

# Banach spaces + Locally convex spaces (6 weeks)

## Lecturers:

Dr Eder Kikianty (University of Pretoria)

Dr Miek Messerschmidt (University of Pretoria)

## Course material:

John Conway. A course in functional analysis. 2nd ed.

Erwin Kreyszig. Introductory functional analysis with applications. 2nd ed.

## Topics:

(week 1-2)

Infinite dimensional spaces and Banach spaces

Hamel Bases (application of Zorn's Lemma)

Schauder Bases

$\ell_p$  direct sums of Banach spaces

Quotients of Banach spaces.

Projections in Banach spaces.

Complemented subspaces vs not complementable subspaces.

Mention Lindenstrauss + Tzafriri characterization of Hilbert spaces.

$\ell_\infty$  /  $c_0$

$c_0$  is not complemented in  $\ell_\infty$ .

(week 3-6)

Topological vector spaces

General locally convex spaces

Hahn Banach Separation Theorem in LCS

Locally convex spaces have rich duals.

$L_p$  with  $0 < p < 1$  is not locally convex. Has a trivial dual.

Metrizable topologies/Non-metrizable topologies

Nets and sequences

When to use sequences, when to use nets.

Weak and Weak\* topologies

Dual pairs

Banach Alaoglu-Theorem

Characterizing reflexivity

Schur property  
Krein Milman Theorem  
Stone-Weierstrass Theorem