Date	Class	Period

Title: The rate of change.

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the variation function.
- 2- Find average rate of change
- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page ()

Lesson activities:

The Variation Function

 $V(h) = f(x_1 + h) - f(x_1)$ the function V is called the variation function of the function f at $x = x_1$

Example

- 1 If $f(x) = 3x^2 + x 2$ and x changes from 2 to 2 + h find the variation function V, then calculate the variation of f when
 - **a** h = 0.3

b h = -0.1

Try to solve

- 1) If $f(x) = x^2 x + 1$ find the variation function V when x = 3 then calculate:
 - **a** V(0.2)

b V(- 0.3)

The Average rate of change function

A (h) =
$$\frac{V(h)}{h} = \frac{f(x_1 + h) - f(x_1)}{h}$$
 or $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

example 👝

- 2) If $f:[0, \infty[\longrightarrow \mathbb{R} \text{ where } f(x) = x^2 + 1 \text{ find } :$
 - **a** The function of the average rate of change of f when x = 2 then calculate A (0.3)
 - **b** The average rate of change of f when x changes from 3 to 4

- 2 If $f(x) = x^2 + 3x 1$ find:
 - **a**) The average rate of change function at x = 2 then find A (0.2)
 - **b** The average rate of change when x changes from 4.5 to 3

The rate of change function

If $f:] a , b[\longrightarrow \mathbb{R} \text{ where } y = f(x) , x_1 , x_1 + h \in] a , b[\text{ then:}$

the function of the rate of change of f at $x_1 = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = \lim_{h \to 0} A(h)$ on condition that the limit exist.



Example

4) Find the function of the rate of change of f when $x = x_1$ then find this rate at the indicated values of x

a
$$f(x) = 3x^2 + 2$$
, $x = 2$

b
$$f(x) = \frac{2}{x-1}, x = 3$$

Try to solve

4) Find the function of the average rate of f where $f(x) = \frac{3}{x-2}$ when x changes from x_1 to x_1 + h then deduce the rate of change of f when x = 5

Date	Class	Period

Title: The first derivative.

Lesson objective: after the end of this lesson the student should be able to

- 4- Find the first derivative.
- 5- Find slop of tangent.
- 6- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page () <u>Lesson activities:</u>

the slope of the tangent at C = $\tan \theta = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$ if exist

the slope of the tangent to the curve of the function f where y = f(x) at the point $(x_1, f(x_1))$ equals the rate of change of f at $x = x_1$

Example

- 1) Find the slope of the tangent to the curve of the function f where $f(x) = 3x^2 5$ at the point A(2, 7) then find the measure of the positive angle which the tangent makes with the positive direction of the x-axis at the point A to the nearest minute.
- Solution

 $\therefore f(2) = 3(2)^2 - 5 = 7 \qquad \therefore \text{ the point A } (2, 7) \text{ lies on the curve of } f$

the slope of the tangent at (x = 2) = the rate of change of f at (x = 2)

=
$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

... the slope of the tangent = $\lim_{h \to 0} \frac{3(2+h)^2 - 5 - 7}{h} = \lim_{h \to 0} \frac{12 h + 3 h^2}{h}$

$$= \lim_{h \to 0} (12 + 3 h) = 12$$

then $\tan \theta = 12$ $\therefore \theta = \tan^{-1}(12) \simeq 85^{\circ} 14^{\circ}$

1) Find the slope of the tangent to the curve of the function f where $f(x) = x^3 - 4$ at the point A (1, -3) then find the measure of the positive angle which the tangent makes with the positive direction of the x-axis at the point A to the nearest minute.

The Derivative Function

If $f:]a, b[\longrightarrow \mathbb{R}, x \in]a, b[$ then the derivative function f':

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
 provided this limit exists.

The derivative function symbols:

If y = f(x) then the first derivative of the function f by the one of the symbols

and read as " y derivative" or " f derivative " y' or *f* '

 $\frac{dy}{dx}$ and read as " dy by d x " or " the derivative of y with respect to x"

Notice that the slope of the tangent to the curve of y = f(x) at the point $(x_1, f(x_1))$ is $f'(x_1)$

Cample Example

2) Using the definition of the derivative. Find the derivative of the function f where $f(x) = x^2 - x + 1$ then find the slope of the tangent at the point (-2, 7)

