The Period of Oscillating Springs/Rubber Bands - What does the Period depend on? (Simple Harmonic Motion)

Purpose

In this activity students will investigate how the attached mass and spring constant (a measure of how stiff the spring or band is) affect the period of an oscillating rubber band or spring. Based on your data, a mathematical model will be developed for the period of a rubber band or spring. Note this activity can and should be done prior to any class discussion of details.

Overview

Students will play the role of 'scientist' by doing this set of experiments *before* the details of simple harmonic motion are discussed or studied in class. Students will be expected to do a short series of controlled experiments to determine what, if any, effect mass and spring constant have on the period of a vertically oscillating spring/rubber band.

The ***period*** of a rubber band is the ***time it takes*** for a hanging rubber band, with a weight on it, to be ***stretched and released, and then get back to where it started (a round trip)***.

For example, students will do a series of trials of measuring the period of oscillation as they vary the mass hanging on the spring. By plotting the period as a function of mass, students should find a non-linear graph – the period is proportional to the square root of mass. The equation for the period of a spring, $T = 2\pi\sqrt{\frac{m}{k}}$

They should find that the spring constant does have a significant effect on the period (they will need multiple springs/rubber bands to do this), but that the period depends on something close to the square-root of the mass on the spring. If you have access to a computer and Google Sheets, Excel, Desmos, or other online graphing programs, a power fit should give something close a power of 0.5. There should be no dependence of the period on the amplitude of oscillation.

Student Outcomes

Learner objectives

Students will:

- ➢ Discover and appreciate the complexity that is often not initially present in everyday, seemingly simple events or phenomena;
- ➢ Identify and investigate individual physical quantities that may have an impact on the nature of a basic spring;
- ➢ If they go on and actually investigate this experimentally, students will begin to learn what a controlled experiment is, how to go about designing and running controlled experiments, learn and create measuring techniques appropriate to , take and organize and record numerous data points, and analyze data;

- ➢ Learn something about periodic motion, and more specifically simple harmonic, phenomena;
- ➢ If experiments are done, and computers and graphing/curve-fitting software available, analyze data to find best-fit mathematical functions and then compare the findings and experimental math model to more accepted theoretical math models you would find in textbooks.

Time

This experiment will take 45-60 minutes.

Level

This experiment is appropriate for physics classes for ages 14-18.

Materials and Tools

Variety of rubber bands or springs (especially different thicknesses, varying stiffness)
Set of known masses or a scale to measure mass
A stand or some means of hanging rubber bands (to hang masses on the rubber band)
Ruler or meter stick
A stopwatch or phone with a stopwatch feature
Graph paper

The basic materials will be a stand or support on which to hang a spring or rubber band, a set of known masses (ideally with hooks to hang on the rubber bands), stopwatches or other timing devices , a balance to measure mass, and a meter stick. If one wants to investigate if spring constant (whether a rubber band is loose or stiff) has an effect on period, multiple springs/rubber bands will be needed by each group of students – it is recommended that four or five rubber bands of varying stiffness be provided, so students have at least five data points to make a graph of period vs. spring constant.

All introductory physics textbooks have more detailed information about oscillating springs and simple harmonic motion, if one wants to get into more details.

Preparation

The main preparation for labs of this type should begin with the first day of class. *Students need to be prepared to actually **do the science**,* and not sit back and digest what is provided from a science teacher. This lab should be the first step of a unit on periodic motion or simple harmonic motion, depending on when the teacher introduces springs.

The lab itself is fairly routine for a physics classroom. Springs or rubber bands should be set out at lab stations. Stopwatches, a meterstick and masses will need to be available. Some teachers may want to use the stopwatch on cellphones. That is the only physical preparation that needs to be done.

But if this type of lab experience is new to students, the teacher should prepare the class by explaining the purpose of doing the lab first, prior to having any class discussions or textbook readings or demonstrations. *Students will be in the position of scientist,* and must develop and collect data from the experiments to come up with their own conclusions and model for what factors determine the period of the spring or rubber band. The student experimental model will then be compared to the accepted model in the textbook, and conclusions and deeper scientific discussions will result from that comparison, as details and the physics behind the pendulum are studied.

Prerequisites

Students do *not* need any prior knowledge about details of periodic motion, the springs/rubber bands or any analysis details about simple harmonic motion. Students should be starting with little, if any, prior knowledge – they are in the role of scientists trying to make the discoveries of the details on their own, from real data.

