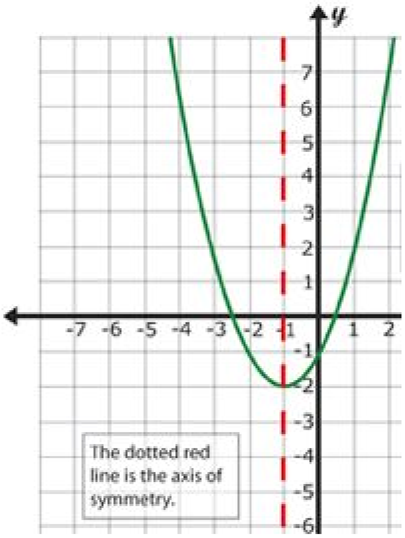
Relations and Functions Review Package

First Multiple Choice Question

Study Notes:



Vertical Line of Symmetry (Axis of Symmetry) -- the vertical line that cuts any graph in half, thus have symmetry on either side of it.

With vertex (h, k), the equation of the axis of symmetry of the parabola is $x = h$.

In the diagram to the right, the equation of the axis of symmetry is

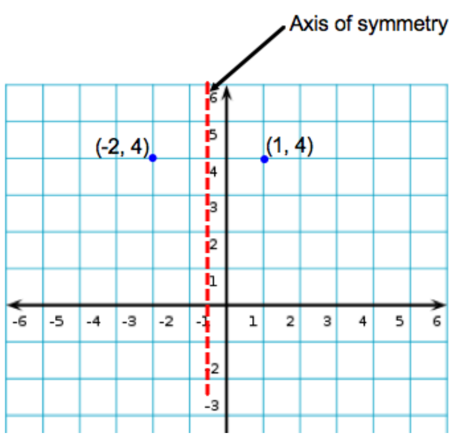
$x = -1$

Sample Question:

The points (–2, 4) and (1, 4) are located on the same parabola. What is the equation for the axis of symmetry for this parabola?

A. $x = -1.5$
B. $x = 0.5$
C. $x = -0.5$
D. $x = -1$

Solution to Sample Question:



We know that axis of symmetry always cuts the graph in half. Drawing a graph we see that the axis of symmetry passes through -0.5.

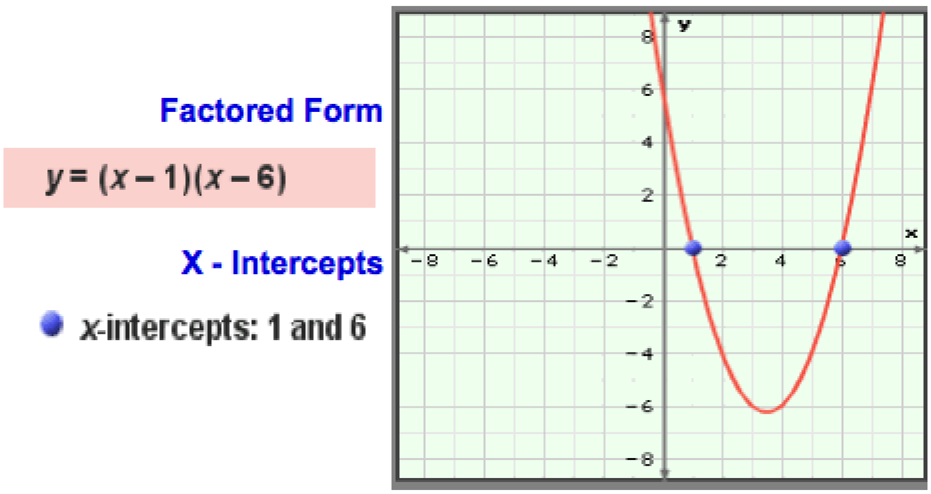
Therefore, the axis of symmetry has the equation $x = -0.5$

The solution is **C**.

