Verona Public School District
Curriculum Overview

AP Statistics

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Verona Public Schools
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Verona Public Schools Mission Statement:

In partnership with a supportive community, we inspire our students to be creative, critical thinkers and compassionate global citizens through dynamic teaching, meaningful curricula, and enriching experiences.

Course Description:

This course is designed to prepare students for the Advanced Placement Statistics Exam. This course is for students that have completed Algebra II and possess sufficient mathematical maturity and quantitative reasoning ability. The topics for this course have been organized into four conceptual themes: Exploring Data, Sampling and Experimentation, Anticipating Patterns, and Statistical Inference. Student must provide their own TI NSpire CAS graphing calculator; these will be an important tool that will routinely be used in instruction. Student must take the AP exam to earn AP course credit.

Prerequisite(s):

Algebra II

Standard 8: Technology Standards

The curricular expectation for the Standard 8: Computer Science and Design Thinking standards in classes that are not specifically focused on computer science or engineering is infusion and integration throughout the curriculum. These are not intended to be standards for separate, stand alone lessons. The computer science and design thinking standards and practices are to be incorporated into other disciplines and contexts as appropriate.

8.1: Computer Science	8.2: Design Thinking
Computing Systems (CS) Networks and the Internet (NI) x Impacts of Computing (IC) x Data & Analysis (DA) x Algorithms & Programming (AP)	x Engineering Design (ED) x Interaction of Technology and Humans (ITH) Nature of Technology (NT) x Effects of Technology on the Natural World (ETW) x Ethics and Culture (EC)

Computer Science and Design Thinking Practices

1. Fostering an Inclusive Computing and Design Culture
2. Collaborating Around Computing and Design
3. Recognizing and Defining Computational Problems
4. Developing and Using Abstractions
5. Creating Computational Artifacts
6. Testing and Refining Computational Artifacts
7. Communicating About Computing and Design

SEL Competencies and Career Readiness, Life Literacies, and Key Skills Practices

The curricular expectation for the Standard 9: Career Readiness, Life Literacies, and Key Skills standards is infusion and integration throughout the curriculum. These are not intended to be standards for separate, stand alone lessons. The CLKS are to be incorporated into other disciplines and contexts as appropriate.

Social and Emotional Learning Core Competencies: *These competencies are identified as five interrelated sets of cognitive, affective, and behavioral capabilities*	**Career Readiness, Life Literacies, and Key Skills Practices:** *Career Readiness, Life Literacies, and Key Skills Practices describe the habits of the mind that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. These practices should be taught and reinforced in all content areas with increasingly higher levels of complexity and expectation as a student advances through a program of study.*
Self-awareness: The ability to accurately recognize one's emotions and thoughts and their influence on behavior. This includes accurately assessing one's strengths and limitations and possessing a well-grounded sense of confidence and optimism.	**CLKS6** Model integrity, ethical leadership, and effective management. **CLKS7** Plan education and career paths aligned to personal goals.
Self-management: The ability to regulate one's emotions, thoughts, and behaviors effectively in different situations. This includes managing stress, controlling impulses, motivating oneself, and setting and working toward achieving personal and academic goals.	**CLKS2** Attend to financial well-being. X **CLKS4** Demonstrate creativity and innovation. X **CLKS5** Utilize critical thinking to make sense of problems and persevere in solving them. X **CLKS8** Use technology to enhance productivity, increase collaboration, and communicate effectively.
Social awareness: The ability to take the perspective of and empathize with others from diverse backgrounds and cultures, to understand social and ethical norms for behavior, and to recognize family, school, and community resources and supports.	X **CLKS1** Act as a responsible and contributing community member and employee. **CLKS6** Model integrity, ethical leadership, and effective management.
Relationship skills: The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. This includes communicating clearly, listening actively, cooperating, resisting inappropriate social pressure, negotiating conflict constructively, and seeking and offering help when needed.	**CLKS6** Model integrity, ethical leadership, and effective management. **CLKS9** Work productively in teams while using cultural global competence.
Responsible decision making: The ability to make constructive and respectful choices about personal behavior and social interactions based on consideration of ethical standards, safety concerns, social norms, the realistic evaluation of consequences of various actions, and the well-being of self and others.	**CLKS3** Consider the environmental, social, and economic impact of decisions. X **CLKS5** Utilize critical thinking to make sense of problems and persevere in solving them. **CLKS6** Model integrity, ethical leadership, and effective management.

Course Materials

Core Instructional Materials: *These are the board adopted and approved materials to support the curriculum, instruction, and assessment of this course.*	**Differentiated Resources**: *These are teacher and department found materials, and also approved support materials that facilitate differentiation of curriculum, instruction, and assessment of this course.*
• AP Statistics Khan Academy • Stats Medic • AP Classroom (College Board) • The Practice of Statistics(Teacher Resources) • The Practice of Statistics (Student Resources) • AP Statistics Past Exam Questions • Stats Monkey • Nobel Stat Man • TI Nspire • TI Nspire Emulator (Teaching Software) • AP Statistics Course and Exam Description	• Albert io • AP Statistics Khan Academy • Stats Medic • Stats Monkey • American Statistical Association • Rossman/Chance Applet • StatKey

Mathematical Practice Standards (Revised for Readability*)
Math Practice 1: Make sense of mathematics. Mathematically proficient students begin a problem with a strategy in mind, but can also revise it until they get the result they are looking for. They feel comfortable representing their thinking using pictures, numbers, symbols, and/or words and can compare their method to other problem-solving strategies.
Math Practice 2: Add or remove context to solve problems. Mathematically proficient students understand what the numbers, symbols, pictures, words, etc. in their work represent. They feel comfortable switching back and forth between a problem's context and its representation and use the form that best fits the situation.
Math Practice 3: Explain and defend your reasoning. Mathematically proficient students can convince others that their reasoning is correct. This includes convincing others who have not solved the problem as well as those who have solved it but reached different conclusions.
Math Practice 4: Ask and answer questions about the world. Mathematically proficient students ask and answer questions about the world. They begin with a question in mind, determine what information is needed to answer it, and get the information. Next, they use that information to create a mathematical representation to answer the question. Then, they verify whether their representation works or needs improvement. If necessary, they repeat this process, adjusting both what information they use and how they use it until they sufficiently answer the question.
Math Practice 5: Use tools to make sense of mathematics. Mathematically proficient students use tools when they are helpful with making sense of mathematics. This includes physical tools (such as rulers, calculators, and manipulatives), virtual tools (such as graphing software and spreadsheets), or self-created tools (such as tables to organize data or estimation to see if an answer is reasonable).
Math Practice 6: Communicate precisely. Mathematically proficient students communicate precisely with others. This includes using proper definitions, defining their variables, specifying their units, and labeling axes.
Math Practice 7: Simplify problems by using their structure. Mathematically proficient students use patterns and structure to strategically transform complicated problems into one or more simpler problems. For example, a student may think of 99 + 46 as 100 + 45 or find the area of a complicated shape by breaking it into multiple simpler shapes.
Math Practice 8: Simplify problems by noticing patterns. Mathematically proficient students notice patterns and use them to simplify problems. For example, a student may notice repeated addition and multiply instead or may create a function to represent a repeated operation.
**Disclaimer These are not the actual Standards for Mathematical Practice. This revised version is Robert Kaplinsky's attempt at making them readable by as many people as possible. Download your copy at https://www.robertkaplinsky.com/smp.*

Unit 1: Exploring One-Variable Data

Unit Duration: ~14-16 Class Periods (15-23% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.A.1 Numbers may convey meaningful information, when placed in context.
VAR-1.B.1 A variable is a characteristic that changes from one individual to another.
VAR-1.C.1 A categorical variable takes on values that are category names or group labels.
VAR-1.C.2 A quantitative variable is one that takes on numerical values for a measured or counted quantity.
UNC-1.A.1 A frequency table gives the number of cases falling into each category. A relative frequency table gives the proportion of cases falling into each category.
UNC-1.B.1 Percentages, relative frequencies, and rates all provide the same information as proportions
UNC-1.B.2 Counts and relative frequencies of categorical data reveal information that can be used to justify claims about the data in context.
UNC-1.C.1 Bar charts (or bar graphs) are used to display frequencies (counts) or relative frequencies (proportions) for categorical data.
UNC-1.C.2 The height or length of each bar in a bar graph corresponds to either the number or proportion of observations falling within each category.
UNC-1.C.3 There are many additional ways to represent frequencies (counts) or relative frequencies (proportions) for categorical data.
UNC-1.D.1 Graphical representations of a categorical variable reveal information that can be used to justify claims about the data in context.
UNC-1.E.1 Frequency tables, bar graphs, or other representations can be used to compare two or more data sets in terms of the same categorical variable.
UNC-1.F.1 A discrete variable can take on a countable number of values. The number of values may be finite or countably infinite, as with the counting numbers.
UNC-1.F.2 A continuous variable can take on infinitely many values, but those values cannot be counted. No matter how small the interval between two values of a continuous variable, it is always possible to determine another value between them.
UNC-1.G.1 In a histogram, the height of each bar shows the number or proportion of observations that fall within the interval corresponding to that bar. Altering the interval widths can change the appearance of the histogram.
UNC-1.G.2 In a stem and leaf plot, each data value is split into a “stem” (the first digit or digits) and a “leaf” (usually the last digit).
UNC-1.G.3 A dotplot represents each observation by a dot, with the position on the horizontal axis corresponding to the data value of that observation, with nearly identical values stacked on top of each other.
UNC-1.G.4 A cumulative graph represents the number or proportion of a data set less than or equal to a given number.
UNC-1.G.5 There are many additional ways to graphically represent distributions of quantitative data.
UNC-1.H.1 Descriptions of the distribution of quantitative data include shape, center, and variability (spread), as well as any unusual features such as outliers, gaps, clusters, or multiple peaks.
UNC-1.H.2 Outliers for one-variable data are data points that are unusually small or large relative to the rest of the data.
UNC-1.H.3 A distribution is skewed to the right (positive skew) if the right tail is longer than the left. A distribution is skewed to the left (negative skew) if the left tail is longer than the right. A distribution is symmetric if the left half is the mirror image of the right half.
UNC-1.H.4 Univariate graphs with one main peak are known as unimodal. Graphs with two prominent peaks are bimodal. A graph where each bar height is approximately the same (no prominent peaks) is approximately uniform.
UNC-1.H.5 A gap is a region of a distribution between two data values where there are no observed data.
UNC-1.H.6 Clusters are concentrations of data usually separated by gaps.
UNC-1.H.7 Descriptive statistics does not attribute properties of a data set to a larger population, but may provide the basis for conjectures for subsequent testing.
UNC-1.I.1 A statistic is a numerical summary of sample data.
UNC-1.I.2 The mean is the sum of all the data values divided by the number of values.
UNC-1.I.3 The median of a data set is the middle value when data are ordered. When the number of data points is even, the median can take on any value between the two middle values. In AP Statistics, the most commonly used value for the median of a data set with an even number of values is the average of the two middle values.
UNC-1.I.4 The first quartile, Q1, is the median of the half of the ordered data set from the minimum to the position of the median. The third quartile, Q3, is the median of the half of the ordered data set from the position of the median to the maximum. Q1 and Q3 form the boundaries for the middle 50% of values in an ordered data set.
UNC-1.I.5 The pth percentile is interpreted as the value that has p% of the data less than or equal to it.
UNC-1.J.1 Three commonly used measures of variability (or spread) in a distribution are the range, interquartile range, and standard deviation.
UNC-1.J.2 The range is defined as the difference between the maximum data value and the minimum data value. The interquartile range (IQR) is defined as the difference between the third and first quartiles: Q3 − Q1. Both the range and the interquartile range are possible ways of measuring variability of the distribution of a quantitative variable
UNC-1.J.3 Standard deviation is a way to measure variability of the distribution of a quantitative variable.
UNC-1.J.4 Changing units of measurement affects the values of the calculated statistics.
UNC-1.K.1 There are many methods for determining outliers. Two methods frequently used in this course are: i. An outlier is a value greater than 1.5 × IQR above the third quartile or more than 1.5 × IQR below the first quartile. ii. An outlier is a value located 2 or more standard deviations above, or below, the mean
UNC-1.K.2 The mean, standard deviation, and range are considered nonresistant (or non-robust) because they are influenced by outliers. The median and IQR are considered resistant (or robust), because outliers do not greatly (if at all) affect their value
UNC-1.L.1 Taken together, the minimum data value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum data value make up the five-number summary. UNC-1.L.2 A boxplot is a graphical representation of the five-number summary (minimum, first quartile, median, third quartile, maximum). The box represents the middle 50% of data, with a line at the median and the ends of the box corresponding to the quartiles. Lines (“whiskers”) extend from the quartiles to the most extreme point that is not an outlier, and outliers are indicated by their own symbol beyond this
UNC-1.M.1 Summary statistics of quantitative data, or of sets of quantitative data, can be used to justify claims about the data in context.
UNC-1.M.2 If a distribution is relatively symmetric, then the mean and median are relatively close to one another. If a distribution is skewed right, then the mean is usually to the right of the median. If the distribution is skewed left, then the mean is usually to the left of the median.
UNC-1.N.1 Any of the graphical representations, e.g., histograms, side-by-side boxplots, etc., can be used to compare two or more independent samples on center, variability, clusters, gaps, outliers, and other features.
UNC-1.O.1 Any of the numerical summaries (e.g., mean, standard deviation, relative frequency, etc.) can be used to compare two or more independent samples.
VAR-2.A.1 A parameter is a numerical summary of a population. VAR-2.A.2 Some sets of data may be described as approximately normally distributed. A normal curve is mound-shaped and symmetric. The parameters of a normal distribution are the population mean, μ, and the population standard deviation, σ.
VAR-2.A.3 For a normal distribution, approximately 68% of the observations are within 1 standard deviation of the mean, approximately 95% of observations are within 2 standard deviations of the mean, and approximately 99.7% of observations are within 3 standard deviations of the mean. This is called the empirical rule.
VAR-2.A.4 Many variables can be modeled by a normal distribution.
VAR-2.B.1 A standardized score for a particular data value is calculated as (data value – mean)/(standard deviation), and measures the number of standard deviations a data value falls above or below the mean.
or below the mean. VAR-2.B.2 One example of a standardized score is a z-score. A z-score measures how many standard deviations a data value is from the mean.
VAR-2.B.3 Technology, such as a calculator, a standard normal table, or computer-generated output, can be used to find the proportion of data values located on a given interval of a normally distributed random variable.
VAR-2.B.4 Given the area of a region under the graph of the normal distribution curve, it is possible to use technology, such as a calculator, a standard normal table, or computer-generated output, to estimate parameters for some populations.
VAR-2.C.1 Percentiles and z-scores may be used to compare relative positions of points within a data set or between data sets

