

PRE-BOARD EXAM 2022-23

CLASS: XII

MATHEMATICS

TIME: 3 hrs.

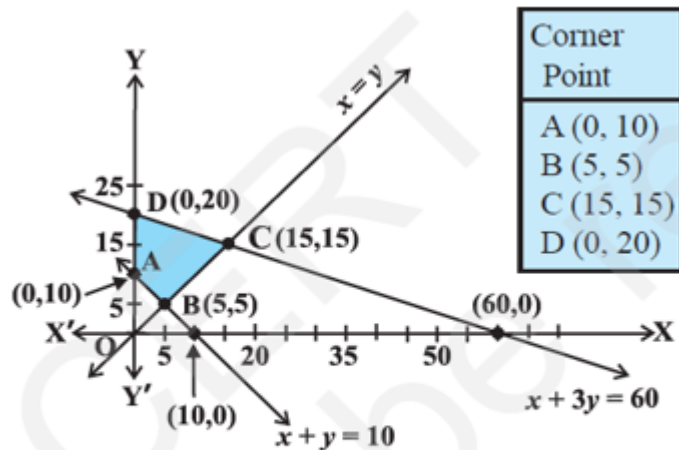
MARKING SCHEME

Max Marks: 80

QUESTION NUMBER	HINTS/SOLUTION	MARK(S)
1	$15p + 15q = 20q$ $3p = q$ CORRECT OPTION-D	1 (d)
2	$7A - I^3 - 3A - 3A - A$ $= -I$ CORRECT OPTION-A	1(a)
3	$a_{32} = 5, A_{32} = 22$ $a_{32} A_{32} = 110$ CORRECT OPTION-D	1(d)
4	$K = \frac{(x+9)(x-3)}{x-3} = 12$ CORRECT OPTION-B	1(b)
5	The strict inequality represents an open half plane and it contains the origin as (0,0) satisfies it. CORRECT OPTION-B	1(b)
6	$f(x) = \sec x$ CORRECT OPTION-B	1(b)
7	The given differential equation is $4 \left(\frac{dy}{dx} \right)^3 \frac{d^2y}{dx^2} = 0$. Here, $m = 2$ and $n = 1$ Hence, $m + n = 3$ CORRECT OPTION-C	1(c)
8	$ydx - xdy = 0 \Rightarrow ydx - xdy = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$ $\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x} + \log K, K > 0 \Rightarrow \log y = \log x + \log K$ $\Rightarrow \log y = \log x K \Rightarrow y = x K \Rightarrow y = \pm Kx \Rightarrow y = Cx$ CORRECT OPTION-A	1(a)
9	$y \sin y \Big _0^x = x \sin x$ CORRECT OPTION-B	1(b)
10	Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k}$ $\frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{ \hat{i} + 2\hat{j} - 3\hat{k} } = \frac{7}{\sqrt{14}}$ CORRECT OPTION-A	1(a)
11	$12.(12)^2 = 1728$ CORRECT OPTION-A	1(a)
12	$-16 = -8 - x^2$ $X = \pm 2\sqrt{2}$ CORRECT OPTION-D	1(d)
13.	$x = 3, y = 3$ $x + y = 6$ CORRECT OPTION-B	1(b)
14	$ \vec{a} \vec{b} \cos \cos \theta = \vec{a} \vec{b} \sin \theta$ $\tan \theta = 1$ $\theta = \frac{\pi}{4}$ CORRECT OPTION-C	1(c)
15	$i.i + j.j + k.(-k)$ $1+1-1 = 1$ CORRECT OPTION-A	1(a)