Solution

$$\therefore f(x) = x^2 - x + 1$$

$$\therefore f(x+h) = (x+h)^2 - (x+h) + 1 = x^2 + 2xh + h^2 - x - h + 1,$$

$$f(x + h) - f(x) = (2x + h - 1) h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f'(x) = \lim_{h \to 0} \frac{(2x+h-1)h}{h}$$

$$f'(x) = \lim_{h \to 0} (2x+h-1) \qquad f'(x) = 2x-1$$

$$f'(x) = \lim_{h \to 0} (2x + h - 1)$$
 $f'(x) = 2x - 1$

:
$$f(-2) = (-2)^2 - (-2) + 1 = 7$$
 : the point $(-2, 7)$ lies on the curve of f

The slope of the tangent at the point (-2, 7) = f'(-2) = 2(-2) - 1 = -5

Try to solve

(2) Using the definition of the derivative to find the derivative of the function f when $f(x) = 3x^2 + 4x + 7$, then find the slope of the tangent to the curve at the point (-1, 6).

Date	Class	Period

Title: Differintiability of a Function

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the first derivative.
- 2- Find slop of tangent.
- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page ()

Lesson activities: Differintiability of a Function

The function f is said to be differentiable at x = a (a belong to the domain of the function) if and only if f'(a) exist where $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Example

3 By using the definition, find the derivative of the function f where $f(x) = \sqrt{x-1}$ then find f'(5)

Solution

$$f(x) = \sqrt{x-1}$$
 ... the domain $f = [1, \infty[$

$$\therefore f(x+h) = \sqrt{x+h-1}$$

$$f(x + h) - f(x) = \sqrt{x + h - 1} - \sqrt{x - 1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

= $\lim_{h\to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$ multiplying by the conjugate of the numerator

$$f'(x) = \lim_{h \to 0} \frac{(x+h-1) - (x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{1}{2\sqrt{x-1}}, x > 1$$

notice that f is not differentiable at x = 1 because of the non-existence of the limit

$$f'(5) = \frac{1}{\sqrt{2 \times 2}} = \frac{1}{4}$$

3 By using the definition, find the derivative of the function f where $f(x) = \sqrt{x+5}$



 \bigcirc Find the derivative function of the function f in each of the following:

a
$$f(x) = 5x + 2$$

b
$$f(x) = 3x^2$$

c
$$f(x) = x^3 - 1$$

d
$$f(x) = x^2 + 2x$$
.

2 Find the first derivative of the function f in each of the following and identify the values of x at which the function is not differentiable:

$$\mathbf{a} f(x) = \underline{1}$$

b
$$f(x) = \frac{1}{x+3}$$

$$\mathbf{c} f(x) = \frac{3}{2x - 5}$$

d
$$\sqrt{x-4}$$

- 3 Find the derivative of the function f where $f(x) = x^3 + 4$ then find the slope of the tangent to the curve at the point (-1, 3) which lies on the curve.
- 4 Find the derivative of the function f where f(x) = ax + b at any point (x, y) where a, $b \in R$.
- 5 Find the slope of the tangent to the curve of the function f where $f(x) = 3x^2 8$ at the point A(2, 4) then find the measure of the positive angle which the tangent makes with the positive direction of x-axis.
- 6 If $f(x) = a x^2 + b$ where a and b are constants. Find:
 - **a** The first derivative of the function f at any point (x, y).
 - **b** The values of a, b if the slope of the tangent at the point (2, -3) which lies on the curve is 12.

Date	Class	Period

Title: Rules of differentiation

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the first derivative.
- 2- Find slop of tangent.
- 3- Build mathematical connections among mathematical knowledge and life situations.

Learning tools and resources: (text. book page (**Lesson activities:**

Derivative of a Function

1 - Derivative of the constant function

If
$$y = c$$
 where: $c \in \mathbb{R}$ then: $\frac{d y}{d x} = 0$

Notice that:

$$y = f(x) = c$$
, $f(x + h) = c$

$$\therefore \frac{\mathrm{d} y}{\mathrm{d} x} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{c - c}{h} = 0 \ (h \neq 0)$$

2 - the derivative of the function $f(x) = x^n$

If
$$y = x^n$$
 where: $n \in \mathbb{R}$ then: $\frac{dy}{dx} = n x^{n-1}$

If
$$y = x$$
 then: $\frac{d y}{d x} = 1$

If
$$y = a x^n$$
 where: $a, n \in \mathbb{R}$ then: $\frac{d y}{d x} = a n x^{n-1}$