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species’ chances to survive and reproduce.

Technology Integration (NJSLS 8):

8.1.12.DA.2: Describe the trade-offs in how and where data is organized and stored.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.2.12.ETW.2: Synthesize and analyze data collected to monitor the effects of a technological product or system on the environment.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following questions...Is my cat old, compared to other cats? How certain are we that what seems to be a pattern is not just a coincidence?

Students will understand that:

1. Interpretation of data is dependent upon the graphical displays and numerical summaries.
2. The Who, What, Where, Why, and How of the data are important information that must be depicted in each given data set.
3. The shape, center, and spread are important characteristics of a distribution.
4. The question to be answered determines the data to be collected and how best to collect it.

Essential Questions:

1. What is data? How do we communicate and understand data?
2. Can you lie with statistics? How and to what extent?
3. How can data analysis be used to predict future happenings?
4. Does the data always lead to the truth?
5. Is all data “created equal”?

Students will know:

1. How to compare distributions or relative positions of points within a distribution.
2. How to interpret statistical calculations and findings to assign meaning or assess a claim.

Students will be able to:

1. Describe data presented numerically or graphically.
2. Construct numerical or graphical representations of distributions.
3. Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
4. Determine relative frequencies, proportions, or probabilities using simulation or calculations.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 1 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 1 Notes
- Unit 1 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
- Stats Monkey
- American Statistical Association
- Rossman/Chance Applet
- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

1. Preferential Seating facing the point of instruction
2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing
5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student’s ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes
5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student’s linguistic ability for activities and assessments

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students’ higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

	22. Utilize alternate reading assignments/materials at the student's reading level. 23. Allow for alternate seating for proximity to peer helper or teacher as necessary 24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary) 25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast. 26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds 27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples. 28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally) **Additional Resources:** ● 20 strategies to Support EAL Children ● What English Language Learners Wish Teachers Knew - Education Week ● A Starting Point: Tips and resources for working with ESL newcomer	

Unit 2: Exploring Two-Variable Data	Unit Duration: ~10-11 Class Periods (5-7% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.D.1 Apparent patterns and associations in data may be random or not.
UNC-1.P.1 Side-by-side bar graphs, segmented bar graphs, and mosaic plots are examples of bar graphs for one categorical variable, broken down by categories of another categorical variable.
UNC-1.P.2 Graphical representations of two categorical variables can be used to compare distributions and/or determine if variables are associated.
UNC-1.P.3 A two-way table, also called a contingency table, is used to summarize two categorical variables. The entries in the cells can be frequency counts or relative frequencies.
UNC-1.P.4 A joint relative frequency is a cell frequency divided by the total for the entire table.
UNC-1.Q.1 The marginal relative frequencies are the row and column totals in a two-way table divided by the total for the entire table.
UNC-1.Q.2 A conditional relative frequency is a relative frequency for a specific part of the contingency table (e.g., cell frequencies in a row divided by the total for that row).
UNC-1.R.1 Summary statistics for two categorical variables can be used to compare distributions and/or determine if variables are associated.
UNC-1.S.1 A bivariate quantitative data set consists of observations of two different quantitative variables made on individuals in a sample or population.
UNC-1.S.2 A scatterplot shows two numeric values for each observation, one corresponding to the value on the x-axis and one corresponding to the value on the y-axis. UNC-1.S.3 An explanatory variable is a variable whose values are used to explain or predict corresponding values for the response variable.
DAT-1.A.1 A description of a scatter plot includes form, direction, strength, and unusual features.
DAT-1.A.2 The direction of the association shown in a scatterplot, if any, can be described as positive or negative.
DAT-1.A.3 A positive association means that as values of one variable increase, the values of the other variable tend to increase. A negative association means that as values of one variable increase, values of the other variable tend to decrease.
DAT-1.A.4 The form of the association shown in a scatterplot, if any, can be described as linear or non-linear to varying degrees.
DAT-1.A.5 The strength of the association is how closely the individual points follow a specific pattern, e.g., linear, and can be shown in a scatterplot. Strength can be described as strong, moderate, or weak.
DAT-1.A.6 Unusual features of a scatter plot include clusters of points or points with relatively large discrepancies between the value of the response variable and a predicted value for the response variable.
DAT-1.B.1 The correlation, r, gives the direction and quantifies the strength of the linear association between two quantitative variables.
DAT-1.B.2 The correlation coefficient can be calculated by a formula. However, the most common way to determine r is by using technology.
DAT-1.B.3 A correlation coefficient close to 1 or −1 does not necessarily mean that a linear model is appropriate.
DAT-1.C.1 The correlation, r, is unit-free, and always between −1 and 1, inclusive. A value of r 0 indicates that ther r 1 e is no linear association. A value of or r 1 indicates that there is a perfect linear association.
DAT-1.C.2 A perceived or real relationship between two variables does not mean that changes in one variable cause changes in the other. That is, correlation does not necessarily imply causation.
DAT-1.D.1 A simple linear regression model is an equation that uses an explanatory variable, x, to predict the response variable, y.
DAT-1.D.2 The predicted response value, denoted by yˆ, is calculated as yˆ = +a bx, where a is the y-intercept and b is the slope of the regression line, and x is the value of the explanatory variable.
DAT-1.D.3 Extrapolation is predicting a response value using a value for the explanatory variable that is beyond the interval of x-values used to determine the regression line. The predicted value is less reliable as an estimate the further we extrapolate.
DAT-1.E.1 The residual is the difference between the actual value and the predicted value: residual = y - y hat .
DAT-1.E.2 A residual plot is a plot of residuals versus explanatory variable values or predicted response values.
DAT-1.F.1 Apparent randomness in a residual plot for a linear model is evidence of a linear form to the association between the variables.
DAT-1.F.2 Residual plots can be used to investigate the appropriateness of a selected model.
DAT-1.G.1 The least-squares regression model minimizes the sum of the squares of the residuals and contains the point (x,y).
DAT-1.G.2 The slope, b, of the regression line can be calculated as b = r (sy/sx) where r is the correlation between x and y, sy is the sample standard deviation of the response variable, y, and sx is the sample standard deviation of the explanatory variable, x.
DAT-1.G.3 Sometimes, the y-intercept of the line does not have a logical interpretation in context.
DAT-1.G.4 In simple linear regression,r2 is the square of the correlation, r. It is also r2 called the coefficient of determination. is the proportion of variation in the response variable that is explained by the explanatory variable in the model.
DAT-1.H.1 The coefficients of the least-squares regression model are the estimated slope and y-intercept.
DAT-1.H.2 The slope is the amount that the predicted y-value changes for every unit increase in x.
DAT-1.H.3 The y-intercept value is the predicted value of the response variable when the explanatory variable is equal to 0. The formula for the y-intercept, a, is a = y - bx
DAT-1.I.1 An outlier in regression is a point that does not follow the general trend shown in the rest of the data and has a large residual when the Least Squares Regression Line (LSRL) is calculated.
DAT-1.I.2 A high-leverage point in regression has a substantially larger or smaller x-value than the other observations have.
DAT-1.I.3 An influential point in regression is any point that, if removed, changes the relationship substantially. Examples include much different slope, y-intercept, and/or correlation. Outliers and high leverage points are often influential.
DAT-1.J.1 Transformations of variables, such as evaluating the natural logarithm of each value of the response variable or squaring each value of the explanatory variable, can be used to create transformed data sets, which may be more linear in form than the untransformed data.
DAT-1.J.2 Increased randomness in residual plots after transformation of data and/or movement of r 2 to a value closer to 1 offers evidence that the least-squares regression line for the transformed data is a more appropriate model to use to predict responses to the explanatory variable than the regression line for the untransformed data.

