Lesson Outline

Title of Lesson: Embodied Slope and Tangent Lines

Abstract:

- In this lesson, students will use embodied techniques to assist their understanding of two important algebra concepts: slope and tangent lines. First, students will utilize a grid to develop an understanding of slope. They will start with a slope value and move from walking along the grid horizontally and vertically to following the diagonal slope line. Finally, they will use these embodied techniques to determine the slope of a given line. For the second part of this lesson, students will use movement to establish an understanding of tangent lines. Students will work with a partner to walk along a high-degree function, stopping at one point to follow the tangent line. Finally, they will combine all of their newfound skills, both in embodiment and math, to determine the slope of a tangent line.

Keywords:

- Algebra, Calculus, Slope, Tangent Line, Grid, Coordinate Plane, Function, 8th Grade, 9th Grade

Goal(s):

- **Begin with the end in mind: Here's where to identify the major takeaway from your lesson. What do you, as the teacher, want students to learn?**
- Students will develop an understanding of slope as a concept and mathematically. Students will understand the connection between the slope equation and the function on the coordinate plane.
  - Students will know how to determine the slope of a line on the coordinate plane.
  - Students will be able to utilize the slope equation.
- Students will develop an understanding of tangent lines as a concept and mathematically.
  - Students will be able to draw a tangent line to a function at a given point. ● Students will understand the connection between slope and tangent lines. Students will understand the difference between slope and tangent lines. ○ Students will be able to determine the slope of a tangent line for a function on the coordinate plane.

Learning Objectives:

- **Here's where to include more specific things to be taught, eg, What will students be able to do/demonstrate as a result ofthis lesson?**
- **If possible, objectives should be measurable.**
- **Rank order the concepts/ideas/skills you want students to learn. Which could be omitted if pressed for time?**

1. Students will be able to utilize the slope equation to determine the slope of a line on the coordinate plane.
2. Students will be able to draw a tangent line to a function at a given point. 3. Students will be able to determine the slope of a tangent line for a function on the coordinate plane
    a. This concept could be omitted if pressed for time, as it is more about connecting the two topics than building an understanding of either original topic.

Assessment:

- **How will you know if your Goals and Learning Objectives have been met? Will you check for understanding throughoutthe class or only atthe end? Will you conduct a formal assessment such as a quiz? Or an informal assessment, such as inviting students to close out class by stating key takeaways?**
- This activity would be done as an introduction to the unit on slope or tangent lines.
- There would be an informal assessment at the end of class.
    - First, students will get in small groups to discuss what they learned and how embodiment assisted this learning. They should come up with a list of key takeaways.
    - Then there will be a class discussion. The instructor should first ask each group to share one key takeaway. Then, the instructor should ask a few questions to test their mathematical understanding and allow students to raise their hands to answer.
- The formal assessment would come later, in the form of a quiz at the end of the unit, which will assess their understanding after more instruction and practice.
    - Another optional form of a formal assessment is to assign a homework assignment on slope/tangent lines. This assignment should involve the coordinate plane and the instructor should encourage them to use their embodiment skills, just downsized.

Purpose/Rationale:

- **What are the reasons for and the significance ofteaching/learning this lesson?**
  - Slope is an extremely important concept for understanding lines and functions. It is the basis of real-world problems that use slope to represent things like speed and other rates of change.
  - Tangent lines are an excellent introduction to derivatives and the changing slope of high-degree functions. This is the basis of calculus, and tangent lines are a great way to start building this understanding prior to calculus courses.
- **If you happen to be aware of specific state standards your lesson addresses, please note them here.**
  - Most states include learning slope in the curriculum for algebra. Not all students take calculus, but if they do tangent lines must be taught.
- **How is this subjecttypically taught, and whatis the value of an embodied approach - other than for creditin STS.014 - given thattraditional methods may be easier to implement?**
  - This subject is typically taught using a chalkboard, or with pen and paper. Students would do the same activity by counting the changes on a coordinate plane on their page.
  - The embodied approach has multiple advantages.
    - It is easier to count the change in the "x" and "y" directions on a larger plane.
    - Students will develop a better understanding of the real-world examples, as they are walking along a life-size grid.
    - Students will be up and moving, which has been shown to make their brains function better and encourage creativity.
    - Students will collaborate with their classmates.
    - Students will not be stuck at their desks doing work on paper alone for the entire class period.
    - It's more fun!