The only prerequisite students should have is knowledge about what a controlled experiment is, and how to go about controlling physical quantities while only varying a single quantity to test how it affects the period of the spring or rubber band. Students should also be aware of how to write up a lab report, using whatever criteria the teacher requires.

Background

Simple harmonic motion is a classic example of **periodic motion**, which is a motion that is repetitive, redundant, keeps repeating itself, keeps going up and down... A spring works whether there is **gravity** or not, because the restoring force is internal, and the oscillations can be used as a clock. The period does not depend on how far the spring or rubber band is stretched. Keep in mind that the term 'simple harmonic' is a subset of periodic motion, due to the restoring force, F, of the spring being directly and linearly dependent on the displacement (or stretch, x) of the spring. This is given by **Hooke's law**, $\mathbf{F} = -k\mathbf{x}$, where k is the spring constant.

For this activity, the teacher does not need to get into any other details about simple harmonic motion. If this is truly new for students, then they will be in a position of investigating the unknown, and they are in the role of the scientist who needs to discover the basic rules for an oscillating spring or rubber band. Once students have their results for this system, it can be compared to the known model in a textbook.

Teaching Notes

To do this activity, or any labs similar to it, the teacher should place a focus on developing an attitude and mindset amongst students that this is going to be the norm in the class. If possible, as many labs should be done with the approach that the lab and the collected data make up the very first part of new units of study. The new approach to science education is getting students to be the scientist; science is an active process. This will begin to take teachers away from beginning a new unit with a lecture or demonstrations that spell out what the details are, or simply writing down notes and have students read about a phenomenon , and finally allowing the lab to simply confirm what was already stated in class – instead, put the lab first, let students discover the relationships, patterns, and details of a phenomenon, and then go to the textbook and compare what students conclude with the accepted.

If there are differences between what students find and the accepted result, this will provide a rich set of questions and discussion points to figure out why the experimental work might differ from what professional scientists have concluded. Students will learn about and actually live the scientific process with this approach, and along the way it builds up the mindset of how to tackle the unknown and how to solve problems of any kind by looking for data and evidence first, from which conclusion should be based.

For this lab, the only demonstration and information the teacher should provide upfront is what the definition of the period of oscillation is, which is the time for one full bounce of the mass attached to the spring (i.e. one round trip). Depending on any previous work done with springs, the teacher may want to remind students how the spring constant is defined, and perhaps how to determine the spring constant. Keep in mind the spring constant, k, is the weight hanging on the spring or rubber band divided by the stretch distance, $k = F/x$.

Teachers should set the expectation that detailed procedures will no longer be provided for labs like this. Students should try to figure out the best techniques, and determine which parameter(s) are being controlled and which is being varied. Students should be allowed to fumble a bit, and use trial and error – after all, this is what professional scientists do when they are investigating the unknown. Experimentation is largely troubleshooting and trial and error.

While students are doing the experiment, it is valuable to have conversations with lab groups. Ask why they are making the measurements they are making. Ask students what some uncertainties are with their measurements or procedures. Let them talk through their thinking, fix misconceptions, ensure they are doing multiple time trials if using stop watches. Make sure they are making good, labeled graphs. This is a process that is more inquiry based and student centered. After doing a few labs this way, students will catch on to the expectations and it will be more comfortable and accepted.

If interested, an extension can be made for students and this lab. Once they have a mathematical model for the period of an oscillating spring, challenge them to use it as a metronome. That is, anyone should be able to ask them to set up the spring to have a specific, arbitrary period so it could be used as a metronome. Students should be able to extrapolate and interpolate from their period-mass graph what mass needs to be added to the spring. One can assess this by measuring with a stopwatch (recommend timing 10-20 full bounces to get the period if using a stopwatch), how accurate the metronome is.

Assessment

Teachers should use whatever their normal grading criteria/rubric is for laboratory reports. While it is nice when students get results that agree well with accepted mathematical results, one should anticipate some discrepancies in this lab, particularly if students are using stopwatches to measure the period of an oscillating spring.

Teachers should assess how students are going about the process of finding a mathematical model in this activity; this relates to the graph that should be made. In some cases, students will have little if any experience doing this type of lab. There is much to learn about creating the initial research questions and purpose, developing experimental designs and measuring techniques, and data collection and organization for analysis – these are all important features the teacher will need to ensure students are learning, in addition to the physics principles of a pendulum.