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.
HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*
HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.

Technology Integration (NJSLS 8):

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.2.12.EC.1: Analyze controversial technological issues and determine the degree to which individuals, businesses, and governments have an ethical role in decisions that are made

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following questions.... Does the fact that the number of shark attacks increases with ice cream sales necessarily mean that ice cream sales cause shark attacks? How might you represent incomes of individuals with and without a college degree to help describe similarities and/or differences between the two groups? How can you determine the effectiveness of a linear model that uses the number of cricket chirps per minute to predict temperature?

Students will understand that:	Essential Questions:
1. Regression is an effective model for prediction. 2. There is a difference between causation and correlation. 3. Scatterplots and other graphs are used to illustrate solutions and solve problems.	1. What does it mean to regress? 2. What is association? What is correlation? How are they connected? 3. Does association imply causation?

4. The way that data is collected, organized, analyzed and displayed influences interpretation. 5. Data is analyzed to verify the truth.	4. How can modeling data help us to understand patterns? 5. Can we use extrapolation to predict the future? 6. Is it possible to test for lack of correlation?
Students will know: 1. How to compare distributions or relative positions of points within a distribution. 2. How to describe data presented numerically or graphically. 3. How to interpret statistical calculations and findings to assign meaning or assess a claim.	**Students will be able to:** 1. Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response. 2. Construct numerical or graphical representations of distributions.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 2 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 2 Notes
- Unit 2 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
- Stats Monkey
- American Statistical Association
- Rossman/Chance Applet
- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

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2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing
5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student's ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes
5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
22. Utilize alternate reading assignments/materials at the student's reading level.
23. Allow for alternate seating for proximity to peer helper or teacher as necessary
24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary)
25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast.
26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples.
28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally)

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

	Additional Resources: • 20 strategies to Support EAL Children • What English Language Learners Wish Teachers Knew - Education Week • A Starting Point: Tips and resources for working with ESL newcomer	

Unit 3: Collecting Data

Unit Duration: ~19-10 Class Periods (12-15% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.E.1 Methods for data collection that do not rely on chance result in untrustworthy conclusions.
DAT-2.A.1 A population consists of all items or subjects of interest.
DAT-2.A.2 A sample selected for study is a subset of the population.
DAT-2.A.3 In an observational study, treatments are not imposed. Investigators examine data for a sample of individuals (retrospective) or follow a sample of individuals into the future collecting data (prospective) in order to investigate a topic of interest about the population. A sample survey is a type of observational study that collects data from a sample in an attempt to learn about the population from which the sample was taken.
DAT-2.A.4 In an experiment, different conditions (treatments) are assigned to experimental units (participants or subjects).
DAT-2.B.1 It is only appropriate to make generalizations about a population based on samples that are randomly selected or otherwise representative of that population.
DAT-2.B.2 A sample is only generalizable to the population from which the sample was selected.
DAT-2.B.3 It is not possible to determine causal relationships between variables using data collected in an observational study.
DAT-2.C.1 When an item from a population can be selected only once, this is called sampling without replacement. When an item from the population can be selected more than once, this is called sampling with replacement.
DAT-2.C.2 A simple random sample (SRS) is a sample in which every group of a given size has an equal chance of being chosen. This method is the basis for many types of sampling mechanisms. A few examples of mechanisms used to obtain SRSs include numbering individuals and using a random number generator to select which ones to include in the sample, ignoring repeats, using a table of random numbers, or drawing a card from a deck without replacement.
DAT-2.C.3 A stratified random sample involves the division of a population into separate groups, called strata, based on shared attributes or characteristics (homogeneous grouping). Within each stratum a simple random sample is selected, and the selected units are combined to form the sample
DAT-2.C.4 A cluster sample involves the division of a population into smaller groups, called clusters. Ideally, there is heterogeneity within each cluster, and clusters are similar to one another in their composition. A simple random sample of clusters is selected from the population to form the sample of clusters. Data are collected from all observations in the selected clusters. DAT-2.C.5 A systematic random sample is a method in which sample members from a population are selected according to a random starting point and a fixed, periodic interval. DAT-2.C.6 A census selects all items/subjects in a population.
DAT-2.D.1 There are advantages and disadvantages for each sampling method depending upon the question that is to be answered and the population from which the sample will be drawn.
DAT-2.E.1 Bias occurs when certain responses are systematically favored over others.
DAT-2.E.2 When a sample is comprised entirely of volunteers or people who choose to participate, the sample will typically not be representative of the population (voluntary response bias). DAT-2.E.3 When part of the population has a reduced chance of being included in the sample, the sample will typically not be representative of the population (undercoverage bias).
DAT-2.E.4 Individuals chosen for the sample for whom data cannot be obtained (or who refuse to respond) may differ from those for whom data can be obtained (nonresponse bias).
DAT-2.E.5 Problems in the data gathering instrument or process result in response bias. Examples include questions that are confusing or leading (question wording bias) and self-reported responses.
DAT-2.E.6 Non-random sampling methods (for example, samples chosen by convenience or voluntary response) introduce potential for bias because they do not use chance to select the individuals.
VAR-3.A.1 The experimental units are the individuals (which may be people or other objects of study) that are assigned treatments. When experimental units consist of people, they are sometimes referred to as participants or subjects. VAR-3.A.2 An explanatory variable (or factor) in an experiment is a variable whose levels are manipulated intentionally. The levels or combination of levels of the explanatory variable(s) are called treatments. VAR-3.A.3 A response variable in an experiment is an outcome from the experimental units that is measured after the treatments have been administered. VAR-3.A.4 A confounding variable in an experiment is a variable that is related to the explanatory variable and influences the response variable and may create a false perception of association between the two.
VAR-3.B.1 A well-designed experiment should include the following: a. Comparisons of at least two treatment groups, one of which could be a control group. b. Random assignment/allocation of treatments to experimental units. c. Replication (more than one experimental unit in each treatment group). d. Control of potential confounding variables where appropriate.
VAR-3.C.1 In a completely randomized design, treatments are assigned to experimental units completely at random. Random assignment tends to balance the effects of uncontrolled (confounding) variables so that differences in responses can be attributed to the treatments.
VAR-3.C.2 Methods for randomly assigning treatments to experimental units in a completely randomized design include using a random number generator, a table of random values, drawing chips without replacement, etc.
VAR-3.C.3 In a single-blind experiment, subjects do not know which treatment they are receiving, but members of the research team do, or vice versa.
VAR-3.C.4 In a double-blind experiment neither the subjects nor the members of the research team who interact with them know which treatment a subject is receiving.
VAR-3.C.5 A control group is a collection of experimental units either not given a treatment of interest or given a treatment with an inactive substance (placebo) in order to determine if the treatment of interest has an effect.
VAR-3.C.6 The placebo effect occurs when experimental units have a response to a placebo.
VAR-3.C.7 For randomized complete block designs, treatments are assigned completely at random within each block.
VAR-3.C.8 Blocking ensures that at the beginning of the experiment the units within each block are similar to each other with respect to at least one blocking variable. A randomized block design helps to separate natural variability from differences due to the blocking variable.
VAR-3.C.9 A matched pairs design is a special case of a randomized block design. Using a blocking variable, subjects (whether they are people or not) are arranged in pairs matched on relevant factors. Matched pairs may be formed naturally or by the experimenter. Every pair receives both treatments by randomly assigning one treatment to one member of the pair and subsequently assigning the remaining treatment to the second member of the pair. Alternately, each subject may get both treatments.
VAR-3.D.1 There are advantages and disadvantages for each experimental design depending on the question of interest, the resources available, and the nature of the experimental units.
VAR-3.E.1 Statistical inference attributes conclusions based on data to the distribution from which the data were collected.
VAR-3.E.2 Random assignment of treatments to experimental units allows researchers to conclude that some observed changes are so large as to be unlikely to have occurred by chance. Such changes are said to be statistically significant.
VAR-3.E.3 Statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.
VAR-3.E.4 If the experimental units used in an experiment are representative of some larger group of units, the results of an experiment can be generalized to the larger group. Random selection of experimental units gives a better chance that the units will be representative.

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.
HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*
HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.
HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.1.12.DA.2: Describe the trade-offs in how and where data is organized and stored.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
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9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
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9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Verona High School — AP Statistics

Transfer Goal:

Students will be able to independently use their learning to investigate the following… What do our data tell us? Why might the data we collected not be valid for drawing conclusions about an entire population?

Students will understand that:

1. Careful planning is essential to obtaining valid data.
2. Clarifying the question leads to the appropriate methodology.
3. The analysis is only as good as the data.
4. Well-designed experiments can allow us to reach appropriate cause-and-effect conclusions.

Essential Questions:

1. What is an experiment?
2. What is bias? How can it be identified? How can it be prevented?
3. To what extent is data biased? To what extent can data be purposely biased? Does size matter?
4. Is all data "created equal"?

Students will know:

1. An appropriate claim or draw an appropriate conclusion.
2. Key and relevant information to answer a question or solve a problem.
3. How to interpret statistical calculations and findings to assign meaning or assess a claim.

Students will be able to:

1. Describe an appropriate method for gathering and representing data.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 3 Test

Other Evidence:

Formal:
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Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 3 Notes
- Unit 3 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

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- Stats Medic
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- The Practice of Statistics(Teacher Resources)
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15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

	26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds 27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples. 28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally) **Additional Resources:** ● 20 strategies to Support EAL Children ● What English Language Learners Wish Teachers Knew - Education Week ● A Starting Point: Tips and resources for working with ESL newcomer	