Cample Example

- 1) find $\frac{dy}{dx}$ for each of the following:
 - (a) y = -3
- \mathbf{c} $\mathbf{v} = 5x$
- **d** $y = \frac{3}{x^2}$ **e** $y = \sqrt{x^3}$



$$\mathbf{c}$$
 :: $y = 5 x$

$$\therefore \frac{dy}{dx} = 5$$

d :
$$y = \frac{3}{x^2} = 3x^{-2}$$
 : $\frac{dy}{dx} = -6x^{-3}$

$$\therefore \frac{dy}{dx} = -6x^{-3}$$

e :
$$y = \sqrt{x^3} = x^{\frac{3}{2}}$$

$$e : y = \sqrt{x^3} = x^{\frac{3}{2}} \qquad \therefore \frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x} \text{ where } x \ge 0$$

1 Find $\frac{dy}{dx}$ for each of the following:

$$\mathbf{a} \quad \mathbf{v} = x^7$$

b
$$v = x^{10}$$

(c)
$$V = x^{\frac{3}{2}}$$

d
$$y = x^{-4}$$

e
$$y = x^{-\frac{5}{3}}$$

e
$$y = x^{-\frac{5}{3}}$$
 f $y = \frac{1}{x^5}$

$$\mathbf{g} \ \mathbf{y} = \sqrt{\chi^9}$$

h
$$y = \sqrt[3]{x^7}$$

i
$$y = 3 x^6$$

$$j y = -2x^{-5}$$

k y =
$$\pi x^{-4}$$

1
$$y = -4 \sqrt[3]{x}$$

The derivative of the sum or difference of two functions

If g and h are two differentiable functions with respect to the variable x, then $g \pm h$ is also differentiable with respect to x and $\frac{d}{dx}(g \pm h) = \frac{dg}{dx} \pm \frac{dh}{dx}$ and in general:

If $f_1, f_2, ..., f_n$ are differentiable functions with respect to the variable x then:

$$\frac{d}{dx} (f_1 \pm f_2 \pm f_3 \pm \dots \pm f_n)(x) = f_1'(x) \pm f_2'(x) \pm f_3'(x) \pm \dots \pm f_n'(x)$$

Example

2 Find $\frac{dy}{dx}$ in each of the following:

a
$$y = 2x^6 + x^{-9}$$

b
$$y = \frac{\sqrt{x} - 2x}{\sqrt{x}}$$

Try to solve

2 Find $\frac{dy}{dx}$ If:

(a)
$$y = 3x^2 + 5$$

b
$$y = 2x^3 - 4x + 7$$

b
$$y = 2 x^3 - 4 x + 7$$
 c $y = 3 x^{-4} + 6 x^{\frac{2}{3}}$

Date	Class	Period

<u>Title:</u> The derivative of the product of two functions:

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the first derivative of the product of two functions.
- 2- Find slop of tangent.
- 3- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page () <u>Lesson activities:</u>

The derivative of the product of two functions:

If g and h are two differentiable functions with respect to the variable x then the function $(g \cdot h)$ is also differentiable with respect to the variable x and $\frac{d}{dx}(g \cdot h) = g \cdot \frac{dh}{dx} + h \cdot \frac{dg}{dx}$



3 Find $\frac{dy}{dx}$ if $y = (x^2 + 1)(x^3 + 3)$ then find $\frac{dy}{dx}$ when x = -1

Try to solve

3 Find
$$\frac{dy}{dx}$$
 if:

a
$$y = (2x + 3)(3x - 1)$$

b
$$y = (2x + 5)^2$$

c
$$y = \sqrt{x} (\sqrt{x} + 4)$$

d
$$y = (\sqrt{x} - 1)(\sqrt{x} + 1)$$

e
$$y = (4x^2 - 1)(7x^3 + x)$$
 then find $\frac{dy}{dx}$ at $x = 1$

The derivative of the quotient of two functions:

If g and h are two differentiable functions with respect to the variable x and h $(x) \neq 0$

then the function $(\frac{g}{h})$ is also differentiable with respect to the variable x and $\frac{d}{dx}(\frac{g}{h}) = \frac{h \frac{dg}{dx} - g \frac{dh}{dx}}{\frac{dx}{dx}}$

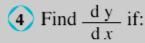
i.e.
$$(\frac{g}{h})' = \frac{hg' - gh'}{h^2}$$



Example

4 Find
$$\frac{dy}{dx}$$
 if $y = \frac{x^2 - 1}{x^3 + 1}$

Try to solve



a
$$y = \frac{2x-1}{x+1}$$

b
$$y = \frac{2x+5}{1-4x}$$

c
$$y = \frac{2x^2 - 1}{x + 5}$$

The derivative of the xomposite function (Chain rule)