If the teacher decides to have students test their spring model and set up the spring as a metronome, assess how accurate it keeps time relative to a standard time keeping device or technique. Students can do this by varying the mass hanging on the spring, and the mass will be determined straight from the period vs. mass graph for that spring.

Additional Information

There is much information about simple harmonic motion in any standard introductory physics textbook. Depending on the level of the class and students, teachers (such as AP level) will want to get into the derivation of the classic result, $T = 2\pi[m/k]^{1/2}$. This is a more straightforward derivation than a pendulum (which involves the small angle approximation). A sample how-to video is at:
http://docvphysics.blogspot.com/2010/04/how-to-get-simple-harmonic-motion.html

If a class wants to do something similar with simple harmonic motion of a physical pendulum, such as an oscillating stick due to a spring, a sample how-to video of the theory is at:
http://docvphysics.blogspot.com/2012/04/shm-of-oscillating-stick-due-to-spring.html

Below is a sample student lab sheet, outlining the goals of the experiments as well as some possible analysis questions.

Period of Oscillating Springs/Rubber bands - Simple Harmonic Motion

Purpose: To better understand simple harmonic motion (e.g. springs) through measurements with an electronic force sensor.

Materials:

Springs or rubber bands
Stand
Known weights or masses
Meter stick

Warning: Please be careful with the springs or rubber bands; do not hang such large weights on them that they stretch too much and permanently deform or break.

Procedure:

To get started, have your experiment set up. A spring/rubber band hangs off whatever stand you are using, and known masses are hung on the rubber band.

1. Do a quick measurement of how far the spring or band stretches for a given mass, and calculate the spring constant, k = weight/(stretch distance). You will need to do this for each spring or band you use.

2. For a given weight, does the period of the oscillating spring vary with amplitude (the distance you stretch the rubber band past the position it is at rest)? Try several different amplitudes and get data to reach a conclusion: Does the amount you stretch a rubber band change the time it takes to bounce up and down? ***Be careful*** *so that when you let the oscillations occur, the weights do not fly off.* ☺

3. Now measure the period for a variety of masses hanging on the spring or rubber band. Your job is to find the experimental relationship between period and mass. Get five or six points to **make a graph** of period (y-axis) vs mass (x-axis). Is this graph linear or more of a curve, based on your data?

4. Measure the period for the same mass, but now on each of four or five different rubber bands with different stiffness. Find the experimental relationship between period and spring constant. **Make a graph** of period (y-axis) vs. spring constant (x-axis) for your data.

5. Combine your results from questions 2, 3 and 4 into a single empirical equation for the period of an oscillating spring or rubber band.

Questions and Analysis:

1. What reason(s) can you think of that could explain why a real spring could become physically deformed if <u>too much</u> weight is hung from it for a large period of time? Hint: Think small.

2. Understanding springs is actually quite important for the modeling of many materials, such as solids and diatomic molecules. It is possible, for example, to treat something like a rubber-band as a bunch of small masses (representing individual atoms and molecules) connected together by springs, one after another. Why does this seem like a reasonable thing to do?

3. Did the period of oscillation for a spring depend on amplitude? Support your conclusion with data.

4. How does your empirical formula for the period of an oscillating spring using data jive with the theoretical result $T = 2\pi\ [m/k]^{1/2}$? Explain.

5. Last part! (about time, right?!)

In the last procedure above, you let a spring oscillate for a longer of time. Qualitatively discuss the graph from that procedure. Specifically, what happens to frequency, period and amplitudes over longer time intervals for oscillating springs when air friction acts on the mass?

<u>Above and Beyond:</u>

Have your teacher ask you to keep time for some arbitrary period value. Using your springs and data/graphs/fits, set up an oscillating spring that will measure the requested period! You are effectively being asked to make a metronome or clock from a rubber band!

For the Teacher:

If available, students can use electronic force sensors to accurately measure the period. Students could use stopwatches to measure the period. Another option is to use a cell phone to take a video of the oscillations, and using video software or frame by frame viewing ($1/30^{th}$ second intervals), students could get a measure of period. By plotting T vs m, students should get something close to a T vs sqrt(m) graph.

Teachers could ask students to plot T^2 vs mass, instead, and get a nearly linear graph.

Students could be asked to then measure period, for a fixed mass, for different springs with different spring constants, and plot T vs k (or T^2 vs. k). They should find an inverse relationship. The actual expression is $T = 2\pi\ [m/k]^{1/2}$.