

Time Allowed: 3 Hours
80

Maximum Marks:

General Instructions: -

1. This Question Paper contains five sections - A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 2 Assertion –Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 source based /case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section – A

(Multiple Choice Questions) Each carries 1 mark

1. Which of the following is a null set?
 (a) $\{x: x \in N, 2x - 1 = 3\}$
 (b) $\{x: x \in N, x^2 < 20\}$
 (c) $\{x: x \text{ is an even prime greater than } 2\}$
 (d) $\{x: x \in Z, 3x + 7 = 1\}$
2. Two finite sets A and B are such that $A \subset B$, then which of the following is not correct?
 (a) $A \cup B = B$ (b) $A \cap B = A$ (c) $A - B = \emptyset$ (d) $B - A = \emptyset$
3. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$, $n(A \cup B) = 110$, then $n(A \cap B)$ is
 (a) 20 (b) 50 (c) 40 (d) 240
4. Let $n(A) = p$ and $n(B) = q$, then the number of non-empty relations from A to B is
 (a) p^q (b) $q^p - 1$ (c) $2^{pq} - 1$ (d) 2^{pq}
5. The domain of the function defined by $f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}$ is equal to
 (a) $(-\infty, -1) \cup (1, 4]$ (b) $(-\infty, -1] \cup (1, 4]$ (c) $(-\infty, -1) \cup [1, 4]$ (d) $(-\infty, -1) \cup [1, 4)$
6. If $[x]^2 - 5[x] + 6 = 0$, where $[.]$ denotes the greatest integer function, then
 (a) $x \in [3, 4]$ (b) $x \in (2, 3]$ (c) $x \in [2, 3]$ (d) $x \in [2, 4)$
7. The value of $8\cos^3 \frac{\pi}{9} - 6\cos \cos \frac{\pi}{9}$ is
 (a) 2 (b) -2 (c) 1 (d) -1
8. Which of the following is not equal to $\cos \cos 2x$
 (a) $\cos^2 x - \sin^2 x$
 (b) $1 - 2\sin^2 x$

(c) $1 - 2\cos^2 x$

(d) $\frac{1-\tan^2 x}{1+\tan^2 x}$

9. The least value of $\sin^2 x + \operatorname{cosec}^2 x$ is

- (a) 0 (b) 2 (c) 3 (d) 1

10. The value of $(1 + i)^4 + (1 - i)^4$ is equal to

- (a) 8 (b) - 8 (c) 4 (d) - 4

11. If $\frac{4}{x-5} < 0$, then x belongs to

- (a) $(-\infty, 5)$ (b) $(-\infty, 9)$ (c) $(-\infty, 5]$ (d) $(-\infty, 9]$

12. The number of ways in which 5 boys and 3 girls can be seated in a row, so that no two girls sit together is

- (a) $8!$ (b) $5! \times 3!$ (c) $3! \times {}^5P_4$ (d) $5! \times {}^6P_3$

13. The number of terms in the expansion of $(x + a)^{53} + (x - a)^{53}$ after simplification is

- (a) 106 (b) 27 (c) 26 (d) 53

14. Distance of a point (3,4,5) from the origin is

- (a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $\sqrt{2}$ (d) $5\sqrt{2}$

15. The number of integers between 100 and 1000 that are not divisible by 7 is

- (a) 128 (b) 771 (c) 899 (d) 772

16. The point $(-2, -3, -4)$ lies in the

- (a) First Octant (b) Seventh Octant (c) Second Octant (d) Eighth Octant

17. If $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$, then $P(A \cup B)$ is equal to

- (a) $\frac{1}{10}$ (b) $\frac{2}{10}$ (c) $\frac{5}{10}$ (d) $\frac{4}{10}$

18. If A & B are mutually exclusive events then

- (a) $P(A) \leq P(B^c)$ (b) $P(A) \geq P(B^c)$ (c) $P(A) < P(B^c)$ (d) $P(A) > P(B^c)$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true and R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. Assertion (A) : If ${}^{2023}C_{2x-2} = {}^{2023}C_x$, then sum of all positive values of x is 677

Reason (R) : If ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$

20. Assertion (A) : The mean deviation of the data 3,4,5,7,8,9 is 2.

Reason (R) : Mean Deviation about mean = $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$

Section – B

(This section comprises of very short answer type questions of 2 marks each.)