If y = f(z) is differentiable with respect to the variable z, and z = g(x) is differentiable with respect to the variable x then y = f(g(x)) is differentiable with respect to the variable x and: $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$



Example

5 If
$$y = (x^2 - 3x + 1)^5$$
 find $\frac{dy}{dx}$

(5) If y =
$$(2 x + 3)^5$$
 find $\frac{d y}{d x}$

the derivative of the function $[f(x)]^n$

If $z = [f(x)]^n$ where f is differentiable with respect to the variable x, n is a real number,

then:
$$\frac{\mathrm{d} z}{\mathrm{d} x} = \mathrm{n} [f(x)]^{n-1} \times f'(x)$$



Example

6 Find
$$\frac{dy}{dx}$$
 if

a
$$y = (6x^3 + 3x + 1)^{10}$$

b
$$y = (\frac{x-1}{x+1})^5$$

6 Find
$$\frac{dy}{dx}$$

a
$$y = (2x^3 - 4x + 1)^6$$

6 Find
$$\frac{dy}{dx}$$
 if **a** $y = (2x^3 - 4x + 1)^9$ **b** $y = (\frac{5x^2}{3x^2 + 2})^3$



o Example

7 If
$$f(x) = \frac{1}{3}x^3 - 2x^2 + 5x - 4$$
 find the values of x which make $f'(x) = 2$

Try to solve

7 Find the values of x which make
$$f'(x) = 7$$
 in each of the following:

a
$$f(x) = x^3 - 5x + 2$$

b
$$f(x) = (x-5)^7$$

Date	Class	Period

Title: Geometric Applications on the Derivative

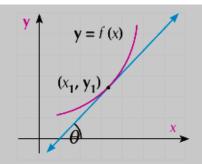
Lesson objective: after the end of this lesson the student should be able to

- 1- Find the first derivative of the product of two functions.
- 2- Find slop of tangent.
- 3- Build mathematical connections among mathematical knowledge and life situations.

Learning tools and resources: (text. book page (**Lesson activities:**

Geometric Applications on the Derivative

In the geometrical explaining of the first derivative of the function f where y = f(x) we found that the slope of the tangent (m) at the point (x_1, y_1) equals the first derivative of the function at that point.



i.e.
$$m = \frac{dy}{dx}$$
 at the point (x_1, y_1) and it can be written at the form
$$m = \left[\frac{dy}{dx}\right]_{(x_1, y_1)}$$

if θ is the measure of the positive angle which the tangent makes with the positive direction and of x-axis, then:

$$m = \tan \theta = \left[\frac{dy}{dx} \right]_{(x_1, y_1)}$$

1 If m₁, m₂ are the slopes of two straight lines L₁, L₂ then:

$$L_1 // L_2$$
 if and only if $m_1 = m_2$ (the condition of parallelism)
, $L_1 \perp L_2$ if and only if $m_1 m_2 = -1$ (the condition of perpendicularity)

The slope of the tangent to a curve at any point on it is known as the slope of the curve at this point, also perpendicular to the tangent to a curve at the point of tangency is known as the normal to the curve at this point.

... the slope of the normal at the point
$$(x_1, y_1) = \frac{-1}{\left[\frac{dy}{dx}\right]_{(x_1, y_1)}}$$

Example

Find the slope of each of the tangent and the normal to the curve $y = 2x^3 - 4x + 5$ at the point (-2, -3) which lies on it.

Try to solve

8 Find the slope of each of the tangent an the normal to the following curves at the indicated points:

a
$$y = x - 7$$
 when $x = -1$

b
$$y = x + \frac{1}{x}$$
 when $x = 4$

c
$$y = \frac{5}{x-3}$$
 when $x = 2$

d
$$y = (x^3 - 2) (x + 1)$$
 when $x = 1$

Example

- (9) Find the points on the curve $y = x^2 6x + 5$ at which:
 - **a** the slope of the tangent = 2
- **b** the tangent is parallel to x axis
- c the tangent is perpendicular to the straight line 4y + x 1 = 0

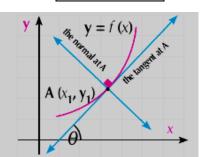
Try to solve

- 9 Find the points which lie on the curve $y = x^3 3x^2$ at which the tangent to the curve :
 - a Parallel to x-axis
- **b** perpendicular to the straight line x + 9y + 3 = 0