Prior Preparation:

- Student Prep
  - The students should be able to do this activity in any type of clothes since they are just walking along a grid.
  - The students may need to be notified of a modified class location. ○ The students should read their textbook chapter on slope/tangent lines.

- Advanced Set Up
  - The instructor will need to set up the grid/coordinate plane on the floor of the classroom. They will do this with tape.
  - The instructor will need to write "x" and "y" values on small pieces of paper and put them into two labeled buckets.
  - The instructor will need to cut the string into many long pieces.
  - The instructor will need to have questions prepared for the informal assessment/class discussion.

Materials/Resources:

● Make a list of needed supplies, facilities, and personnel if applicable. ● Whatis the likely costfor this lesson, and can it be adapted for schools with limited resources?

- Supplies
  - Tape
  - String
  - 2 buckets/baskets
  - Paper
- Facilities
  - A room with empty space for the grid (either the classroom with desks moved or in another empty room like the gym)
- Personnel
  - One instructor
  - An additional helper would be useful (TA, another teacher)
- Cost
  - The cost should be low since there are few materials needed.
  - If there are not enough resources to make the grid, this can be replicated on a dry-erase board or small paper version on their desks, where students use small game pieces to do the "walking." Instead of string, the lines can be drawn on.

Bibliography and Sources for Further Reading:

- **What books/articles/videos or other sources did you use as background for designing this lesson?**
- **Where might someone interested in the topic go for further information?**
- Bibliography
  - https://blogs.ams.org/matheducation/2016/02/08/learning-mathematics-through-embodied-activities/

    - https://edrl.berkeley.edu/wp-content/uploads/2021/03/BlessingBecky.2010.MACSME.thesis.MathMoves.pdf
    - https://ccl.northwestern.edu/2020/Abrahamson-et-al.2020.Frontiers.pdf
- Further Reading
    - http://torus.math.uiuc.edu/eggmath/Expon/tanslope.html
    - https://machinelearningmastery.com/a-gentle-introduction-to-slopes-and-tangents/
    - https://www.youtube.com/watch?v=7EFYoQ6H7Tw

6-STEP PROCEDURE FOR EXPERIENTIAL EDUCATION:

#1 Introduction: "Set Up for Success" - *10 minutes*

- **How will you motivate/frame your lesson so that students wantto learn the material?**
    - The students should already have an understanding of slope. The instructor should explain that this lesson will show the students how to determine the slope for a line on the coordinate plane. The instructor should elaborate and explain how slope is important for every type of linear function, and is also important in other high-level topics, like calculus.
    - Then, the instructor should introduce the topic of tangent lines. After a basic understanding is established, they should explain how tangent lines are the basis of calculus (derivatives), and try to explain how important calculus is in a broad sense.
- **Introduce content and vocabulary necessary for the lesson. Note here any readings, videos, handouts, models, etc. you might use in class.**
    - The slope equation should be on the board.
    - A conceptual drawing of tangent lines should be on the board, along with the definition of a tangent line.
        - This drawing should feature a function on the coordinate plane and a tangent line at a given point.
- **Set expectations by describing how you've structured the lesson in terms of doing v.talking. Will students do multiple activities (Step 2) before moving on to analysis (Steps 3-6)? Iteratively run through doing and analysis for each activity in turn (Steps 2-6, followed by Steps 2-6, followed by Steps 2-6)? Whatformat makes sense for your lesson?**
    - Students will do multiple activities with a partner/group. There are four activities in total. The instructor should give an overall explanation of what