21. If $A = \{x: x \in R, x \text{ is the root of the equation } x^3 - x = 0\}$,
 $B = \{x: x \in R, x \text{ is the root of } x^3 + 2x^2 - x - 2 = 0\}$

Then find the values of (i) $A \cup B$ (ii) $A \cap B$

22. Prove that

$$\tan \tan 2023x - \tan \tan 2022x - \tan \tan x = \tan \tan 2023x \cdot \tan \tan 2022x \cdot \tan \tan x$$

OR

If $\tan \tan (A + B) = p$ and $\tan \tan (A - B) = q$ then prove that

$$\tan \tan 2A = \frac{p+q}{1-pq}$$

23. If $(x + iy)^3 = u + iv$ then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

24. Find the sum of n terms of the following series $7+77+777+7777+\dots$

25. Mean and Standard deviation of 100 observations are 50 and 4 respectively. Find the sum of all the observations and the sum of the squares of the observations.

OR

Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, x_3, \dots, x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, \dots, ax_n$ ($a \neq 0$) are $a\bar{x}$ and $a^2 \sigma^2$.

Section – C

(This section comprises of short answer type questions of 3 marks each)

26. IQ of a person is given the formula $Q = \frac{MA}{CA} \times 100$; where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 year children, find the range of mental age.

27. Find the coefficient of a^4 in the product $(1 + 2a)^4 (2 - a)^5$ using binomial theorem

28. If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \operatorname{cosec} \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$.

29. Find the equation of the circle which passes through the point (1,1) and Centre lies at the point of intersection of lines $x + y = 4$ and $x - y = 0$.

OR

If the eccentricity of the ellipse is $\frac{5}{8}$ and distance between its foci is 10. Find the equation of ellipse.

30. If $y = (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1) \sin \sin x + 1$ then find $\frac{dy}{dx}$.

OR

Find the value of $\frac{\tan \tan x - \sin \sin x}{x^3}$

31. Find the Mean deviation about the median for the following data :

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

Section – D

(This section comprises of long answer type questions of 5 marks each)

32. Find the domain and range of $y = \frac{1}{2 - \sin 3x}$.

33. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$ then find the values of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, $\tan \frac{x}{2}$.

OR

Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

34. If a and b are the roots of $x^2 - 3x + p = 0$ and c and d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP. Then prove that $\frac{q+p}{q-p} = \frac{17}{15}$.

35. Find the derivative of $f(x) = \sin \sin (x^2 + 1)$ using first principle.

OR

(i) Find the derivative of $\frac{\sin^2 x}{1 + \cos x}$ w.r.t. x 2M

(ii) Find the value of $\frac{\sin x - \cos x}{x - \frac{\pi}{4}}$ 3M

Section – E

(This section comprises of 3 case-study/passage-based questions with sub parts. First two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36. During the Mathematics class, A teacher clears the concept of permutations and combinations to the 11th class students. After the class was over he asks the students some more questions.



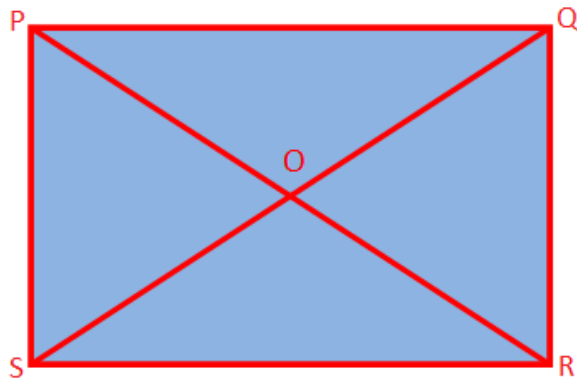
On the basis of the information given above answer the following:-

- (a) Find the number of arrangements of the letters of the word INDEPENDENCE.
- (b) In How many of these do the words begin with I and end in P.
- (c) In How many of these do all the vowels never occur together.

OR

In How many of these do all the four E's do not occur together

37. For an EMC project students need rectangular sheets, therefore they made Eco friendly rectangular sheets PQRS from the paper waste such that on the Cartesian plane equation of QR is $3x + 4y = 12$ and point P is (2,4) while point R is $(\frac{16}{5}, a)$.



On the basis of the information given above answer the following:-

- (a) Find the equation of PS.
- (b) Find the value of a.
- (c) Find the area of rectangular sheet.

OR

Find the perimeter of rectangular sheet.

38. On her winter vacations, Ayesha visits four cities (Delhi, Mumbai, Goa and Bangalore) in random order.



On the basis of the information given above answer the following

- (a) What is the probability that she visits Delhi before Goa and Goa before Mumbai?

OR

What is the probability that she visits Delhi First and Mumbai last?

- (b) What is the probability that she visits Delhi just before Mumbai?