Equation of the Tangent to a Curve

 $\mathbf{2}^{\mathrm{nd}}\,\mathrm{sec}$

If (x_1, y_1) is a point lie on the curve of the function f where y = f(x), and m is the slope of the tangent to the curve at this point, then the equation of the tangent at the point (x_1, y_1) is



$$y - y_1 = m(x - x_1)$$

Note: the equation of the normal to the curve at the point (x_1, y_1) is:

$$y - y_1 = \frac{-1}{m} (x - x_1)$$

Example

10) Find the equations of the tangent and the normal to the curve $y = 2x^2 - 5x + 1$ at the point on it whose abscissa = 2

Try to solve

10 Find the equations of the tangent and the normal to the following curves at the indicated points:

a
$$y = x^3 + x^2 - 1$$
, $x = -2$

b
$$y = (3x-5)^7, x = 2$$



11) If the straight line $y = ax^3 + bx^2$ touches the curve y = 8x + 5 at the point (-1, -3) find the values of a, b.

Try to solve

11 Find the values of the constants a, b given that the tangent to the curve $y = x^2 + ax + b$ at the point (1, 3) which lies on it is 5.

Date	Class	Period

<u>Title:</u> Exercise (Equation of tangent)

<u>Lesson objective:</u> after the end of this lesson the student should be able to

1- Find

2- Build mathematical connections among mathematical knowledge and life situations.



Choose thr correct answer:

- 1) The rate of change of $x^3 + 4$ with respect to x when x = 2 equals
 - **a** 4

b 8

 \mathbf{c} $\frac{1}{4}$

- **d** 12
- 2 The slope of the tangent to the curve $y = 3x x^3$ at x = 0 equals
 - **a** 3

- **b** zero
- **c** 3

- **d** 6
- 3 The tangent to the curve $y = x^2 8x + 2$ is parallel to x-axis at $x = x^2 8x + 2$
 - **a** -8

b 2

c 4

- **d** zero
- 4 The straight line x + y = 5 touches the curve $y = 3x^2 + 5x + 1$ at $x = 3x^2 + 5x + 1$
 - **a** 1

b 5

c 3

d -1

17 $y = \frac{1}{2}x^4 - \frac{2}{3}x^3 + 7x - 9$

19 $y = \frac{4x^2 - 3x + 2\sqrt{x}}{x}$

23 $y = (x^2 - 7) (2x + 3)$

21) $y = \frac{4x^2 - x + 3}{\sqrt{x}}$

Find the first derivative with respect to x of each of the following.

$$\mathbf{16} \ \mathbf{y} = x^3 + 3x^2 - 5$$

$$\sqrt{18} \text{ y} = 2x^6 + 3\sqrt{x}$$

20
$$y = x (3x^2 - \sqrt{x})$$

$$22 y = (x^2 + 1) (2x + 5)$$

24
$$y = (x^2 + 3)(x^3 - 3x + 1)$$

26
$$y = \frac{5x - 2}{5x + 1}$$

25
$$y = (x^2 - \sqrt{x})(x^2 + 2\sqrt{x})$$

$$y = \frac{x-2}{x+5}$$

28 Find the value of
$$\frac{dy}{dx}$$
 of each of the following at the indicated point:

a
$$y = (x^2 - 2)^7$$
 at $x = 0$

b
$$y = (x^2 - x + 1)^{-4}$$
 at $x = 1$

29 Find
$$\frac{dy}{dx}$$
 of each the following:

a
$$y = (x + 3)^7$$

b
$$y = (2x^2 - 3)^4$$

$$\mathbf{c}$$
 $\mathbf{v} = (x^3 + x - 1)^5$

d
$$y^3 = 3x^2 + 1$$

e
$$y = \sqrt[3]{(2x^3 - 4x + 7)^2}$$

f
$$y = z^2$$
 , $z = 3x^2 + 2$

30 Find the measure of the positive angle which the tangent to the curve $y = x^2 + \frac{1}{1} - 1$

Date	Class	Period

Title: Integration.

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the.
- 2- Find.
- 3- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page (<u>Lesson activities:</u>



The function F is called antiderivative of the function f if F '(x) = f(x) for all x in the domain of f.



1 Prove that the function F where F $(x) = \frac{1}{2} x^4$ is antiderivative of the function f where $f(x) = 2x^3$.



The indefinite Integral

The set of all antiderivatives of the function f is called the indefinite integral of this function and denoted by $\int f(x) dx$ [is read as integral of f(x) with respect to x]



If F'(x) = f(x) then $\int f(x) dx = F(x) + c$ where c is an arbitrary constant.