Unit 4: Probability, Random Variables, and Probability Distributions

Unit Duration: ~18-20 Class Periods (10-20% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.F.1 Patterns in data do not necessarily mean that variation is not random.
UNC-2.A.1 A random process generates results that are determined by chance.
UNC-2.A.2 An outcome is the result of a trial of a random process.
UNC-2.A.3 An event is a collection of outcomes.
UNC-2.A.4 Simulation is a way to model random events, such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance. Record the counts of simulated outcomes and the count total.
UNC-2.A.5 The relative frequency of an outcome or event in simulated or empirical data can be used to estimate the probability of that outcome or event.
UNC-2.A.6 The law of large numbers states that simulated (empirical) probabilities tend to get closer to the true probability as the number of trials increases.
VAR-4.A.1 The sample space of a random process is the set of all possible non-overlapping outcomes.
VAR-4.A.2 If all outcomes in the sample space are equally likely, then the probability of an event E will occur is defined as the fraction: (the number of outcomes in event E) / (the total outcomes in the sample space).
VAR-4.A.3 The probability of an event is a number between 0 and 1, inclusive.
VAR-4.A.4 The probability of the complement of an event E, E' or EC , (i.e., not E) is equal to 1 - P(E).
VAR-4.B.1 Probabilities of events in repeatable situations can be interpreted as the relative frequency with which the event will occur in the long run.
VAR-4.C.1 The probability that events A and B both will occur, sometimes called the joint probability is the probability of the intersection of A and B, denoted P(A∩B) .
VAR-4.C.2 Two events are mutually exclusive or disjoint if they cannot occur at the same time. So P(A∩B)= 0.
VAR-4.D.1 The probability that event A will occur given that event B has occurred is called a conditional probability
VAR-4.D.2 The multiplication rule states that the probability that event A and B both will occur is equal to the probability that event A will occur multiplied by the probability that event B will occur, given that A has occurred.
VAR-4.E.1 Events A and B are independent if, and only if, knowing whether event A has occurred (or will occur) does not change the probability that event B will occur.
VAR-4.E.2 If, and only if, events A and B are independent, then P(A|B) = P(A), P(B|A) = P(B) = and P(A ∩B) = P(A) * P(B). .
VAR-4.E.3 The probability that event A or event B (or both) will occur is the probability of the union of A and B.
VAR-4.E.4 The addition rule states that the probability that event A or event B or both will occur is equal to the probability that event A will occur plus the probability that event B will occur minus the probability that both events A and B will occur.
VAR-5.A.1 The values of a random variable are the numerical outcomes of random behavior.
VAR-5.A.2 A discrete random variable is a variable that can only take a countable number of values. Each value has a probability associated with it. The sum of the probabilities over all of the possible values must be 1.
VAR-5.A.3 A probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.
VAR-5.A.4 A cumulative probability distribution can be represented as a table or function showing the probability of being less than or equal to each value of the random variable.
VAR-5.B.1 An interpretation of a probability distribution provides information about the shape, center, and spread of a population and allows one to make conclusions about the population of interest.
VAR-5.C.1 A numerical value measuring a characteristic of a population or the distribution of a random variable is known as a parameter, which is a single, fixed value.
VAR-5.C.2 Find the mean, or expected value, for a discrete random variable X.
VAR-5.C.3 Find the standard deviation for a discrete random variable X.
VAR-5.D.1 Parameters for a discrete random variable should be interpreted using appropriate units and within the context of a specific population.
VAR-5.E.1 For random variables X and Y and real numbers a and b, find the mean.
VAR-5.E.2 Two random variables are independent if knowing information about one of them does not change the probability distribution of the other.
VAR-5.E.3 For independent random variables X and Y and real numbers a and b, find the mean and the variance.
VAR-5.F.1 For Y = a + bX , the probability distribution of the transformed random variable, Y, has the same shape as the probability distribution for X, so long as a>0 and b>0. The mean of $\boldsymbol{\mu}$y = a +b$\boldsymbol{\mu}$x. The standard deviation of Y is $\boldsymbol{\sigma}$y=|b|$\boldsymbol{\sigma}$x.
UNC-3.A.1 A probability distribution can be constructed using the rules of probability or estimated with a simulation using random number generators.
UNC-3.A.2 A binomial random variable, X, counts the number of successes in n repeated independent trials, each trial having two possible outcomes (success or failure), with the probability of success p and the probability of failure 1 – p.
UNC-3.B.1 The probability that a binomial random variable, X, has exactly x successes for n independent trials when the probability of success is p, is calculated with the binomial formula , x = 0, 1, 2, . . . , n. This is the binomial probability function.
UNC-3.C.1 If a random variable is binomial, its mean, μ x, is np and its standard deviation, σ x, is $\sqrt{np(1-p)}$.
UNC-3.D.1 Probabilities and parameters for a binomial distribution should be interpreted using appropriate units and within the context of a specific population or situation.
UNC-3.E.1 For a sequence of independent trials, a geometric random variable, X, gives the number of the trial on which the first success occurs. Each trial has two possible outcomes (success or failure) with the probability of success p and the probability of failure 1 – p.
UNC-3.E.2 The probability that the first success for repeated independent trials with probability of success p occurs on trial x is calculated as P(X=x) = (1-p)x-1p, x=1,2,3,.... This is the geometric probability function.
UNC-3.F.1 If a random variable is geometric, its mean, x μ , is (1/p) and its standard deviation, x is $\sqrt{\frac{(1-p)}{p}}$.
UNC-3.G.1 Probabilities and parameters for a geometric distribution should be interpreted using appropriate units and within the context of a specific population or situation.

Interdisciplinary Standards (NJSLS):

HS-LS3-3. Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a Population.
HS-LS4-3. Apply concepts of statistics and probability to support explanations that organisms with an advantageous heritable trait tend to increase in proportion to organisms lacking this trait.

Technology Integration (NJSLS 8):

8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.
8.1.12.AP.8: Evaluate and refine computational artifacts to make them more usable and accessible
8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following…How can an event be both random and predictable? About how many rolls of a fair six-sided die would we anticipate it taking to get three 1s?

Students will understand that:

1. Probability models are useful tools for making decisions and predictions.
2. The notion and behavior of a random variable is foundational to understanding probability distributions.
3. Probability is based on relative frequencies.

Essential Questions:

1. What is the probability of understanding probability?
2. How can we base decisions on chance?
3. How can probability be used to simulate events and to predict future happenings?
4. What are the benefits of simulating events as opposed to gathering real data?
5. Is independence desirable?

4. The Law of Large Numbers is an important concept when simulating probability experiments but should be interpreted carefully.
5. Randomness and probability are the theoretical bases of statistical theory.
6. Probability models are useful tools for making decisions and predictions.
7. Probability is the basis of statistical inference.
8. The notion and behavior of a random variable is foundational to understanding probability distributions.

6. What is randomness?
7. How can modeling predict the future?
8. To what extent does our world exhibit binomial and geometric phenomena?
9. When is probability a sure thing?
10. How can we base decisions on chance?
11. Is anything in nature truly random?

Students will know:

1. How to Interpret statistical calculations and findings to assign meaning or assess a claim.
2. Parameters for probability distribution.
3. Various probability distributions

Students will be able to:

1. Determine relative frequencies, proportions, or probabilities using simulation or calculations.
2. Construct numerical or graphical representations of distributions.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 4 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 4 Notes
- Unit 4 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
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- American Statistical Association
- Rossman/Chance Applet
- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

1. Preferential Seating facing the point of instruction
2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing
5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student's ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes
5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
22. Utilize alternate reading assignments/materials at the student's reading level.
23. Allow for alternate seating for proximity to peer helper or teacher as necessary
24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary)
25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast.
26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

	hesitant to verbalize what they do not understand at first, so monitor and give examples. 28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally) **Additional Resources:** • 20 strategies to Support EAL Children • What English Language Learners Wish Teachers Knew - Education Week • A Starting Point: Tips and resources for working with ESL newcomer	

Unit 5: Sampling Distributions

Unit Duration: ~10-12 Class Periods (7-12% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.G.1 Variation in statistics for samples taken from the same population may be random or not.

VAR-6.A.1 A continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.

VAR-6.A.2 A continuous random variable with a normal distribution is commonly used to describe populations. The distribution of a normal random variable can be described by a normal, or "bell-shaped," curve.

VAR-6.A.3 The area under a normal curve over a given interval represents the probability that a particular value lies in that interval.

VAR-6.B.1 The boundaries of an interval associated with a given area in a normal distribution can be determined using z-scores or technology, such as a calculator, a standard normal table, or computer-generated output.

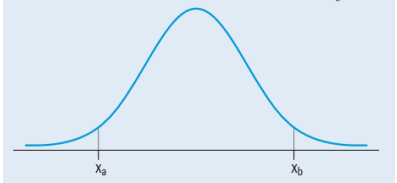
VAR-6.B.2 Intervals associated with a given area in a normal distribution can be determined by assigning appropriate inequalities to the boundaries of the intervals:

a. $P(X < x_a) = \frac{p}{100}$ means that the lowest $p\%$ of values lie to the left of x_a.

b. $P(x_a < X < x_b) = \frac{p}{100}$ means that $p\%$ of values lie between x_a and x_b.

c. $P(X > x_b) = \frac{p}{100}$ means that the highest $p\%$ of values lie to the right of x_b.

d. To determine the most extreme $p\%$ of values requires dividing the area associated with $p\%$ into two equal areas on either extreme of the distribution: $P(X < x_a) = \frac{1}{2}\frac{p}{100}$ and $P(X > x_b) = \frac{1}{2}\frac{p}{100}$ means that half of the $p\%$ most extreme values lie to the left of x_a and half of the $p\%$ most extreme values lie to the right of x_b.



VAR-6.C.1 Normal distributions are symmetrical and "bell-shaped." As a result, normal distributions can be used to approximate distributions with similar characteristics

UNC-3.H.1 A sampling distribution of a statistic is the distribution of values for the statistic for all possible samples of a given size from a given population.

UNC-3.H.2 The central limit theorem (CLT) states that when the sample size is sufficiently large, a sampling distribution of the mean of a random variable will be approximately normally distributed. UNC-3.H.3 The central limit theorem requires that the sample values are independent of each other and that n is sufficiently large.

UNC-3.H.4 A randomization distribution is a collection of statistics generated by simulation assuming known values for the parameters. For a randomized experiment, this means repeatedly randomly reallocating/reassigning the response values to treatment groups. UNC-3.H.5 The sampling distribution of a statistic can be simulated by generating repeated random samples from a population.

UNC-3.I.1 When estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator is equal to the population parameter.

UNC-3.J.1 When estimating a population parameter, an estimator exhibits variability that can be modeled using probability.

UNC-3.J.2 A sample statistic is a point estimator of the corresponding population parameter.

UNC-3.K.1 For independent samples (sampling with replacement) of a categorical variable from a population with population proportion, p, the sampling distribution of the sample proportion $\hat{p}$, has a mean, $\mu_{\hat{p}}$ =p and a standard deviation, $\boldsymbol{\sigma}\,\hat{p} = \sqrt{\frac{p(1-p)}{n}}$.

UNC-3.K.2 If sampling without replacement, the standard deviation of the sample proportion is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.

UNC-3.L.1 For a categorical variable, the sampling distribution of the sample proportion, $\hat{p}$ will have an approximate normal distribution, provided the sample size is large enough: np ≥ 10 and n (1 - p)≥ 10

UNC-3.M.1 Probabilities and parameters for a sampling distribution for a sample proportion should be interpreted using appropriate units and within the context of a specific population.
Determine parameters of a sampling distribution for a difference in sample proportions. [Skill 3.B]

UNC-3.N.1 For a categorical variable, when randomly sampling with replacement from two independent populations with population proportions p1 and p2, the sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ has mean $\mu_{\hat{p}_1 - \hat{p}_2}$ = p1 - p2 and standard deviation, $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$.

UNC-3.N.2 If sampling without replacement, the standard deviation of the difference in sample proportions is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.

UNC-3.O.1 The sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ will have an approximate normal distribution provided the sample sizes are large enough:

$n_1p_1 \geq 10\ and\ n_1(1 - p_1) \geq 10; n_2p_2 \geq 10\ and\ n_2(1 - p_2) \geq 10.$

UNC-3.P.1 Parameters for a sampling distribution for a difference of proportions should be interpreted using appropriate units and within the context of a specific populations.

UNC-3.Q.1 For a numerical variable, when random sampling with replacement from a population with mean μ and standard deviation, σ , the sampling distribution of the sample mean has mean x $\mu_{\bar{x}}$=μ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

UNC-3.Q.2 If sampling without replacement, the standard deviation of the sample mean is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.

UNC-3.R.1 For a numerical variable, if the population distribution can be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled with a normal distribution.