16	$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$ $p = \frac{1}{5}$ CORRECT OPTION-A	1(a)
17	$\vec{b} = \frac{\vec{a} + \vec{oc}}{2}$ $\vec{oc} = 2\vec{b} - \vec{a}$ CORRECT OPTION-B	1(b)
18	$\frac{dy}{dx} = \frac{3t^2}{2t}$ $= \frac{3t}{2}$ CORRECT OPTION-D	1(d)
19	Let, $f(x)=f(y) \Rightarrow x = y$. $\therefore f(x)$ is injective. Not surjective because co-domain $\neq$ range CORRECT OPTION-A	1(a)
20	$\vec{r} = -\vec{i} + \vec{j} + \vec{k}$ Direction cosines $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ If (a,b,c) are d.r then d.c are $\left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$ CORRECT OPTION-C	1(c)
21	$\frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$ $= \frac{-\pi}{3}$ (OR) $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}$	1 1 1 1
22	$\vec{a} + \vec{b} = 4i + (2 + \mu)j + 12k$ $\vec{a} - \vec{b} = 2i + (2 - \mu)j + 6k$ $8 + 4 - \mu^2 + 72 = 0$ $\mu = \pm 2\sqrt{21}$	(1/2) (1/2) (1/2) (1/2)
23	$\hat{n} = \frac{i-2j+2k}{3}$ Required vector = $5(i - 2j + 2k)$	1 1
24	$\log y = \sin x \cdot \log(\sin x)$ $\frac{1}{y} \frac{dy}{dx} = \cos \cos x [1 + \log(\sin \sin x)]$ $\frac{dy}{dx} = (\sin \sin x)^{\sin \sin x} \cdot \cos \cos x [1 + \log(\sin \sin x)]$	(1/2) 1 (1/2)
25	$R(x) = 26x + 26$ $M(x) = 26(7) + 26$ $= 208$	1 (1/2) (1/2)
26	$\int \frac{dx}{\sqrt{-(x^2+2x-3)}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{2}\right) + C \quad \left[\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \right]$	(1+1) (1)

27



Minimum value of z is 60 at (5,5)

3 lines (1.5)	Corner points (0.5)
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1

28

$$1. \quad \frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$
$$= 1 + \frac{5}{x-2} + \frac{10}{x-3}$$

$$\therefore I = x - 5 \log (x - 2) + 10 \log (x - 3) + c$$

1

1

1

29

$$\text{Let } I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \quad \dots(i)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}}{\sqrt{\sin(\frac{\pi}{6} + \frac{\pi}{3} - x)} + \sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{..(ii).}$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\pi/6}^{\pi/3} dx$$

$$= [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

Hence, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \frac{\pi}{12}$

(1)

(1)

(0.5)

(0.5)

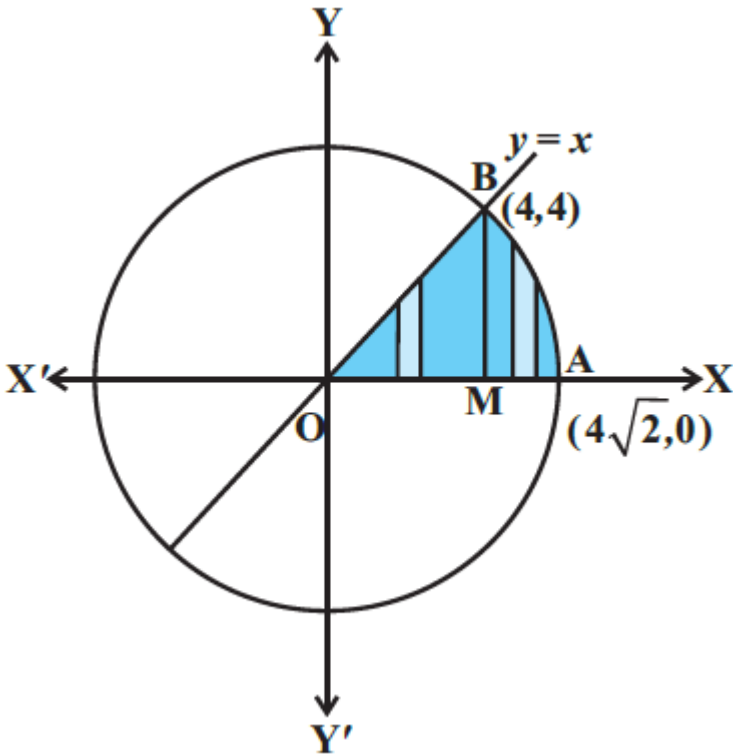
30.

