VCE School Assessed Coursework: SAC

Students Name:

Sacred Heart College Yarrawonga

VCE Study:	Further Mathematics
Unit:	4
Outcomes:	1, 2 and 3
Assessment Task	Core SAC – Matrices SAC Multiple Choice
Date:	Tuesday 21st June 2022
Time:	5 mins reading 50 mins writing
Instructions:	Students to circle the correct answer on the answer sheet.
Conditions:	Silent, individual work
Permitted Materials:	Pens, Pencils, Ruler, Eraser, TIInspire CAS calculator, Bound Notes
Marks allocated:	20

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the room.

I understand I must not intentionally or unintentionally disclose any details on this SAC or imply what is or is not included, or in any way gain an unfair advantage for myself over other students. If I do, I understand that disciplinary action will occur and my result will be downgraded. In fairness to fellow students it is my responsibility to inform the VCE office if I am aware that information about the SAC is being passed on, or that a student has gained unfair advantage.

Student Signature: Date:

Matrices Multiple Choice SAC

Question 1

Consider the following four matrices.

$$\begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 7 & 3 & 1 \\ 8 & 2 & 4 \end{bmatrix}$$
$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -3 & 6 \\ 5 & 8 \end{bmatrix}$$

The number of these matrices that have an inverse is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 2

Richie purchases 2 kg of carrots and 5 kg of potatoes from his greengrocer.
The cost, in dollars per kg, of carrots and potatoes, is shown in the table below.

Item	Cost ($'s per kg)
carrots	3
potatoes	4

The total cost of Richie's purchase can be determined by calculating

A. $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 5 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \end{bmatrix} \times \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Question 3.

George used a permutation matrix to scramble his password of 3P*K$ to become $3KP*.
The permutation matrix George used was

A. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Question 4

A family group including Grandma (G), Uncle Harry (H), Aunty Iris (I), cousin Jess (J) and grandchild Kieran (K), catch up using a Zoom meeting.
Technical problems mean that not everyone can communicate directly.
The matrix below shows which family members can communicate directly with each other.

$$\begin{array}{cc} & \textit{receiver} \\ & \begin{array}{ccccc} G & H & I & J & K \end{array} \\ \textit{sender} \begin{array}{c} G \\ H \\ I \\ J \\ K \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A ‘1’ in the matrix indicates a direct link exists between the sender and the receiver.
A ‘0’ in the matrix indicates no direct link exists between the sender and the receiver.
Grandma has some news to tell Kieran. The number of different ways in which this can happen is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 5

Matrices A, B, C and D are defined as follows-

$$A=\begin{bmatrix} 1 & 6 & 3 \\ 4 & 0 & 9 \end{bmatrix}, \quad B=\begin{bmatrix} 4 & 7 \\ 6 & 1 \end{bmatrix}, \quad C=\begin{bmatrix} 4 & 1 \\ 6 & 0 \\ 0 & 2 \end{bmatrix}, \quad D=\begin{bmatrix} 5 & 4 \\ 3 & 8 \\ 0 & 1 \end{bmatrix}$$

Which of the following matrix product is not defined?

A. $C \times D$
B. $D \times B$
C. $C \times B$
D. $A \times D$
E. $D \times A$

Question 6

For the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, AA^T is equal to:

A. $\begin{bmatrix} -1 \\ 7 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 2 \\ 2 & 25 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 10 \\ 2 & 25 \end{bmatrix}$

D. $\begin{bmatrix} 13 & 10 \\ 10 & 17 \end{bmatrix}$

E. $\begin{bmatrix} 4 & 1 \\ 9 & 16 \end{bmatrix}$

Question 7

The following transition matrix shows the expected change from a choice of tea and coffee for morning breakfast.

$$\begin{array}{cccc} & \textit{This week} & & \\ & \begin{array}{cc} \textit{Tea} & \textit{Coffee} \end{array} & & \\ T = & \begin{bmatrix} 0.65 & 0.30 \\ 0.35 & 0.70 \end{bmatrix} & \begin{array}{c} \textit{Tea} \\ \textit{Coffee} \end{array} & \textit{Next week} \end{array}$$

In week 1, 150 customers drink tea and 160 customers drink coffee.