UNC-3.R.2 For a numerical variable, if the population distribution cannot be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled approximately by a normal distribution, provided the sample size is large enough, e.g., greater than or equal to 30.

UNC-3.S.1 Probabilities and parameters for a sampling distribution for a sample mean should be interpreted using appropriate units and within the context of a specific population.

UNC-3.T.1 For a numerical variable, when randomly sampling with replacement from two independent populations with population means μ1 and μ2 and population standard deviations σ1 and σ2, the sampling distribution of the difference in sample means has mean $\mu_{(\bar{x}_1, \bar{x}_2)} = \mu_1 - \mu_2$ and standard deviation $\sigma_{(\bar{x}_1, \bar{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$.

UNC-3.T.2 If sampling without replacement, the standard deviation of the difference in sample means is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.

UNC-3.U.1 The sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ can be modeled with a normal distribution if the two population distributions can be modeled with a normal distribution.

UNC-3.U.2 The sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ can be modeled approximately by a normal distribution if the two population distributions cannot be modeled with a normal distribution but both sample sizes are greater than or equal to 30.

UNC-3.V.1 Probabilities and parameters for a sampling distribution for a difference of sample means should be interpreted using appropriate units and within the context of a specific population.

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.

HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Verona High School — AP Statistics

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
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9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following…How likely is it to get a value this large just by chance? How can we anticipate patterns in the values of a statistic from one sample to another?

Students will understand that:

1. Many discrete phenomena may be described and thus predicted by binomial and geometric models.
2. The normal distribution and central limit theorem are essential to analyzing samples of data.
3. Variation can be expected in the results of random samples and is affected by the design of the sample or experiment.

Essential Questions:

1. How can modeling predict the future?
2. How does the normal distribution apply to the real world?
3. Does the Central Limit Theorem test one's limit?

Students will know:

1. Various probability distributions.
2. How to interpret statistical calculations and findings to assign meaning or assess a claim.
3. parameters for probability distributions

Students will be able to:

1. Determine relative frequencies, proportions, or probabilities using simulation or calculations.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 5 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 5 Notes
- Unit 5 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

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- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
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Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

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24. Provide the opportunity to revise for partial credit on wrong answers

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
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15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
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17. Simplify the language, format, and directions of the assessment
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Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
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15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
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Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

25. Modify homework, tests, or quizzes

Additional Resources:

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26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples.
28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally)

Additional Resources:

- 20 strategies to Support EAL Children
- What English Language Learners Wish Teachers Knew - Education Week
- A Starting Point: Tips and resources for working with ESL newcomer

Unit 6: Inference for Categorical Data: Proportions	Unit Duration: ~16-18 Class Periods (12-15% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.H.1 Variation in shapes of data distributions may be random or not.
UNC-4.A.1 The appropriate confidence interval procedure for a one-sample proportion for one categorical variable is a one sample z-interval for a proportion.
UNC-4.B.1 In order to make assumptions necessary for inference on population proportions, means, and slopes, we must check for independence in data collection methods and for selection of the appropriate sampling distribution.
UNC-4.B.2 In order to calculate a confidence interval to estimate a population proportion, p, we must check for independence and that the sampling distribution is approximately normal. a. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that 10%nN ≤ , where N is the size of the population. b. To check that the sampling distribution of $\widehat{p}$ is approximately normal (shape): i. For categorical variables, check that both the number of successes,n$\widehat{p}$, and the number of failures,n(1-$\widehat{p}$)are at least 10 so that the sample size is large enough to support an assumption of normality.
UNC-4.C.1 Based on sample data, the standard error of a statistic is an estimate for the standard deviation for the statistic.
UNC-4.C.2 A margin of error gives how much a value of a sample statistic is likely to vary from the value of the corresponding population parameter.
UNC-4.C.3 For categorical variables, the margin of error is the critical value (* z) times the standard error (SE) of the relevant statistic.
UNC-4.C.4 The formula for margin of error can be rearranged to solve for n, the minimum sample size needed to achieve a given margin of error. For this purpose, use a guess for $\widehat{p}$or use $\widehat{p} = 0.5$ in order to find an upper bound for the sample size that will result in a given margin of error.
UNC-4.D.1 In general, an interval estimate can be constructed as point estimate ± (margin of error).
UNC-4.D.2 Critical values represent the boundaries encompassing the middle C% of the standard normal distribution, where C% is an approximate confidence level for a proportion.
UNC-4.E.1 Confidence intervals for population proportions can be used to calculate interval estimates with specified units.
UNC-4.F.1 A confidence interval for a population proportion either contains the population proportion or it does not, because each interval is based on random sample data, which varies from sample to sample.
UNC-4.F.2 We are C% confident that the confidence interval for a population proportion captures the population proportion.
UNC-4.F.3 In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the population proportion.
UNC-4.F.4 Interpreting a confidence interval for a one sample proportion should include a reference to the sample taken and details about the population it represents.
UNC-4.G.1 A confidence interval for a population proportion provides an interval of values that may provide sufficient evidence to support a particular claim in context.
UNC-4.H.1 When all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases.
UNC-4.H.2 For a given sample, the width of the confidence interval for a population proportion increases as the confidence level increases.
UNC-4.H.3 The width of a confidence interval for a population proportion is exactly twice the margin of error.
VAR-6.D.1 The null hypothesis is the situation that is assumed to be correct unless evidence suggests otherwise, and the alternative hypothesis is the situation for which evidence is being collected.
VAR-6.D.2 For hypotheses about parameters, the null hypothesis contains an equality reference (=, ≥, or ≤), while the alternative hypothesis contains a strict inequality (<, >, or ≠). The type of inequality in the alternative hypothesis is based on the question of interest. Alternative hypotheses with < or > are called one-sided, and alternative hypotheses with ≠ are called two sided. Although the null hypothesis for a one sided test may include an inequality symbol, it is still tested at the boundary of equality.
VAR-6.D.3 The null hypothesis for a population proportion is: Ho: p = po, where po is the null hypothesized value for the population proportion.
VAR-6.D.4 A one-sided alternative hypothesis for a proportion is either $H_a: p < p_o$ or $H_a: p > p_o$. A two-sided alternative hypothesis is $H_a: p \neq p_o$.
VAR-6.D.5 For a one-sample z-test for a population proportion, the null hypothesis specifies a value for the population proportion, usually one indicating no difference or effect.
VAR-6.E.1 For a single categorical variable, the appropriate testing method for a population proportion is a one-sample z-test for a population proportion.
VAR-6.F.1 In order to make statistical inferences when testing a population proportion, we must check for independence and that the sampling distribution is approximately normal: a. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that n ≤10% N. b. To check that the sampling distribution of $\widehat{p}$ is approximately normal (shape): i. Assuming that H0 is true, verify that both the number of successes, npo and the number of failures, n(1-po) are at least 10 so that that the sample size is large enough to support an assumption of normality.
VAR-6.G.1 The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or when a probability model is assumed to be true, a theoretical distribution (z).
VAR-6.G.2 When using a z-test, the standardized test statistic can be written: test statistic = (sample statistic-null value of the parameter)/ (standard deviation of the statistic) This is called a z-statistic for proportions.
VAR-6.G.4 A p-value is the probability of obtaining a test statistic as extreme or more extreme than the observed test statistic when the null hypothesis and probability model are assumed to be true. The significance level may be given or determined by the researcher.
DAT-3.A.1 The p-value is the proportion of values for the null distribution that are as extreme or more extreme than the observed value of the test statistic. This is: a. The proportion at or above the observed value of the test statistic, if the alternative is >. b. The proportion at or below the observed value of the test statistic, if the alternative is <. c. The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is ≠.
DAT-3.A.2 An interpretation of the p-value of a significance test for a one-sample proportion should recognize that the p-value is computed by assuming that the probability model and null hypothesis are true, i.e., by assuming that the true population proportion is equal to the particular value stated in the null hypothesis.
DAT-3.B.1 The significance level,α , is the predetermined probability of rejecting the null hypothesis given that it is true.
DAT-3.B.2 A formal decision explicitly compares the p-value to the significance level, α . If the p-value ≤ α , reject the null hypothesis. If the p-value > α, fail to reject the null hypothesis.
DAT-3.B.3 Rejecting the null hypothesis means there is sufficient statistical evidence to support the alternative hypothesis. Failing to reject the null means there is insufficient statistical evidence to support the alternative hypothesis.
DAT-3.B.4 The conclusion about the alternative hypothesis must be stated in context.
DAT-3.B.5 A significance test can lead to rejecting or not rejecting the null hypothesis, but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
DAT-3.B.6 Small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis and probability model were true, and so provide evidence for the alternative. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
DAT-3.B.7 p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis and probability model were true, so do not provide convincing statistical evidence for the alternative hypothesis nor do they provide evidence that the null hypothesis is true.
DAT-3.B.8 A formal decision explicitly compares the p-value to the significance α . If the p-value ≤α , then reject the null hypothesis$Ho: p = p_o$. If the p-value > α , then fail to reject the null hypothesis.
DAT-3.B.9 The results of a significance test for a population proportion can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
UNC-5.A.1 A Type I error occurs when the null hypothesis is true and is rejected (false positive).
UNC-5.A.2 A Type II error occurs when the null hypothesis is false and is not rejected (false negative).
UNC-5.B.1 The significance level, α , is the probability of making a Type I error, if the null hypothesis is true.
UNC-5.B.2 The power of a test is the probability that a test will correctly reject a false null hypothesis. UNC-5.B.3 The probability of making a Type II error = 1 - power.
UNC-5.C.1 The probability of a Type II error decreases when any of the following occurs, provided the others do not change: i. Sample size(s) increases. ii. Significance level (α) of a test increases. iii. Standard error decreases. iv. True parameter value is farther from the null.
UNC-5.D.1 Whether a Type I or a Type II error is more consequential depends upon the situation.
UNC-5.D.2 Since the significance level is the probability of a Type I error, the consequences of a Type I error influence decisions about a significance level.
UNC-4.I.1 The appropriate confidence interval procedure for a two-sample comparison of proportions for one categorical variable is a two-sample z-interval for a difference between population proportions.
UNC-4.J.1 In order to calculate confidence intervals to estimate a difference between proportions, we must check for independence and that the sampling distribution is approximately normal.
UNC-4.L.1 Confidence intervals for a difference in proportions can be used to calculate interval estimates with specified units.
UNC-4.M.1 In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference in population proportions.
UNC-4.M.2 Interpreting a confidence interval for difference between population proportions should include a reference to the sample taken and details about the population it represents.
UNC-4.N.1 A confidence interval for difference in population proportions provides an interval of values that may provide sufficient evidence to support a particular claim in context.
Identify the null and alternative hypotheses for a difference of two population proportions. [Skill 1.F]
VAR-6.H.1 For a two-sample test for a difference of two proportions, the null hypothesis specifies a value of 0 for the difference in population proportions, indicating no difference or effect.
VAR-6.I.1 For a single categorical variable, the appropriate testing method for the difference of two population proportions is a two-sample z-test for a difference between two population proportions.