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin \frac{y}{x}}$$

(0.5)

(0.5)

	$\text{put } \frac{y}{x} = v$ $v + x \cdot \frac{dv}{dx} = v - \frac{1}{\sin v}$ $-\int \sin v \, dv = \int \frac{1}{x} \, dx$ $\cos v = \log x + c$ $\cos \left(\frac{y}{x} \right) = \log x + c$ (OR) $ydx + (x - y^2)dy = 0$ Reducing the given differential equation to the form $\frac{dx}{dy} + Px = Q$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$ $\text{I.F} = e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$ The general solution is given by $x \cdot \text{IF} = \int Q \cdot \text{IF} dy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C, \text{ which is the required general solution}$	(0.5) (0.5) (0.5) (0.5) (0.5) (0.5) (1) (1) (0.5)
31	A: both are girls B: older is a girl $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ $= \frac{\frac{1}{4}}{\frac{2}{4}}$ $= \frac{1}{2}$ (OR) $\sum P(X) = 1$ $\Rightarrow 10k^2 + 9k - 1 = 0$ $k = \frac{1}{10}$	(0.5) (1) (1) (0.5) (0.5) (1.5) (1)
32	$ A = 10$ $A^{-1} = \frac{1}{10} \begin{pmatrix} -6 & 25 & -24 & -12 & 40 & -38 & 10 & -40 & 40 \end{pmatrix}$ $x = 1, y = \frac{1}{2}, z = -1$ (OR) $AB = 6I$ $A^{-1} = \frac{1}{6} B$	(1) (3) (1) (2) (1)

	$X = \frac{1}{6}(2 \ 2 \ -4 \ -4 \ 2 \ -4 \ 2 \ -1 \ 5)(3 \ 17 \ 7)$ $x = 2, y = -1, z = 4$	(1) (1)
33	$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$ $\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{59}$ $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 10$ $d = \frac{10}{\sqrt{59}}$ (OR) $\vec{m} = \begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ $= 24\hat{i} + 36\hat{j} + 72\hat{k}$ Required equation is $(\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$	(1) (1) (1) (1) (1) (2) (2) (1)
34	Proving reflexive, symmetric and transitive Elements related to 1 are {1,5,9}	(4.5) (0.5)
35	 To find point of intersection as (4,4) $\text{Required area} = \int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} \, dx.$ $= 8 + (4\pi - 8)$	(1) (1) (1/2 + 1/2) (1/2 + 1) (1/2)

	= 4π sq. units										
36	(1) f(x) being a polynomial function, it is differentiable in (0,12) (2) f(x) = - 0.2 x + 1.2 = - 0.2(x – 6) f(x) = 0 ⇒ x = 6 (3). <table border="1"> <thead> <tr> <th>In the Interval</th><th>f'(x)</th><th>Conclusion</th></tr> </thead> <tbody> <tr> <td>(0, 6)</td><td>+ve</td><td>f is strictly increasing in [0, 6]</td></tr> <tr> <td>(6, 12)</td><td>-ve</td><td>f is strictly decreasing in [6, 12]</td></tr> </tbody> </table>	In the Interval	f'(x)	Conclusion	(0, 6)	+ve	f is strictly increasing in [0, 6]	(6, 12)	-ve	f is strictly decreasing in [6, 12]	(1) (0.5) (0.5) (1) (1)
In the Interval	f'(x)	Conclusion									
(0, 6)	+ve	f is strictly increasing in [0, 6]									
(6, 12)	-ve	f is strictly decreasing in [6, 12]									
37	(1) $y = \sqrt{4a^2 - x^2}$ (2) $A = \frac{x\sqrt{4a^2 - x^2}}{2}$ (3) $\frac{dy}{dx} = \frac{-x^2 + 2a^2}{\sqrt{4a^2 - x^2}}$ (4) $x = y = \sqrt{2} a$ and $\frac{d^2y}{dx^2} < 0$ when $x = \sqrt{2} a$	(1) (1) (1) (1)									
38.	(1) $P(E_1) = \frac{3}{5}$ (2) $P(E/E_1) = 1$ (3) $P(E_1/E) = \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$ $= \frac{9}{11}$	(1) (1) (1) (1)									