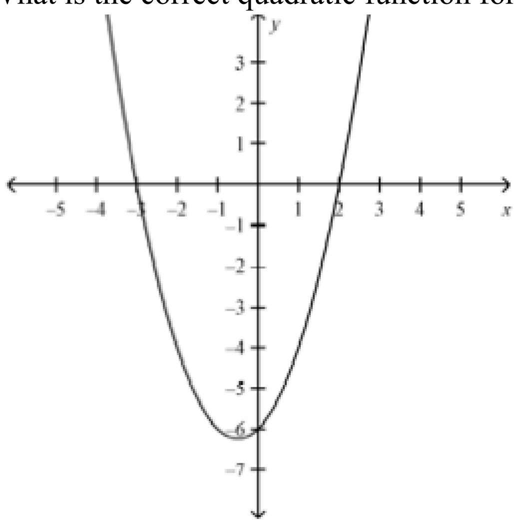
Practice Question: Page 386 (#3)

Second Multiple Choice Question

Study Notes:
When a quadratic function is written in factored form, each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.



Sample Question:
What is the correct quadratic function for this parabola?



A. $f(x)$ □ $(x$ □ $2)(x$ □ $3)$
B. $f(x)$ □ $(x$ □ $2)(x$ □ $3)$
C. $f(x)$ □ $(x$ □ $2)(x$ □ $3)$
D. $f(x)$ □ $(x$ □ $2)(x$ □ $3)$

Solution to Sample Question:
The zeros of the graph are -3 and + 2. We can find the factors from the zeros.

$x = -3$ gives us a factor of $(x + 3)$
$x = 2$ gives us a factor of $(x - 2)$

Therefore the quadratic function that describes the graph above is $y = (x + 3)(x - 2)$. So the answer is **B**.

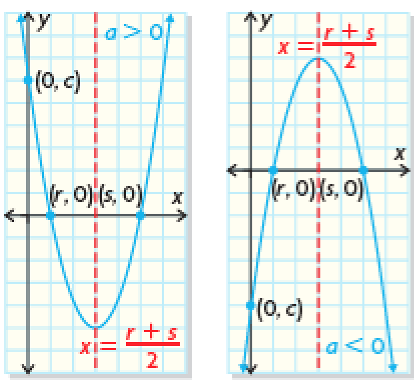
Practice Question: Page 387 (#12)

Third Multiple Choice Question

Study Notes:

A quadratic function that is written in the form $y = a(x - r)(x - s)$ has the following characteristics:

- The x-intercepts of the graph of the function are $x = r$ and $x = s$.
- The linear equation of the axis of symmetry is $x = \frac{r + s}{2}$
- The y-intercept, c, is $c = ars$.



• If a quadratic function has only one x-intercept, the factored form can be written as follows:

$$f(x) = a(x - r)(x - r)$$
$$f(x) = a(x - r)^2$$

Sample Question:

Which set of data is correct for the quadratic relation $f(x) = \square 2(x \square 1)(x \square 5)$?

	x-intercepts	*y*-intercept	Axis of Symmetry	Vertex
A.	(1, 0), (5, 0)	$y = -10$	$x = 3$	(3, 8)
B.	(–1, 0), (–5, 0)	$y = 10$	$x = -3$	(–3, 64)
C.	(–1, 0), (–5, 0)	$y = -10$	$x = 3$	(3, –8)
D.	(1, 0), (5, 0)	$y = 10$	$x = -3$	(3, 64)

A. Set A.
B. Set B.
C. Set D.
D. Set C.

Solution to Sample Question:
The *x* - intercepts (or zeros) come from the factors.

$x - 1 = 0$ has an x-intercept of (1, 0) and $x - 5 = 0$ has an *x* -intercept of (5, 0)

This information alone eliminates distractor B and C.

The axis of symmetry has the equation $x = \frac{r + s}{2}$

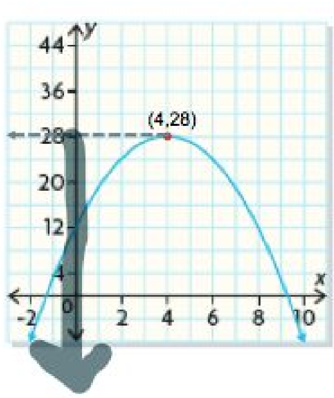
$x = \frac{1 + 5}{2}$, so $x = 3$

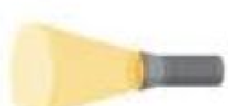
This now eliminates distractor D so the answer is **A**.