VAR-6.J.1 In order to make statistical inferences when testing a difference between population proportions, we must check for independence and that the sampling distribution is approximately normal.
DAT-3.C.1 An interpretation of the p-value of a significance test for a difference of two population proportions should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population proportions are equal to each other.
DAT-3.D.1 A formal decision explicitly compares the p-value to the significance. If the p-value ≤ α , then reject the null hypothesis, $H_o: p_1 = p_2$ *or* $H_o: p_1 - p_2 = 0$. If the p-value>α , then fail to reject the null hypothesis.
DAT-3.D.2 The results of a significance test for a difference of two population proportions can serve as the statistical reasoning to support the answer to a research question about the two populations that were sampled.

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.
HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*
HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.
HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.
8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.
8.2.12.ITH.2: Propose an innovation to meet future demands supported by an analysis of the potential costs, benefits, trade-offs and risks, related to the use of the innovation.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following…When can we use a normal distribution to perform inference calculations involving population proportions? How can we narrow the width of a confidence interval? If the proportion of subjects who experience serious side effects when taking a new drug is smaller than the proportion of subjects who experience serious side effects when taking a placebo, how can we determine if the difference is statistically significant?

Students will understand that:

1. Statistical inference guides the selection of appropriate models.
2. Inference is based upon chance.
3. Confidence intervals are effective tools for estimation.
4. Tests of significance and confidence intervals drive decision making in our world.
5. Error analysis is a critical component of significance testing.

Essential Questions:

1. How much evidence do you need before you are able to make a reasonable conjecture?
2. Is it reasonable to think that different people require different amounts of convincing?
3. How is statistical inference used to draw conclusions from data?
4. How is probability used to express the strength of our conclusions?
5. What is inference?
6. To what extent should decisions be made based on chance?

Students will know:

1. When inference procedures apply in a given situation.
2. How to interpret statistical calculations and findings to assign meaning or assess a claim.
3. When to make an appropriate claim or draw an appropriate conclusion.
4. Key and relevant information to answer a question or solve a problem.
5. Relative frequencies, proportions, or probabilities using simulation or calculations.

Students will be able to:

1. Identify an appropriate inference method for confidence intervals.
2. Construct a confidence interval, provided conditions for inference are met.
3. Justify a claim based on a confidence interval.
4. Identify null and alternative hypotheses.
5. Identify an appropriate inference method for significance tests.
6. Calculate a test statistic and find a p-value, provided conditions for inference are met.
7. Justify a claim using a decision based on significance tests.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 6 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 6 Notes
- Unit 6 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
- Stats Monkey
- American Statistical Association
- Rossman/Chance Applet

- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

1. Preferential Seating facing the point of instruction
2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing
5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student's ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes
5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
22. Utilize alternate reading assignments/materials at the student's reading level.
23. Allow for alternate seating for proximity to peer helper or teacher as necessary
24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary)
25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast.
26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples.
28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally)

Additional Resources:

- 20 strategies to Support EAL Children
- What English Language Learners Wish Teachers Knew - Education Week
- A Starting Point: Tips and resources for working with ESL newcomer

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

Unit 7: Inference for Quantitative Data: Means

Unit Duration: ~14-16 Class Periods (10-18% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.I.1 Random variation may result in errors in statistical inference.

VAR-7.A.1 When s is used instead of σ to calculate a test statistic, the corresponding distribution, known as the t-distribution, varies from the normal distribution in shape, in that more of the area is allocated to the tails of the density curve than in a normal distribution.

VAR-7.A.2 As the degrees of freedom increase, the area in the tails of a t-distribution decreases.

UNC-4.O.1 Because σ is typically not known for distributions of quantitative variables, the appropriate confidence interval procedure for estimating the population mean of one quantitative variable for one sample is a one-sample t-interval for a mean.

UNC-4.O.2 For one quantitative variable, X, that is normally distributed, the distribution of $t = -\frac{\bar{x}-\mu}{\frac{s_x}{\sqrt{n}}}$ is a t-distribution with n - 1 degrees of freedom.

UNC-4.O.3 Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for confidence intervals proceeds as for a population mean.

UNC-4.P.1 In order to calculate confidence intervals to estimate a population mean, we must check for independence and that the sampling distribution is approximately normal: a. To check for independence:

i. Data should be collected using a random sample or a randomized experiment.

ii. When sampling without replacement, check that $n \leq 10\%N$, where N is the size of the population.

b. To check that the sampling distribution of $\bar{x}$ is approximately normal (shape):

i. If the observed distribution is skewed, n should be greater than 30.

ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.

UNC-4.Q.1 The critical value t* with n - 1 degrees of freedom can be found using a table or computer-generated output.

UNC-4.Q.2 The standard error for a sample mean is given by $\frac{s}{\sqrt{n}}$, where s is the sample standard deviation.

UNC-4.Q.3 For a one-sample t-interval for a mean, the margin of error is the critical value (t*) times the standard error (SE), which equals $t^*\left(\frac{s}{\sqrt{n}}\right)$.

UNC-4.R.1 The point estimate for a population mean is the sample mean, $\bar{x}$.

UNC-4.R.2 For the population mean for one sample with unknown population standard deviation, the confidence interval is $\bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right)$.

UNC-4.S.1 A confidence interval for a population mean either contains the population mean or it does not, because each interval is based on data from a random sample, which varies from sample to sample.

UNC-4.S.2 We are C% confident that the confidence interval for a population mean captures the population mean.

UNC-4.S.3 An interpretation of a confidence interval for a population mean includes a reference to the sample taken and details about the population it represents.

UNC-4.T.1 A confidence interval for a population mean provides an interval of values that may provide sufficient evidence to support a particular claim in context.

UNC-4.U.1 When all other things remain the same, the width of a confidence interval for a population mean tends to decrease as the sample size increases.

UNC-4.U.2 For a single mean, the width of the interval is proportional to $\frac{1}{\sqrt{n}}$.

UNC-4.U.3 For a given sample, the width of the confidence interval for a population mean increases as the confidence level increases.

VAR-7.B.1 The appropriate test for a population mean with unknown standard deviation is a one-sample t-test for a population mean.

VAR-7.B.2 Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for significance testing proceeds as for a population mean.

VAR-7.C.1 The null hypothesis for a one-sample t-test for a population mean is $H_o: \mu = \mu_o$, where μ_o is the hypothesized value. Depending upon the situation, the alternative hypothesis is $\mu < \mu_o$, $\mu > \mu_o$, or $\mu \neq \mu_o$.

VAR-7.C.2 When finding the mean difference, μ_d, between values in a matched pair, it is important to define the order of subtraction.

VAR-7.D.1 In order to make statistical inferences when testing a population mean, we must check for independence and that the sampling distribution is approximately normal:

a. To check for independence:

i. Data should be collected using a random sample or a randomized experiment.

ii. When sampling without replacement, check that n≤10%N .

b. To check that the sampling distribution of $\bar{x}$ is approximately normal (shape):

i. If the observed distribution is skewed, n should be greater than 30.

ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.

VAR-7.E.1 For a single quantitative variable when random sampling with replacement from a population that can be modeled with a normal distribution with mean μand standard deviation σ , the sampling distribution of $t = -\frac{\bar{x}-\mu}{\frac{s_x}{\sqrt{n}}}$ is t-distribution with n – 1 degrees of freedom.

DAT-3.E.1 An interpretation of the p-value of a significance test for a population mean should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population mean is equal to the particular value stated in the null hypothesis.

DAT-3.F.1 A formal decision explicitly compares the p-value to the significance . If the p-value ≤ α , then reject the null hypothesis, $H_o: \mu = \mu_o$. If the p-value > α , then fail to reject the null hypothesis.

DAT-3.F.2 The results of a significance test for a population mean can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.

UNC-4.V.1 Consider a simple random sample from population 1 of size n_1, mean μ_1, and standard deviation σ_1 and a second simple random sample from population 2 of of size n_2, mean μ_2, and standard deviation σ_2. If the distributions of populations 1 and 2 are normal or if both n_1 and n_2 are greater than 30, then the sampling distribution of the difference of means, $\overline{x_1} - \overline{x_2}$is also normal. The mean for the sampling distribution of $\overline{x_1} - \overline{x_2}$ is $\mu_1 - \mu_2$. The standard deviation of $\overline{x_1} - \overline{x_2}$ is

$$\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}.$$

UNC-4.V.2 The appropriate confidence interval procedure for one quantitative variable for t two independent samples is a two-sample -interval for a difference between population means.

UNC-4.W.1 In order to calculate confidence intervals to estimate a difference of population means, we must check for independence and that the sampling distribution is approximately normal:

a. To check for independence:

i. Data should be collected using two independent, random samples or a randomized experiment.

ii. When sampling without replacement, check that $n_1 < 10\%N_1$ and $n_2 < 10\%N_2$.

b. To check that the sampling distribution of $\overline{x_1} - \overline{x_2}$ should be approximately normal (shape):

i. If the observed distributions are skewed, both n_1 and n_2 should be greater than 30.

UNC-4.X.1 For the difference of two sample means, the margin of error is the critical value (t*) times the standard error (SE) of the difference of two means.

UNC-4.X.2 The standard error for the difference in two sample means with sample standard deviations, $s_1 and\ s_2$is

$$\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}.$$

UNC-4.Y.1 The point estimate for the difference of two population means is the difference in sample means, $\overline{x_1} - \overline{x_2}$.

UNC-4.Y.2 For a difference of two population means where the population standard deviations are not known, the confidence interval is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

± *t are the critical values for the central C% of a t-distribution with appropriate degrees of freedom that can be found using technology.

UNC-4.Z.1 In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference of population means. UNC-4.Z.2 An interpretation for a confidence interval for the difference of two population means should include a reference to the samples taken and details about the populations they represent.

UNC-4.AA.1 A confidence interval for a difference of population means provides an interval of values that may provide sufficient evidence to support a particular claim in context.

UNC-4.AB.1 When all other things remain the same, the width of the confidence interval for the difference of two means tends to decrease as the sample sizes increase.

VAR-7.F.1 For a quantitative variable, the appropriate test for a difference of two population means is a two-sample t-test for a difference of two population means.

VAR-7.G.1 The null hypothesis for a two-sample t-test for a difference of two population means, $H_o: \mu_1 = \mu_2$. The alternative hypothesis is $\mu_1 > \mu_2$,$\mu_1 < \mu_2$, or $\mu_1 \neq \mu_2$.

VAR-7.H.1 In order to make statistical inferences when testing a difference between population means, we must check for independence and that the sampling distribution is approximately normal:

a. Individual observations should be independent:

i. Data should be collected using simple random samples or a randomized experiment.

ii. When sampling without replacement, check that $n_1 < 10\%N_1$ and $n_2 < 10\%N_2$.

b. The sampling distribution of $\overline{x_1} - \overline{x_2}$ should be approximately normal (shape):

i. If the observed distributions are skewed, both n_1 and n_2 should be greater than 30..

ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers. This should be checked for BOTH samples.

VAR-7.I.1 For a single quantitative variable, data collected using independent random samples or a randomized experiment from two populations, each of which can be modeled with a normal distribution, the sampling distribution of

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

is an approximate t-distribution with degrees of freedom that can be found using technology. The degrees of freedom fall between the smaller of n_1−1 and n_1 −1 and $n_1 + n_2 - 2$.