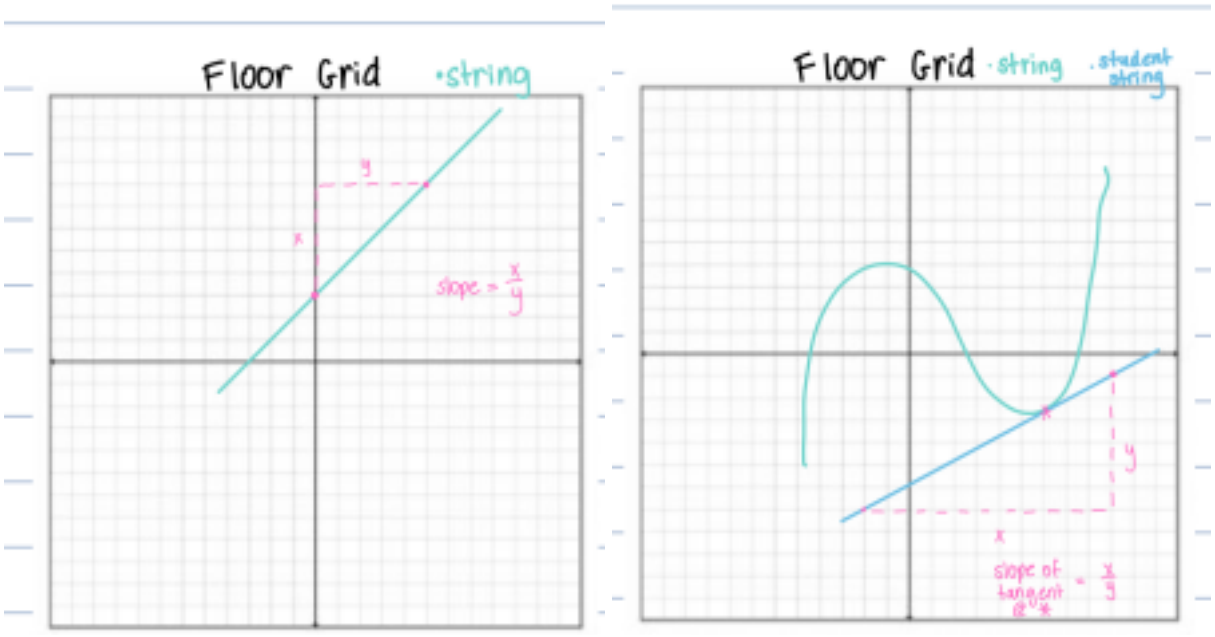
they are doing to begin. Then, before each activity, they should give an in-depth explanation and demonstration.
    - After the four activities (rounds of Step 2), the students will do the analysis (Steps 3-6) reflecting on all of the activities.

#2 Exploration: *"Do it" - 40 minutes(10, 5, 15, 10)*

- **Perform or do an activity or activities. Whatis the activity or activities to be encountered in today's class? Break down the lesson into specific steps so that another instructor can replicate your plan.**
- **Slope I**
    - Students should be put in pairs

    - Each student will choose one value from the "x" bucket and one value from the "y" bucket.
    - One student will choose a point on the grid.
    - The student will walk along the grid in the "x" and "y" directions according to the values they chose, starting at the point they chose and marking their endpoint.
    - The student will then walk diagonally from their starting point to their endpoint.
    - The other student should repeat this process with their values and starting point.
- **Slope II**
    - The instructor will use the string to make a line on the grid. This line should connect two points that have a normal slope.
    - The students will choose two integer points on the line.
    - The students will walk along the grid in the "x" and "y" directions to get from one point to the other to determine these values.
    - The students will determine the slope using the slope equation.
- **Tangent Lines**
    - The instructor will use the string to make a line on the grid. This line should represent a function of degree 2 or 3.
    - Each student should walk along the line to develop an understanding of the function.
    - One student should walk along the line slowly until their partner says stop. ○ When the partner says stop, their partner should mark the point on the line with a new string.
    - Then, the student should continue walking in the same direction (now going off of the original line), marking this new line with the new string. This new

line represents the tangent line.
  - Repeat with the second student.
- **Combination**
  - Students should use the walking method from slope II in order to find the slope of their tangent line (which they marked with a string).
  - The students will choose two integer points on the line.
  - The students will walk along the grid in the "x" and "y" directions to get from one point to the other to determine these values.
  - The students will determine the slope using the slope equation.
  - Repeat for the second student.



- **What are the features ofthe activity or activities, eg, individual or group experience, unfamiliar or familiar to learners, likely level of (dis)comfort or difficulty for learners?**
  - Most of the embodied parts of this activity are individual, except for following the partner's instruction in the tangent line activity. However, the students are encouraged to work with their partners to determine the slope, as this mathematical part can be challenging.
  - Most of the embodied parts of this activity should be familiar and comfortable, as the students are simply walking along a specified route. However, conceptually and mathematically they should experience some discomfort as they begin to understand the concept of slope and tangent lines. Luckily, the math is pretty straightforward once you understand the concept.
  - Embodiment: Familiar/Comfortable, Concept & Math: Familiar, Medium

Difficulty

- **Note how much assistance from the facilitator/teacher/coach is likely needed.**
  - First, the instructor will need to lay out strings for each group at the beginning of both the Slope II and Tangent Line activities.
  - The instructor should continuously check on the groups. Hopefully, a clear explanation at the beginning will make for smoother activities. It may be helpful to demonstrate the different activities as the students progress through them, instead of giving all of the directions at once.