Example

- 2 Verify each of the following:
 - **a** $\int x^7 \, dx = \frac{1}{8} x^8 + c$

b $\int (7x^6 + \frac{4}{x^3}) dx = x^7 - \frac{2}{x^2} + c$

Try to solve

- 2 Verify each of the following:
 - **a** $\int x^{-4} dx = \frac{-1}{3} x^{-3} + c$

b $\int x \sqrt{1+x^2} \, dx = \frac{1}{3}(1+x^2)^{\frac{3}{2}} + c$

Example

- \bigcirc Using the principle definition of integration, find the antiderivative of the function f where:
 - **a** $f(x) = 5x^4$

- **b** $f(x) = 18 x^{\circ}$
- $c f(x) = -3 x^{-4}$

Solution

- **a** $F(x) = \int 5x^4 dx = x^5 + c$
- **b** $F(x) = \int 18 x^{\circ} dx = \int 3(6 x^{5}) dx = 3 x6 + c$
- c $F(x) = \int -3x^{-4} dx = x^{-3} + c$

Date	Class	Period

<u>Title:</u> Integration.

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the.
- 2- Find.
- 3- Build mathematical connections among mathematical knowledge and life situations.

Learning tools and resources: (text. book page (

Lesson activities:

Rule (1):

 $\int x^{n} dx = \frac{1}{n+1} x^{n+1} + c \quad \text{where} \quad c \text{ is constant }, \text{ n is a rational number }, \text{ n} \neq -1$



Example

Find:





c
$$\int x^{\frac{2}{5}} dx$$

d
$$\int \frac{1}{\sqrt[5]{x^3}} dx$$

- 4 Find:
 - $\mathbf{a} \int x^8 \, \mathrm{d} x$
- **b** $\int x^{\frac{2}{3}} dx$ **c** $\int x^{\frac{7}{9}} dx$
- **d** $\int \sqrt[7]{x^5} \, \mathrm{d}x$

Rule (2):

 $\int a \cdot f(x) dx = a \int f(x) dx$ where f is a constant real number

Example

5 **a**
$$\int 3x^3 dx = 3 \int x^3 dx = 3 \times \frac{1}{4} x^4 + c = \frac{3}{4} x^4 + c$$

Corollary

$$\int a dx = a x + c$$

then:

$$\int 5 \, dx = 5 \, x + c \,, \int -9 \, dt = -9 \, t + c$$
$$\int dx = x + c \,, \int \sqrt{7} \, dz = \sqrt{7} \, z + c$$

Try to solve

- 5 Find each of:

 - **a** $\int 3 x^4 dx$ **b** $\int -2 z dz$ **c** $\int -x dx$
- **d** $\int -\frac{\mathrm{d}x}{5}$

Rule (3):

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

Example

6 Find: **a** $\int (4 x + 3x^2) dx$

b $\int \frac{x^2 - 2x - 8}{x + 2} dx$

6 Find:

a
$$\int (3x^2 + 2x - 1) dx$$

c
$$\int 2x(x+3) dx$$

b
$$\int (\frac{1}{x^2} + \sqrt{x} + 3) \, dx$$

d
$$\int \frac{4x^2 - 9}{2x - 3} dx$$

Rule (4):

$$\int [(ax + b)^n dx = \frac{1}{a(n+1)} (ax + b)^{n+1} + c, n \neq -1$$

Example

7 Find:

a
$$\int (3-2x)^5 dx$$

b
$$\int (2x-7)^{-3} dx$$

$$\int \frac{7}{\sqrt{3x-4}} \, dx, x > \frac{4}{3}$$

Date	Class	Period

<u>Title:</u> Integration.

Lesson objective: after the end of this lesson the student should be able to

- 1- Find the.
- 2- Find.
- 3- Build mathematical connections among mathematical knowledge and life situations.

<u>Learning tools and resources</u>: (text. book page ()
<u>Lesson activities:</u>

Exercises 3 - 4



Find:

$$\int x^2 dx$$

$$(2) \int x^7 dx$$

$$3$$
 $\int 8x \, dx$

4
$$\int -4 x^3 dx$$

$$\int 9x^8 dx$$

6
$$\int 12 x^{-4} dx$$

$$\sqrt{7}$$
 $\int 5 \, dx$

(8)
$$\int (5\sqrt[3]{x}) dx$$

9
$$\int -3 x^7 dx$$

$$\int -\frac{8}{5} x^{-3} dx$$

$$\int \frac{7}{2} t^6 dt$$

$$(12) \int \frac{12}{5} f^5 df$$

 $\mathbf{2}^{\mathbf{nd}}$ sec

Date	Class	Period

Title: probability

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<u>Learning tools and resources</u>: (text. book page (Lesson activities:

definition

Sample space (outcomes space)

The sample space of a random experiment is the set of all possible outcomes for this experiment, it denoted by (S)

Remarks:

- The number of elements in the sample space of a random experiment is denoted by n(S).
- ➤ The sample space will be finite if the number of its elements is finite and it will be infinite if the number of its elements is infinite. We will deal with the finite sample space.