DAT-3.G.1 An interpretation of the p-value of a significance test for a two-sample difference of population means should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population means are equal to each other.

DAT-3.H.1 A formal decision explicitly compares the p-value to the significance . If the p-value ≤ α , then reject the null hypothesis. If the p-value > α , then fail to reject the null hypothesis.

DAT-3.H.2 The results of a significance test for a two-sample test for a difference between two population means can serve as the statistical reasoning to support the answer to a research question about the populations that were sampled

Verona High School — AP Statistics

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.
HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*
HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.
HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.
8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.
8.2.12.ITH.2: Propose an innovation to meet future demands supported by an analysis of the potential costs, benefits, trade-offs and risks, related to the use of the innovation.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.
CLKS4 Demonstrate creativity and innovation.
CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.
CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following…How do we know whether to use a t-test or a z-test for inference with means? How can we make sure that samples are independent? Why is it inappropriate to accept a hypothesis as true based on the results of statistical inference testing?

Students will understand that:

1. Confidence intervals are effective tools for estimating the mean or proportion of a population.
2. Significance tests determine the likelihood of a sample.
3. The analysis is only as good as the data.
4. Significance tests determine the likelihood of a sample.
5. Confidence intervals are effective tools for estimating the mean or proportion of a population.
6. Significance tests determine the likelihood of a sample.
7. The analysis is only as good as the data.
8. Inference is a tool for validating a claim about a population parameter.
9. Inference is a tool for estimating an unknown population parameter.

Essential Questions:

1. To what extent are significance tests reliable?
2. How can one prepare for errors from significance tests?
3. What makes an argument statistically convincing?
4. What is significant about significance?
5. What does it mean to be 95% confident ?
6. How do you determine if there is a statistical significance?
7. What does it mean to make an inference?
8. What is a confidence interval?
9. What makes an argument statistically convincing?

Students will know:

1. The inference methods for confidence intervals.
2. Which inference procedures apply in a given situation.
3. How to interpret statistical calculations and findings to assign meaning or assess a claim.
4. Null and alternative hypotheses.

Students will be able to:

1. Describe probability distributions.
2. Construct a confidence interval, provided conditions for inference are met.
3. Justify a claim based on a confidence interval.
4. Make an appropriate claim or draw an appropriate conclusion.
5. Calculate a test statistic and find a p-value, provided conditions for inference are met.
6. Justify a claim using a decision based on significance tests.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 7 Test

Other Evidence:

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 7 Notes
- Unit 7 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
- Stats Monkey
- American Statistical Association
- Rossman/Chance Applet
- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

1. Preferential Seating facing the point of instruction
2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)

5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student's ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
22. Utilize alternate reading assignments/materials at the student's reading level.
23. Allow for alternate seating for proximity to peer helper or teacher as necessary
24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary)
25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast.
26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples.
28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally)

Additional Resources:

- 20 strategies to Support EAL Children
- What English Language Learners Wish Teachers Knew - Education Week
- A Starting Point: Tips and resources for working with ESL newcomer

5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

Unit 8: Inference for Categorical Data: Chi-Square

Unit Duration: ~10-11 Class Periods (2-5% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.J.1 Variation between what we find and what we expect to find may be random or not.

VAR-8.A.1 Expected counts of categorical data are counts consistent with the null hypothesis. In general, an expected count is a sample size times a probability. VAR-8.A.2 The chi-square statistic measures the distance between observed and expected counts relative to expected counts. VAR-8.A.3 Chi-square distributions have positive values and are skewed right. Within a family of density curves, the skew becomes less pronounced with increasing degrees of freedom.

VAR-8.B.1 For a chi-square goodness-of-fit test, the null hypothesis specifies null proportions for each category, and the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis.

VAR-8.D.1 Expected counts for a chi-square goodness-of-fit test are (sample size) (null proportion).

VAR-8.E.1 In order to make statistical inferences for a chi-square test for goodness of fit we must check the following:

a. To check for independence:

i. Data should be collected using a random sample or randomized experiment.

ii. When sampling without replacement, check that $n \leq 10\%N$.

b. The chi-square test for goodness of fit becomes more accurate with more observations, so large counts should be used (shape).

i. A conservative check for large counts is that all expected counts should be greater than 5.

The test statistic for the chi-square test for goodness of fit is

$$\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count},$$

with

$$degrees\ of\ freedom = number\ of\ categories - 1.$$

VAR-8.F.2 The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or, when a probability model is assumed to be true, a theoretical distribution (chi-square).

VAR-8.G.1 The p-value for a chi-square test for goodness of fit for a number of degrees of freedom is found using the appropriate table or computer generated output.

DAT-3.I.1 An interpretation of the p-value for the chi-square test for goodness of fit is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.

DAT-3.J.1 A decision to either reject or fail to reject the null hypothesis is based on comparison of the p-value to the significance level, α.

DAT-3.J.2 The results of a chi-square test for goodness of fit can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.

VAR-8.H.1 The expected count in a particular cell of a two-way table of categorical data can be calculated using the formula:

$$expected\ count = \frac{(row\ total)(column\ total)}{table\ total}.$$

VAR-8.I.1 The appropriate hypotheses for a chi-square test for homogeneity are:

H_o:There is no difference in distributions of a categorical variable across populations or treatments.

H_a:There is a difference in distributions of a categorical variable across populations or treatments.

VAR-8.I.2 The appropriate hypotheses for a chi-square test for independence are:

H_o: There is no association between two categorical variables in a given population or the two categorical variables are independent.

H_a:Two categorical variables in a population are associated or dependent.

VAR-8.J.1 When comparing distributions to determine whether proportions in each category for categorical data collected from different populations are the same, the appropriate test is the chi-square test for homogeneity.

VAR-8.J.2 To determine whether row and column variables in a two-way table of categorical data might be associated in the population from which the data were sampled, the appropriate test is the chi-square test for independence.

VAR-8.K.1 In order to make statistical inferences for a chi-square test for two-way tables (homogeneity or independence), we must verify the following:

a. To check for independence:

i. For a test for independence: Data should be collected using a simple random sample.

ii. For a test for homogeneity: Data should be collected using a stratified random sample or randomized experiment.

iii. When sampling without replacement, check that $n \leq 10\%N$.

b. The chi-square tests for independence and homogeneity become more accurate with more observations, so large counts should be used (shape).

i. A conservative check for large counts is that all expected counts should be greater than 5.

VAR-8.L.1 The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic:

$$\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count},$$

with

$$degrees\ of\ freedom = number\ of\ categories - 1.$$

VAR-8.M.1 The p-value for a chi-square test for independence or homogeneity for a number of degrees of freedom is found using the appropriate table or technology.

VAR-8.M.2 For a test of independence or homogeneity for a two-way table, the p-value is the proportion of values in a chi-square distribution with appropriate degrees of freedom that are equal to or larger than the test statistic.

DAT-3.K.1 An interpretation of the p-value for the chi-square test for homogeneity or independence is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.

DAT-3.L.1 A decision to either reject or fail to reject the null hypothesis for a chi-square test for homogeneity or independence is based on comparison of the p-value to the significance level, α .

DAT-3.L.2 The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to a research question about the population that was sampled (independence) or the populations that were sampled (homogeneity).

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.

HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.

HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*

HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.

HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.

8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.

8.2.12.ITH.2: Propose an innovation to meet future demands supported by an analysis of the potential costs, benefits, trade-offs and risks, related to the use of the innovation.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.

CLKS4 Demonstrate creativity and innovation.

CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.

CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

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9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Verona High School — AP Statistics

Transfer Goal:

Students will be able to independently use their learning to investigate the following…How does increasing the degrees of freedom influence the shape of the chi-square distribution? Why is it inappropriate to use statistical inference to justify a claim that there is no association between variables?

Students will understand that:

1. Significance tests can determine the likelihood of a sample from a series of proportions.
2. Significance tests can determine whether two variables are independent.
3. Inference is a tool for validating a claim about a population parameter.

Essential Questions:

1. How can we verify that two variables are independent?
2. How does one distinguish among the various tests of significance?
3. What does it mean to make an inference?
4. How can decisions be based on chance?
5. What makes an argument statistically convincing?
6. How do we make a declaration of independence statistically?
7. Is independence desirable?

Students will know:

1. Null and alternative hypotheses.
2. The appropriate inference method for significance tests.
3. The inference procedures that apply in a given situation

Students will be able to:

1. Describe probability distributions.
2. Determine relative frequencies, proportions, or probabilities using simulation or calculations.
3. Calculate a test statistic and find a p-value, provided conditions for inference are met.
4. Interpret statistical calculations and findings to assign meaning or assess a claim.
5. Justify a claim using a decision based on significance tests

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

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- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 8 Test

Other Evidence:

Formal:
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Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

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Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
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5. Provide model sentence frames and sentence starters for both oral responses and written responses
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10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
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13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
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20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
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1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
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11. Provide Mentorships
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13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

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- Practical Recommendations and Interventions: Gifted Student

	24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary) 25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast. 26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds 27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples. 28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally) **Additional Resources:** ● 20 strategies to Support EAL Children ● What English Language Learners Wish Teachers Knew - Education Week ● A Starting Point: Tips and resources for working with ESL newcomer	

Unit 9: Inference for Quantitative Data: Slopes

Unit Duration: ~7-8 Class Periods (2-5% AP Exam Weighting)

Stage 1: Desired Results

Established Subject Area Goals (College Board):

VAR-1.K.1 Variation in points' positions relative to a theoretical line may be random or non-random.

UNC-4.AC.1 Consider a response variable,y, that is normally distributed with standard deviation,σ. The standard deviation σ can be estimated using the standard deviation of the residuals $s = \sqrt{\frac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}}$.

UNC-4.AC.2 For a simple random sample of n observations, let b represent the slope of a sample regression line. Then the mean of the sampling distribution of b equals the population mean slope: $\mu_b = \beta$. The standard deviation of the sampling distribution for β is $\sigma_b = \frac{\sigma}{\sigma_x\sqrt{n}}$ where $\sigma_x = \sqrt{\frac{\Sigma\left(x_i - \bar{x}\right)^2}{n}}$.

UNC-4.AC.3 The appropriate confidence interval for the slope of a regression model is a t-interval for the slope.

UNC-4.AD.1 In order to calculate a confidence interval to estimate the slope of a regression line, we must check the following:

a. The true relationship between x and y is linear. Analysis of residuals may be used to verify linearity.

b. The standard deviation for y, σ_y does not vary with x. Analysis of residuals may be used to check for approximately equal standard deviations for all x.

c. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that n ≤10%N.

d. For a particular value of x, the responses (y-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality. i. If the observed distribution is skewed, n should be greater than 30.

UNC-4.AE.1 For the slope of a regression line, the margin of error is the critical value (t*) times the standard error (SE) of the slope.

UNC-4.AE.2 The standard error for the slope of a regression line with sample standard deviation, s, is $SE = \frac{s}{s_x\sqrt{n-1}}$, where s is the estimate of σand s_x is the sample standard deviation of the x values.

UNC-4.AF.1 The point estimate for the slope of a regression model is the slope of the line of best fit, b.

UNC-4.AF.2 For the slope of a regression model, the interval estimate is $b \pm t^*\left(SE_b\right)$.