The number of customers who drink coffee in week 3 is:

A. 167

B. 143

C. 166

D. 144

E. 53

Question 8

The transpose of matrix $M = \begin{bmatrix} 7 & 1 & 4 \\ 3 & 6 & 2 \end{bmatrix}$, is :

A. $\begin{bmatrix} 3 & 6 & 2 \\ 7 & 1 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 6 & 3 \\ 4 & 1 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 7 & 1 \\ 4 & 3 \\ 6 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 7 & 3 \\ 1 & 6 \\ 4 & 2 \end{bmatrix}$

E. $\begin{bmatrix} 3 & 7 \\ 6 & 1 \\ 2 & 4 \end{bmatrix}$

Question 9

If matrix $K = \begin{bmatrix} E \\ D \\ I \\ T \end{bmatrix}$ and matrix $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, then what word is formed by the matrix product $P \times K$?

A. $\begin{bmatrix} E \\ D \\ I \\ T \end{bmatrix}$

B. $\begin{bmatrix} D \\ E \\ T \\ I \end{bmatrix}$

C. $\begin{bmatrix} T \\ I \\ D \\ E \end{bmatrix}$

D. $\begin{bmatrix} D \\ I \\ E \\ T \end{bmatrix}$

E. $\begin{bmatrix} T \\ I \\ E \\ D \end{bmatrix}$

Question 10

If A is a 4×3 matrix and B is 2×4 matrix, what will be the order of the product matrix of these two matrices that is defined ?

A. 4×4
B. 3×2
C. 2×3
D. 2×4
E. 4×2

The following information relates to Questions 18 and 19

Kelly invested \$3000 in a term deposit that earns 4% per annum compounded quarterly.

Question 18

The recurrence relation that models this investment is:

A. $V_0 = 3000, \quad V_{n+1} = 1.04V_n$
B. $V_0 = 3000, \quad V_{n+1} = 0.01V_n$
C. $V_0 = 3000, \quad V_{n+1} = 0.04V_n$
D. $V_0 = 3000, \quad V_{n+1} = 4V_n$
E. $V_0 = 3000, \quad V_{n+1} = 1.01V_n$

Question 11

In a particular situation, the following matrix recurrence relation applies

$$S_{n+1} = TS_n + B$$

Where $T = \begin{bmatrix} 0.75 & 0.3 \\ 0.25 & 0.7 \end{bmatrix}$ and $B = \begin{bmatrix} 25 \\ 20 \end{bmatrix}$.

The value of S_0 if $S_1 = \begin{bmatrix} 130 \\ 115 \end{bmatrix}$ is :

A. $\begin{bmatrix} 126 \\ 119 \end{bmatrix}$

B. $\begin{bmatrix} 151 \\ 139 \end{bmatrix}$

C. $\begin{bmatrix} 100 \\ 100 \end{bmatrix}$

D. $\begin{bmatrix} 132 \\ 113 \end{bmatrix}$

E. $\begin{bmatrix} 107 \\ 93 \end{bmatrix}$

Question 12

The matrix S below shows the number of sick days taken by four employees Alan (A), Bert (B), Carrie (C), and Dan (D) in 2013, 2014 and 2015.

$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ S = & \left[\begin{array}{cccc} 3 & 5 & 2 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 5 & 1 \end{array}\right] \begin{array}{c} 2013 \\ 2014 \\ 2015 \end{array} \end{array}$$

The term s_{ij} represents the element in row i and column j of the matrix.

The number of sick days taken by Carrie in 2014 is represented by

A. s_{22}

B. s_{23}

C. s_{24}

D. s_{32}

E. s_{33}

Question 13

The total cost of two hamburgers and three drinks is \$12.00.
The total cost of three hamburgers and five drinks is \$19.00.
Let h be the cost of a hamburger and d be the cost of a drink.

The matrix $\begin{bmatrix} d \\ h \end{bmatrix}$ is equal to :

A. $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} d \\ h \end{bmatrix}$

B. $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 12 \\ 19 \end{bmatrix}$

C. $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 19 \\ 12 \end{bmatrix}$

D. $\begin{bmatrix} 3 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 12 \\ 19 \end{bmatrix}$

E. $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 12 \\ 19 \end{bmatrix}$

Question 14

A podiatry practice has two podiatrists Jane (J) and Sudhir (S) who jointly look after 260 patients who have regular monthly appointments.
The transition matrix T, below, shows the way patients change their preference for the two podiatrists from one month to the next.

$$\begin{array}{cc} & \text{this month} \\ & \begin{array}{cc} J & S \end{array} \\ T = & \begin{bmatrix} 0.75 & 0.15 \\ 0.25 & 0.85 \end{bmatrix} \begin{array}{c} J \\ S \end{array} \quad \text{next month} \end{array}$$

Jane and Sudhir joined the practice at the same time and they were each assigned 130 of these patients. Over the long term, the number of these patients who are expected to choose Sudhir for their monthly appointment is closest to

A. 98
B. 111
C. 130
D. 163
E. 178

Question 15

Four sets of simultaneous linear equations are shown in the table below.