First: Tossing a coin

Second: Rolling a die



2 A bag contains three identical balls: the first is red, the second is white and the third is yellow. Write down the sample space if you draw two balls, one after the other, while reputing the drawn ball before drawing the other one (with replacement) and observing the succession of colors.

Try to solve

2 A box contains three identical balls numbered from 1 to 3. Two balls are drawn one after another with replacement and observing the number on the ball. Write down the sample space and find the number of its elements.

definition

The event

second

An event is a subset from the sample space.

The simple event

Is a subset from the sample space that contains only one element.

definition

The certain event

It is an event whose elements are the elements of the sample space S. And it is an event that must occurs in each time we do the experiment.

The impossible event

It is an event that has no elements and is denoted by the symbol ϕ . And it is an event that must not occur each time we do the experiment.

Example

3 In the experiment of throwing a coin several times, the experiment will stop if a head or three tails appear.

Write down the sample space, and then determine the following events:

- A The appearance of head at most
- C The appearance of two tails at least
- B The appearance of head at least
- D The appearance of two heads at least

Solution

From the drawing, we find:

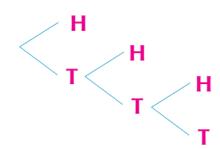
$$S = \{H, (T, H), (T, T, H), (T, T, T)\}$$

$$A = \{H,(T, H),(T, T, H),(T, T, T)\} = S$$

$$B = \{H,(T, H),(T, T, H)\}$$

$$C = \{(T, T, H), (T, T, T)\}$$

D =
$$\{ \} = \phi$$
 the impossible event



Date	Class	Period

Title: probability

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- 1- Find the.
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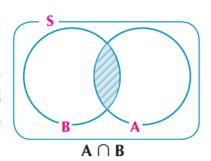
<u>Learning tools and resources</u>: (text. book page (

Lesson activities:

Operation of the events

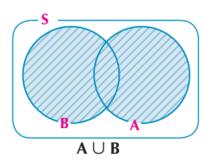
First: Intersection

The intersection of the two events A and B is the event $A \cap B$ which contains all elements of the sample space that belong to both A and B and means the occurrence of A and B (the occurrence of the two events at the same time).



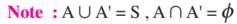
Second: Union

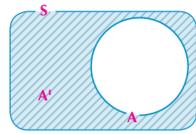
The union of the two events A and B is the event $A \cup B$ which contains all elements of the sample space that belong to A or B or both of them and means the occurrence of A or B (the occurrence of one of them at least).



Third: Complement

The event A` is called the complementary event of the event A, where A` contains all elements of the sample space that does not belong to the event A, and means non - occurrence of the event A.



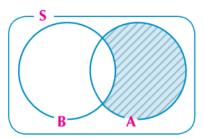


Fourth: Difference

The event A - B contains all elements of the sample space that belong to A and does not belong to B it also contains the same elements of the event $A \cap B'$

and it means the occurrence of A and non-occurrence of B (occurrence of A only).

$$A - B = A \cap B' = A - (A \cap B)$$

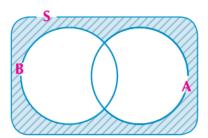


Fifth: De morgan's laws

A and B are the two events from the sample space S, then:

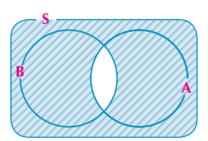
(first)
$$A' \cap B' = (A \cup B)'$$

which means the event of (non-occurrence of any of the two events) or the event of (the non-occurrence of A and the non-occurrence of B)



(second)
$$A' \cup B' = (A \cap B)'$$

which means the event of (non-occurrence of any of the two events all together) or the event of (the occurrence of one of the events at most)



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Mutually exclusive events

definition

- Ye Two events A and B are said to be mutually exclusive if $A \cap B = \phi$
- Several events are said to be mutually exclusive if and only if each two by two are mutually exclusive events.

Example

4 Two distinct dice are tossed and observing the numbers on the upper faces.

First: represent the sample space geometrically, and then write down the following two events.