UNC-4.AG.1 In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the slope of the regression model, i.e., the true slope of the population regression model.UNC-4.AG.2 An interpretation for a confidence interval for the slope of a regression line should include a reference to the sample taken and details about the population it represents.

UNC-4.AH.1 A confidence interval for the slope of a regression model provides an interval of values that may provide sufficient evidence to support a particular claim in context.

UNC-4.AI.1 When all other things remain the same, the width of the confidence interval for the slope of a regression model tends to decrease as the sample size increases.

VAR-7.J.1 The appropriate test for the slope of a regression model is a t-test for a slope.

VAR-7.K.1 The null hypothesis for a t-test for a slope is $H_o: \beta = \beta_o$,where β is the hypothesized value from the null hypothesis. The alternative hypothesis is: $H_a: \beta < \beta_o$, $H_a: \beta > \beta_o$, $or\ H_a: \beta \neq \beta_o$.

VAR-7.L.1 In order to make statistical inferences when testing for the slope of a regression model, we must check the following:

a. The true relationship between x and y is linear. Analysis of residuals may be used to verify linearity.

b. The standard deviation for y, σ_y, does not vary with x. Analysis of residuals may be used to check for approximately equal standard deviations for all x.

c. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that n< 10%N.

d. For a particular value of x, the responses (y-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality.

i. If the observed distribution is skewed, n should be greater than 30. ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.

VAR-7.M.1 The distribution of the slope of a regression model assuming all conditions are satisfied and the null hypothesis is true (null distribution) is a t-distribution.

VAR-7.M.2 For simple linear regression when random sampling from a population for the response that can be modeled with a normal distribution for each value of the explanatory variable, the sampling distribution of $t = \frac{b-\beta}{SE_b}$ has a t -distribution with degrees of freedom equal to $n - 2$. When testing the slope in a simple linear regression model with one parameter, the slope, the test for the slope has $df = n - 1$.

DAT-3.M.1 An interpretation of the p-value of a significance test for the slope of a regression model should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population slope is equal to the particular value stated in the null hypothesis.

DAT-3.N.1 A formal decision explicitly compares the p-value to the significance α . If the p-value ≤ α , then reject the null hypothesis, $H_o: \beta = \beta_o$. If the p-value > α , then fail to reject the null hypothesis.

DAT-3.N.2 The results of a significance test for the slope of a regression model can serve as the statistical reasoning to support the answer to a research question about that sample.

Interdisciplinary Standards (NJSLS):

HS-LS2-6. Evaluate claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.

HS-PS4-4. Evaluate the validity and reliability of claims in published materials of the effects that different frequencies of electromagnetic radiation have when absorbed by matter.

HS-LS2-7. Design, evaluate, and refine a solution for reducing the impacts of human activities on the environment and biodiversity.*

HS-LS2-8. Evaluate evidence for the role of group behavior on individual and species' chances to survive and reproduce.

HS-LS4-6. Create or revise a simulation to test a solution to mitigate adverse impacts of human activity on biodiversity.*

Technology Integration (NJSLS 8):

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.

8.1.12.IC.3: Predict the potential impacts and implications of emerging technologies on larger social, economic, and political structures, using evidence from credible sources.

8.2.12.ITH.2: Propose an innovation to meet future demands supported by an analysis of the potential costs, benefits, trade-offs and risks, related to the use of the innovation.

21st Century Skills Integration (NJSLS 9):

CLKS1 Act as a responsible and contributing community member and employee.

CLKS4 Demonstrate creativity and innovation.

CLKS5 Utilize critical thinking to make sense of problems and persevere in solving them.

CLKS8 Use technology to enhance productivity, increase collaboration and communicate effectively.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.IML.2: Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

9.4.12.IML.8: Evaluate media sources for point of view, bias, and motivations

Transfer Goal:

Students will be able to independently use their learning to investigate the following…How can there be variability in slope if the slope statistic is uniquely determined for a line of best fit? When is it appropriate to perform inference about the slope of a population regression line based on sample data? Why do we not conclude that there is no correlation between two variables based on the results of a statistical inference for slopes?

Students will understand that:	Essential Questions:
1. Significance tests can determine the likelihood of a sample from a series of proportions. 2. Significance tests can determine whether two variables are independent. 3. Significance tests can determine the likelihood of a bivariate sample's slope. 4. Regression is an instrument used to generalize relationships for bivariate data. 5. Inference is a tool for validating a claim about a population parameter.	1. How can we test the slope of a correlation? 2. How do we use a model to make statistical inference? 3. How can decisions be made based on chance? 4. What makes an argument statistically convincing? 5.
Students will know:	**Students will be able to:**
1. The appropriate inference method for confidence intervals.	1. Construct a confidence interval, provided conditions for inference are met.

2. Which inference procedures that apply in a given situation. 3. Null and alternative hypotheses. 4.	2. Interpret statistical calculations and findings to assign meaning or assess a claim. 3. Justify a claim based on a confidence interval. 4. Make an appropriate claim or draw an appropriate conclusion. 5. Calculate a test statistic and find a p-value, provided conditions for inference are met. 6. Justify a claim using a decision based on significance tests.

Stage 2: Acceptable Evidence

Transfer Task and Unit Assessments:

- Free Response Questions via collegeboard
- Progress Checks via AP Classroom
- Quizzes/Unit Test via Khan Academy
- StatsMedic Assessments
- Unit 9 Test

Other Evidence

Formal:
Providing written/oral response to the EQs
Passing all quizzes on basic concepts in the unit.
Successful completion of Unit Test

Informal:
AP Classroom Progress Checks
AP Classroom Question Bank
Performance on Khan Academy Assignments

Stage 3: Activities to Foster Learning

- StatsMedic Activities
- Khan Academy Videos
- Unit 9 Notes
- Unit 9 Practice Packet
- Review Student Responses to Free Response Questions
- Performing Simulations via StatKey

Reference Materials

- AP Statistics Khan Academy
- Stats Medic
- AP Classroom (College Board)
- The Practice of Statistics(Teacher Resources)
- The Practice of Statistics (Student Resources)
- AP Statistics Past Exam Questions
- Stats Monkey
- Nobel Stat Man
- AP Statistics Course and Exam Description
- Stats Monkey
- American Statistical Association
- Rossman/Chance Applet
- StatKey

Accommodations and Modifications

Differentiation for Students with IEPs, 504s, and/or Students at Risk of Failure (IEP/504/RF)

1. Preferential Seating facing the point of instruction
2. Allow for movement breaks, as necessary
3. Provide visual cues for orally presented information when possible
4. Provide study guides or a test outline prior to testing
5. Break long term assignments into more manageable segments
6. Extended time on tests, quizzes, and projects, as necessary
7. Partial credit for missing/incomplete work
8. Modify pace of instruction to allow for additional processing time
9. Allow for repetition and/or clarification of directions, as needed
10. Modify the number of choices on tests/quizzes
11. Adjust test format to student's ability level
12. Provide study guides, if available
13. Provide class notes, if available
14. Provide assessments with fewer questions on page
15. Preferential seating to allow better visibility to copy notes
16. Breakdown tasks into manageable units
17. Minimize auditory distractions
18. Provide a word bank, if possible, for short answer or fill-in tasks on assignments and tests
19. Periodically review notes for accuracy and completeness
20. Teachers will reword, clarify, or repeat instructions or items on assignments and/or tests as needed.
21. Student will be redirected when necessary
22. Provide frequent feedback and check often for understanding
23. Monitor on-task performance
24. Provide the opportunity to revise for partial credit on wrong answers
25. Modify homework, tests, or quizzes

Additional Resources:

- Khan Academy
- Pearsonsuccessnet.com
- www.deltamath.com
- www.albert.io
- www.desmos.com

Differentiation for English Language Learners

1. Provide alternate ways for the student to respond (verbal/pictographic answers instead of written)
2. Substitute a hands-on activity or use of different media in projects for a written activity
3. Provide word banks / word walls
4. Prepare and distribute advance notes
5. Provide model sentence frames and sentence starters for both oral responses and written responses
6. Provide additional time to complete assessments and assignments
7. Model and use gestures to aid in understanding
8. Model tasks by giving one or two examples before releasing students to work independently
9. Present instructions both verbally and visually
10. Simplify written and verbal instructions
11. Allow students to use eDictionaries
12. Avoid slang and idiomatic expressions.
13. Speak clearly and naturally, and try to enunciate words, especially their ending sounds.
14. Provide Sensory Supports (Real-life objects, Manipulatives, Pictures & photographs, Illustrations, Diagrams, & drawings, Magazines & newspapers, Physical activities, Videos & films, Broadcasts, Models & figures)
15. Provide Graphic Supports (Charts, Graphic organizers, Tables, Graphs, Timelines, Number lines)
16. Provide Interactive Supports(Pair or Partner work, Group work, Peer Mentor)
17. Simplify the language, format, and directions of the assessment
18. Accept correct answers on test or worksheets in any written form such as lists, phrases, or using inventive spelling
19. Allow editing and revision before grading
20. Design projects and assessment for student that require reduced sentence or paragraph composition
21. Give alternative homework or class work assignments suitable to the student's linguistic ability for activities and assessments
22. Utilize alternate reading assignments/materials at the student's reading level.
23. Allow for alternate seating for proximity to peer helper or teacher as necessary
24. Assist student in building a picture file of key vocabulary (Pics4Learning, Webster's Visual Dictionary Online, ClipArt Etc, Shahi Visual Dictionary)
25. When showing video used Closed Captioning. Some videos also allow for a slower replay so the speech is not as fast.
26. Provide wait-time sufficient for English language learners who are trying to translate terms while formulating an explanation - Sufficient wait time is often said to be about 7-10 seconds
27. Check for understanding consistently - ask students one-on-one what their questions are, monitor their progress on independent work and redirect as needed. They may not understand or be hesitant to verbalize what they do not understand at first, so monitor and give examples.
28. Support use of student's primary language by translating key words in directions, or key vocabulary terms or giving students opportunities to communicate in their primary language (written or orally)

Additional Resources:

- 20 strategies to Support EAL Children

Differentiation for Enrichment (G&T):

1. Provide more challenging reading materials
2. Focus on the overall trends, patterns and themes rather than small details and facts
3. Study problems that do not have a clear solution
4. Allow a variety of acceptable products (using Multiple Intelligences, for example)
5. Offer leveled projects (For an A..., For a B..., etc.)
6. Involve the student in creating the scoring guide
7. Assign tasks that are authentic and for a real audience
8. Match the product to the outcomes being met
9. Use topics of interest to the student, relevant to how the world works, complex and worthwhile
10. Provide outside research projects
11. Provide Mentorships
12. Provide cumulative assessments
13. Allow students to create their own problems
14. Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning.
15. Invite students to explore different points of view on a topic of study and compare the two.
16. Encourage students to explore concepts in depth and encourage independent studies or investigations.
17. Reduced amounts of drill and practice

Additional Resources:

- Serving Gifted Students in General Ed Classrooms
- Practical Recommendations and Interventions: Gifted Student

	• What English Language Learners Wish Teachers Knew - Education Week • A Starting Point: Tips and resources for working with ESL newcomer	