Practice Questions: Page 386 (#7)

Fourth Multiple Choice Question

Study Notes:
Finding Range





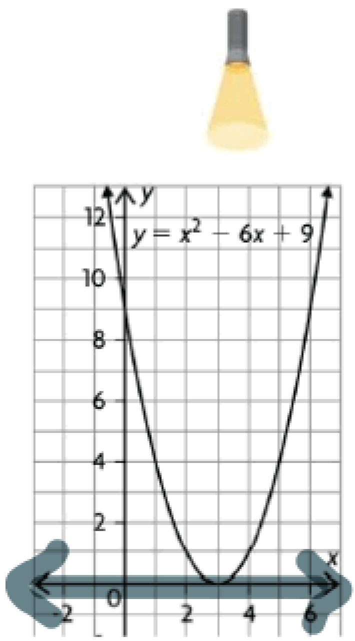
To find the range of a function shine a flashlight from the side and see what is in the shadow. Lets look at an example.

$\{y \;/\; y \le 28, y \in R\}$

We can see from this picture that 28 is maximum *y* - value and all other values of *y* are smaller. Therefore, we can write the range like

This is a shorter version that says: " *y* values such that *y* is less than or equal 28 and *y* is a member of Real numbers.

Finding Domain
To find the domain of a function shine a flashlight from the top or bottom and see what is in the shadow. Lets look at an example.



We can see from this picture that there are no restrictions on the x values. It appears that the graph continues on left and right. Therefore, the domain is

$\{x / x \in R\}$

This is a shorter version that says: " *x* is a member of Real numbers". This means there are no restrictions.

For all quadratic functions, the domain is the set of real numbers, and the range is a subset of real numbers.

Sample Question:
Which quadratic function **does not** have the domain $x \in R$ and the range y ≥ –6?

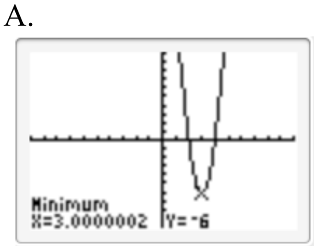
a. $y = 6(x-4)(x-2)$

b. $y = 0.5(x-3)^2 - 6$

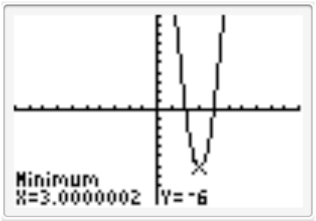
c. $y = 0.5x^2 - 2x - 4$

d. $y = x^2 - 2x - 6$

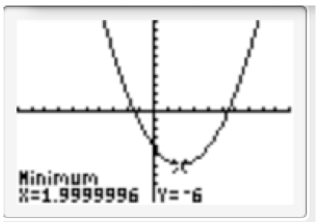
Solution to Sample Question:
Graph each equation:
A.



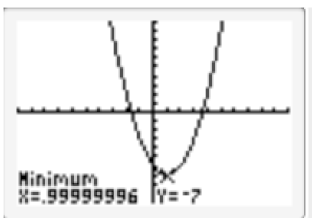
B.



c.



d.



From the graphs above you can see that the graph that DOES NOT have a range of $y \geq -6$, is **D**

Practice Questions: Page 386 (#4)

Fifth Multiple Choice Question

Study Notes:
The form of the quadratic function that you use to model a given situation depends on what you know about the relationship:

- Use the vertex form when you know the vertex and an additional point on the parabola.
- Use the factored form when you know the x-intercepts and an additional point on the parabola.

Quadratic Function	Algebraic Strategy to Determine the Vertex
Standard form: $f(x) = ax^2 + bx + c$	Use partial factoring to determine two points on the parabola with the same y-coordinate, then the axis of symmetry, and then the y-coordinate of the vertex: $f(x) = x(ax + b) + c$
Factored form: $f(x) = a(x - r)(x - s)$	Set each factor equal to zero to determine the zeros. Use the zeros to determine the equation of the axis of symmetry, then determine the y-coordinate of the vertex.
Vertex form: $f(x) = a(x - h)^2 + k$	The vertex is (h, k).

All maximum/minimum problems can be solved using graphing technology, if you know the quadratic function that models the situation.

Sample Question:
A punter kicks a football 48 m to another player who catches it. The path of the football is defined by the function $h(x) = -\frac{1}{30}(x - 24)^2 + 19.2$, where x is the horizontal distance, measured in meters, from the kicker.

a) Determine the axis of symmetry of the parabola.

b) What was the highest point of the football's path?

c) How high was the football from the ground when it was 6 m horizontally from the receiving player?

d) What is the range for this function? Justify your answer.

