba	b^3	b^3	a

Example 2:

Does PCP with two lists $x = (b, a, aba, bb)$ and $y = (ba, ba, ab, b)$ have a solution?

Solution: Now we have to find out such a sequence that strings formed by x and y are identical. Such a sequence is 1, 2, 1, 3, 3, 4. Hence from x and y list

1	2	1	3	3	4		1	2	1	3	3
b	a	b	aba	aba	bb	=	ba	ba	ba	ab	ab

Example 3:

Obtain the solution for the following system of posts correspondence problem. $A = \{100, 0, 1\}$, $B = \{1, 100, 00\}$

Solution: Consider the sequence 1, 3, 2. The string obtained from $A = babababb$. The string obtained from $B = bababbbb$. These two strings are not equal. Thus if we try various combination from both the sets to find the unique sequence, we could not get such a sequence. Hence there is no solution for this system.

Example 4:

Obtain the solution for the following system of posts correspondence problem, $X = \{100, 0, 1\}$, $Y = \{1, 100, 00\}$.

Solution: The solution is 1, 3, 1, 1, 3, 2, 2. The string is

$X_1X_3X_1X_1X_3X_2X_2 = 100 + 1 + 100 + 100 + 1 + 0 + 0 = 1001100100100$
 $Y_1Y_3Y_1Y_1Y_3Y_2Y_2 = 1 + 00 + 1 + 1 + 00 + 100 + 100 = 1001100100100$