A "appearance of the same numbers on the two faces"

B "appearance of two numbers their sum equals 7"

Second: Are A, B mutually exclusive? Justify your answer

Solution

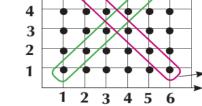
First : The elements of the sample space are ordered pairs, their number = $6^2 = 36$

the opposite figure is the geometrical representation of the sample space where every element is represented by a point

$$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$B = \{(6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6)\}$$

Second: $A \cap B = \phi$ $A \cap B = \phi$ $A \cap B = \phi$



Date	Class	Period

Title: probability

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<u>Learning tools and resources</u>: (text. book page ()

Lesson activities:

Calculation of probability:

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$$P\left(A\right) = \frac{n\left(A\right)}{n\left(S\right)} = \frac{\text{The number of the outcomes leads to the occurence of the event } A}{\text{The number of all possible outcomes for the random experiment}}$$

Example

- 5 If a ball drawn from a box contains 10 identical balls, 5 of them are white, 2 are red and the rest are green .Find the probability of the following events:
 - A the event "the drawn ball is red"
 - B the event "the drawn ball is red or green"
 - C the event "the drawn ball is not green"

Solution

The probability that the drawn ball is red = $P(A) = \frac{\text{The number of red balls}}{\text{The number of all balls}} = \frac{2}{10} = 0.2$

The probability that the drawn ball is red or green = $\frac{\text{The number of red balls + green balls}}{\text{The number of all balls}}$ $= \frac{2+3}{10} = \frac{5}{10} = 0.5$

The probability that the drawn ball is not green = P(C) = The probability that the drawn ball is red or white = $\frac{2+5}{10}$ = 0.7

Axioms of probability

- **1-** For every event $A \subset S$ there exists a real number called probability of event A, and denoted by P(A) Where : $0 \le P(A) \le 1$
- **2-** P(S) = 1
- 3- If $A \subset S$, $B \subset S$ and A, B are mutually exclusive events, then: $P(A \cup B) = P(A) + P(B)$

From the previous axioms we notice that:

The first axiom means that the probability of the occurrence of any event is a real number belongs to the interval [0, 1]

The second axiom means that the probability of the sure event = 1

The third axiom is called the sum of probabilities of mutually exclusive events rule which is circulating for a finite number of mutually exclusive events.

 $P(A_1 \cup A_2 \cup A_3 \cup ... \cup A_n) = P(A_1) + P(A_2) + P(A_3) + ... + P(A_n)$ where A_1, A_2, A_3 , ..., A_n are two by two mutually exclusive events

Example

6 If A, B are two events in a sample space S of a random experiment, such that:

$$P(A) = \frac{3}{8}$$
, $P(B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, Find:

- **a** $P(A \cup B)$ **b** P(A')
- **c** P(A B)
- **d** $P(A' \cap B')$

Example

- 7 If A, B are two events in a sample space S of a random experiment, such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$, $P(A - B) = \frac{3}{8}$ Find:

- (a) $P(A \cap B)$ (b) $P(A \cup B)$ (c) $P(A' \cap B')$ (d) $P(A' \cup B)$

Try to solve

- 7) In the previous question, find the following probabilities:
 - **a** P(A')

b $P(A' \cup B')$

 $P(B \cap A')$

Example

- 8 If A and B are two events in a sample space S of a random experiment, such that $P(A') = \frac{1}{3}P(A)$, $P(B) = \frac{1}{2}$, $P(A' \cup B') = \frac{5}{8}$ Find:
 - a The probability of occurrence of one of the two events at least.
 - **b** The probability of occurrence of one of the two events at most.
 - The probability of occurrence of the event B only
 - d The probability of occurrence of only one of the two events.

Try to solve

- (8) If A and B are two events in a sample space S of a random experiment, such that P(A) = 0.8, P(B) = 0.6 , $P(A \cup B)' = 0.1$ Find The probability of the following events:
 - **a** The occurrence of one of the two events at least.
 - **b** The occurrence of the event A only
 - **c** The occurrence of only one of the two events

Example

9 A and B are two events in a sample space S of a random experiment, where:

 $P(B) = 3 P(A), P(A \cup B) = 0.72, find: P(A), P(B)$

First: if A, B are mutually exclusive events. **Second:** if $A \subset B$

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 $2^{\mathrm{nd}}\,\mathrm{sec}$

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<u>Learning tools and resources</u>: (text. book page (<u>Lesson activities:</u>