Solutions to Sample Question:
a. When the quadratic is in vertex form we simply use the *h* to find the equation of the axis of symmetry. Therefore, $x = 24$ is the equation of the axis of symmetry.

b. The highest point (maximum value) comes from the *k* of the vertex (h, k)

c. When we want to find out how high the football is after travelling 6 m horizontally, we substitute 6 in for x.

$$h(x) = -\frac{1}{30}(x-24)^2 + 19.2$$
$$h(x) = -\frac{1}{30}(6-24)^2 + 19.2$$
$$h(x) = -\frac{1}{30}(-18)^2 + 19.2$$
$$h(x) = -\frac{1}{30}(324) + 19.2$$
$$h(x) = -10.8 + 19.2$$
$$h(x) = 8.4$$

The football will be 8.4 *m* in the air.

Practice Question: Page 387 (#11)

Sixth Multiple Choice Question

Study Notes:

Purplemath Click the icon to link to notes on Solving Quadratic Equations using Graphing

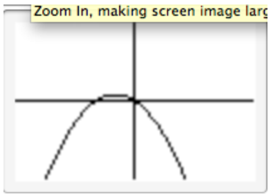
Sample Question:

1. Solve □5x^2 □ x □ 0 by graphing the corresponding function and determining the zeros.

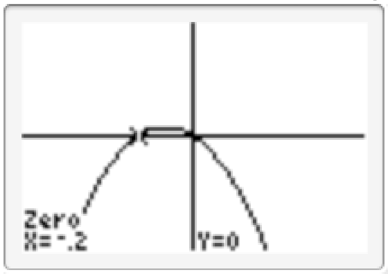
   a. x □ 0, x □ □0.2
   b. x □ 0, x □ 5
   c. x □ 0, x □ □5
   d. x □ 0, x □ 0.2

Solution to Sample Question

Graph the quadratic equation.
From the graph, we can see that one x-intercept is 0.



Let’s calculate the other using the ZERO function.



Remember that the x-intercepts of the graph are also the solutions to the quadratic. Therefore, the solutions are (0, 0) and (-0.2, 0) so the answer is **A**.

Practice Questions: Page 436 (#1 and 2)

Seventh Multiple Choice Question

Study Notes:

Math TV - Algebra - Quadratic Equations - Factoring
Math TV - Algebra - Quadratic Equations - Quadratic in Form
Brightstorm - Solving Quadratic Equations by Factoring

Sample Question:

Solve $2x^2 \square 7x \square 6$ by factoring.

A. $x \square 2,\ x \square \frac{3}{2}$

B. $x \square \square 2,\ x \square \square \frac{3}{2}$

C. $x \square 6,\ x \square \square 1$

D. $x \square 2,\ x \square 3$

Solution to Sample Question:
Manipulate the quadratic to standard form

$$2x^2 \square 7x \square 6 \square 0$$

Factor

$$(2x \square 3)(x \square 2) \square 0$$

Use the zero property to solve

$$2x \square 3 \square 0, \text{ so } x \square \frac{3}{2}$$

or

$$x \square 2 \square 0, \text{ so } x \square 2$$

The answer is **A**

Practice Question: Page 436 (#3)

Eighth Multiple Choice Question

Study Notes:

Solving Quadratic Equations using the Quadratic Formula

To solve a quadratic equation by using square roots:

Step 1: Write the equation in the form $ax^2 + bx + c = 0$
Step 2: Identify the values for a, b and c and substitute the values into the quadratic equation.
Step 3: Solve for x.
Step 4: Check your answers by substituting the all solutions into the original equation.

Sample Question:

Solve $x^2 \square 2x \square 4$ using the quadratic formula.

a. $x = 1 + \sqrt{20}, x = 1 - \sqrt{20}$
b. $x = -1 + \sqrt{20}, x = -1 - \sqrt{20}$
c. $x = -1 + \sqrt{5}, x = -1 - \sqrt{5}$
d. $x = 1 + \sqrt{5}, x = 1 - \sqrt{5}$

Solution to Sample Question:

Write the quadratic in standard form

$$x^2 - 2x = 4$$

$$x^2 - 2x - 4 = 0$$

Use the quadratic formula. Substitute $a = 1$, $b = -2$ and $c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{4 + 16}}{2}$$
$$x = \frac{2 \pm \sqrt{20}}{2}$$
$$x = \frac{2 \pm \sqrt{20}}{2}$$
$$x = \frac{2 \pm 2\sqrt{5}}{2}$$
$$x = 1 \pm \sqrt{5}$$

Practice Questions: Page 436 (#4)

First Numerical Response Question

Sample Question:
A quadratic function has an equation that can be written in the form $f(x) = a(x - r)(x - s)$. The graph of the function has x-intercepts at (–4, 0) and (2, 0) and passes through the point (–2, –8). Write the equation of the function.