$3x+6y=4$ $2x+5y=-1$	$x-y=3$ $x+y=5$	$2x-3y=8$ $-4x+6y=7$	$-2x+4y=6$ $7x+14y=1$

The number of these sets of equations that have a unique solution is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 16

Matrix P is a column matrix. Element a_{ij} is the element in row i and column j of matrix P.

The elements of this matrix can be found using the rule $a_{ij} = 2i + j$

Matrix P could be

A. $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

E. $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

Question 17

A principal makes weekly visits to each of the five Year 6 classes, 6A, 6B, 6C, 6D and 6E.
He visits a different class each weekday. On Thursdays he visits 6B.
The transition matrix that ensures he visits 6E on Tuesdays is

A.

$$\begin{array}{c} \text{one week day} \\ \begin{array}{ccccc} 6A & 6B & 6C & 6D & 6E \end{array} \end{array}$$
$$T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{l} 6A \\ 6B \\ 6C \text{ next week day} \\ 6D \\ 6E \end{array}$$

B.

$$\begin{array}{c} \text{one week day} \\ \begin{array}{ccccc} 6A & 6B & 6C & 6D & 6E \end{array} \end{array}$$
$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} 6A \\ 6B \\ 6C \text{ next week day} \\ 6D \\ 6E \end{array}$$

C.

$$\begin{array}{c} \text{one week day} \\ \begin{array}{ccccc} 6A & 6B & 6C & 6D & 6E \end{array} \end{array}$$
$$T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} 6A \\ 6B \\ 6C \text{ next week day} \\ 6D \\ 6E \end{array}$$

D.

$$\begin{array}{c} \text{one week day} \\ \begin{array}{ccccc} 6A & 6B & 6C & 6D & 6E \end{array} \end{array}$$
$$T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} 6A \\ 6B \\ 6C \text{ next week day} \\ 6D \\ 6E \end{array}$$

E.

$$\begin{array}{c} \text{one week day} \\ \begin{array}{ccccc} 6A & 6B & 6C & 6D & 6E \end{array} \end{array}$$
$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} 6A \\ 6B \\ 6C \text{ next week day} \\ 6D \\ 6E \end{array}$$

Question 18

As part of their community service certificate, students each week visit a garden project (G), a maintenance project (M) or a reading project (R).
Students can change projects from one week to the next week.
Let P_n be the state matrix that gives the number of students attending each of the projects n weeks after the visits begin.
The number of students expected at each of the projects can be determined by the matrix recurrence relation given by

$$P_{n+1} = TP_n, \qquad \text{where } T = \begin{array}{c} \text{this week} \\ \begin{array}{ccc} G & M & R \end{array} \\ \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.7 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ G \\ M \\ R \end{array} \text{ next week}$$

The state matrix P_3 shows the number of students at the projects three weeks after the visits begin.

$$P_3 = \begin{bmatrix} 26 \\ 37 \\ 48 \end{bmatrix}$$

The number of students expected to visit the maintenance project two weeks after the visits begin, can be found in the state matrix P_2 and is closest to

A. 23
B. 26
C. 31
D. 36
E. 53

Question 19

Matrix A is 2×3 matrix.
Matrix B is a 2×1 matrix.
The transpose of matrix C can be written as C^T.
Given that the matrix expression $A^T + (B \times C^T)^T$ is defined, what is the order of matrix C?

A. 1×2
B. 1×3
C. 2×2
D. 2×3
E. 3×1

Question 20

Matrix X is multiplied by the inverse of matrix Y, that is, Y^{-1} .
The result is cI where c is a non-zero scalar and I is the identity matrix.
Matrix Y is equal to

A. $\frac{1}{c}X$

B. $\frac{1}{c}X^{-1}$

C. cX

D. cX^{-1}

E. c^2XY^{-1}

SHC Further Mathematics Unit 4 Matrices

Answer Sheet for Multiple Choice Questions

Name: ___________________________

*****Please circle the correct answer.**

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E