  - Additionally, the students will likely need help with the mathematical steps, at least the first couple of times they do it.
  - Ideally, the instructor would have another TA/educator to help.
- **How can this be adapted to differentlearners?**
  - For students who are not able to do the walking portion, you can create a small grid on the desk and give them a small game piece to "walk" along the grid with their hands. This is also a good adaptation for a classroom that lacks the space for a grid on the floor.
  - Students who are already familiar with slope can skip to the tangent line part and you can give them more difficult lines/functions with the string. They should spend more time understanding tangent lines and the connection between tangent lines and slope.
- **What could go wrong, and what will you do to preventthis from occurring?** ○ There may be too many students to all fit in the grid at once. In this case, there can be groups instead of partners who all work together with one line for each activity. Alternatively, the instructor could make multiple smaller grids.
  - In groups or partnerships, one student could be dominating the activity. In this case, the instructor can assign specific jobs to each student.

 # 3 Sharing: *"What Happened" - 5 minutes*

- **Invite participants to talk abouttheir experience, eg, results, reactions and observations, what was (un)expected aboutthe experience.**
- **Since you will notlikely run your lesson, offer your best guesses about student responses.**
  - Students will likely say it was harder than expected to create the tangent line.
  - Students will likely say that walking in the "x" and "y" directions made it much easier to utilize the slope formula.
  - Students will likely say that it was more difficult than expected to find the

slope if they didn't pick two points that were integers.
    - Students will hopefully say that the combination activity was easier than the other activities since they had practice through those activities.

#4 Processing: *"What'sImportant?" - 5 minutes*

- **Discuss, analyze, and reflect upon the experience to connectitto course content/goals.**
- **Identify any key points that didn't come outin student discussion**

- **Since you will notlikely run your lesson, offer some thoughts as to what you want students to discover.**
    - If you know two points on a line you can determine the slope by examining the change in the "x" and "y" directions.
    - Any two points on the line will produce the same slope value.
    - It is easier to use integer points to determine the slope.
    - The slope is always a fraction that represents the change in y over the change in x.
    - The embodied method is the same as the slope method if you count in your head on a coordinate plane.
    - You can use the slope method for any straight line.
    - Functions that are not straight lines have different slope values at different points on the function. Tangent lines can be used along with the slope equation to determine this value.

#5 Generalizing:"*So What?" - 5 minutes*

- **Connectthe experience with real world examples, eg, why students should care aboutthis activity beyond the class context.**
    - Building roads, roofs, and bridges
    - Identifying the steepness of a hill (runners, skiers, snowboarders)
    - Creating roller coasters
    - Building handicap ramps
- **Identify any key points that didn't come outin student discussion. ● Since you will notlikely run your lesson, offer your response to the "So What?" question.**
    - The slope of a line never changes. Functions of any degree higher than 1 do not have a consistent slope. This is why we need tangent lines.
    - For the walking method, it is crucial to use integer points to find your slope. However, as long as you know the exact point on the line, any points will work with the slope equation. No matter what two points you pick on a

straight line, you should get the same slope value.
○ Describe how the slope equation is a mathematical representation of the embodied method.
○ Tangent lines represent the slope at a specific point on the function. ○ There are infinite points on a function, and thus there are infinite tangent lines.
○ All tangent lines have a consistent slope.
○ What happens if a line is completely vertical or horizontal (no change in x or y direction)? You cannot use the slope equation. Completely horizontal lines

have a slope of 0, as there is no change in y. Completely vertical lines have an undefined slope, as there is no change in x.

#6 Application and Wrap-Up: *"Now What?" - 5-10 minutes*

- **Answer questions and summarize takeaways (or ask students to summarize to check for understanding).**
  - ○ After establishing the key takeaways (first by asking the students what they came up with as groups, then filling in the blanks that they missed), the instructor should answer any questions the students may have.
  - ○ Then, the instructor should test their understanding by asking them some mathematical questions about slope and tangent lines. This should only be a few questions, one per activity, and should take 5 minutes or so.
- **Discuss what students can do with whatthey have learned.**
- **Imagine how it could be refined as well as adapted to a different situation.** ○ This can be done with lines/functions on a coordinate plane, which is how students will use it most.
  ○ In calculus, these concepts are used for extremely complex functions, in order to find the slope at different points in the function. (Give an example.) ○ This can be used in real life as well, as long as you establish a grid/plane. Refer to the real-world applications discussed previously.