Solution to Sample Question:
Substitute the x-intercepts for r and s.

$$y = a(x - (-4))(x - 2)$$
$$y = a(x + 4)(x - 2)$$

In order to find a, you must substitute another point in (-2, -8)

$$y = a(x + 4)(x - 2)$$
$$y = a(-2 + 4)(-2 - 2)$$
$$-8 = a(2)(-4)$$
$$-8 = -8a$$
$$a = 1$$

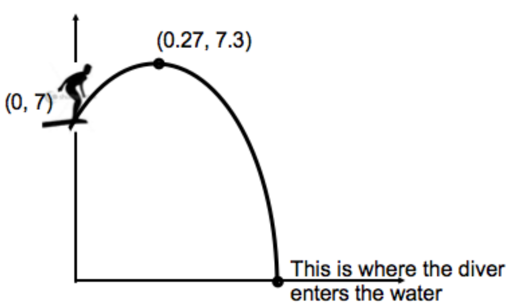
Substitute the value of *a* into $y = a(x + 4)(x - 2)$ to write the equation of the function.
$y = (x + 4)(x - 2)$ or $y = x^2 + 2x - 8$.

Practice Question: Page 387 (#13)

Second Numerical Response Question

Sample Question:
Ben dives off a 7 m platform. He reaches a maximum height of 7.3 m after 0.27 s. How long does it take him to reach the water?
A. 0.52 s
B. 0.69 s
C. 0.78 s
D. 1.32 s

Solution to Sample Question:
To reach the water you want to find the zero of the quadratic function. Let's create the quadratic function first using the information given. Use the graph



From the graph we can determine the vertex to be (0.27, 7.3) and we know another point on the graph (0, 7).
Substitute the vertex in for the *h* and the *k*.

$$y = a(x - h)^2 + k$$
$$y = a(x - 0.27)^2 + 7.3$$

Substitute (0, 7) for *x* and *y*

$$7 = a(0 - 0.27)^2 + 7.3$$

$$7 = a(0.27)^2 + 7.3$$

$$7 - 7.3 = 0.0729a$$

$$-0.3 = 0.0729a$$

$$\frac{-0.3}{0.0729} = \frac{0.0729a}{0.0729}$$

$$-41.2 = a$$

Now that you solved for a substitute in to complete the quadratic function.

$$y = -41.2(x - 0.27)^2 + 7.3$$

To reach the water we are actually the zero of the function. Using the graphing calculator we determine the zero to be: 0.69 **so answer B**.

Practice Question: Page 436 (#8)

First Written Response Question

Study Notes:

Solving Quadratic Equations using the Quadratic Formula

To solve a quadratic equation by using square roots:

Step 1: Write the equation in the form $ax^2 + bx + c = 0$
Step 2: Identify the values for a, b and c and substitute the values into the quadratic equation.
Step 3: Solve for x.
Step 4: Check your answers by substituting the all solutions into the original equation.

Sample Question:

Solve the following quadratic equation.

$$9x^2 - 12x + 1 = 0$$

a. Write your solutions as exact values **(radical)** in simplified form.

b. Calculate your solutions rounded to the nearest hundredth.

Solution to Sample Question:

a. $a = 9$, $b = -12$ and $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(1)}}{2(9)}$$

$$x = \frac{(12) \pm \sqrt{144 - 36}}{18}$$

$$x = \frac{12 \pm \sqrt{108}}{18}$$

$$x = \frac{12 \pm \sqrt{36 \cdot 3}}{18}$$

$$x = \frac{12 \pm 6\sqrt{3}}{18}$$

$$x = \frac{2 \pm \sqrt{3}}{3}$$

b.

$$x = \frac{2 + \sqrt{3}}{3}$$

$$x \approx 1.24$$

$$x = \frac{2 - \sqrt{3}}{3}$$

$$x \approx 0.09$$

Practice Questions: Page 436 (#4)