

PRE-BOARD EXAM 2022-23

CLASS: XII

MATHEMATICS

TIME: 3 hrs.

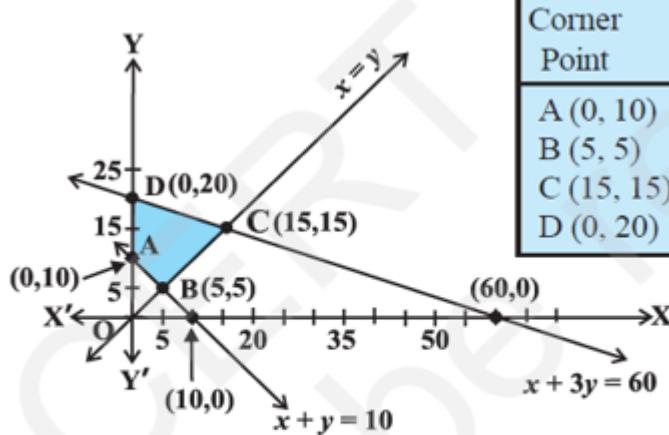
MARKING SCHEME

Max Marks: 80

QUESTION NUMBER	HINTS/SOLUTION	MARK(S)
1	$15p + 15q = 20q$ $3p = q$ CORRECT OPTION-D	1(d)
2	$7A - I^3 - 3A - 3A - A$ = - INCORRECT OPTION-A	1(a)
3	$a_{32} = 5, A_{32} = 22$ $a_{32} A_{32} = 110$ CORRECT OPTION-D	1(d)
4	$K = \frac{(x+9)(x-3)}{x-3} = 12$ CORRECT OPTION-B	1(b)
5	The strict inequality represents an open half plane and it contains the origin as $(0,0)$ satisfies it. CORRECT OPTION-B	1(b)
6	$f(x) = \sec x$ CORRECT OPTION-B	1(b)
7	The given differential equation is $4 \left(\frac{dy}{dx} \right)^3 \frac{d^2y}{dx^2} = 0$. Here, $m = 2$ and $n = 1$ Hence, $m + n = 3$ CORRECT OPTION-C	1(c)
8	$ydx - xdy = 0 \Rightarrow ydx - xdy = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$ $\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x} + \log K, K > 0 \Rightarrow \log y = \log x + \log K$ $\Rightarrow \log y = \log x K \Rightarrow y = x K \Rightarrow y = \pm Kx \Rightarrow y = Cx$ CORRECT OPTION-A	1(a)
9	$y \sin y / x = x \sin x$ CORRECT OPTION-B	1(b)
10	Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k}$ $\frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{ \hat{i} + 2\hat{j} - 3\hat{k} } = \frac{7}{\sqrt{14}}$ CORRECT OPTION-A	1(a)
11	$12.(12)^2 = 1728$ CORRECT OPTION-A	1(a)
12	$-16 = -8 - x^2$ $x = \pm 2\sqrt{2}$ CORRECT OPTION-D	1(d)
13.	$x = 3, y = 3$ $x + y = 6$ CORRECT OPTION-B	1(b)
14	$ \vec{a} \vec{b} \cos \theta = \vec{a} \vec{b} \sin \theta$ $\tan \theta = 1$ $\theta = \frac{\pi}{4}$ CORRECT OPTION-C	1(c)
15	$i.i + j.j + k.(-k)$ $1+1-1 = 1$ CORRECT OPTION-A	1(a)

16	$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$ $p = \frac{1}{5}$ CORRECT OPTION-A	1(a)
17	$\vec{b} = \frac{\vec{a} + \vec{c}}{2}$ $\vec{oc} = 2\vec{b} - \vec{a}$ CORRECT OPTION-B	1(b)
18	$\frac{dy}{dx} = \frac{3t^2}{2t}$ $= \frac{3t}{2}$ CORRECT OPTION-D	1(d)
19	Let, $f(x)=f(y) \Rightarrow x = y$. $\therefore f(x)$ is injective. Not surjective because co-domain \neq range CORRECT OPTION-A	1(a)
20	$\vec{r} = -\vec{i} + \vec{j} + \vec{k}$ Direction cosines $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ If (a,b,c) are d.r then d.c are $\left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$ CORRECT OPTION-C	1(c)
21	$\frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$ $= \frac{-\pi}{3}$ (OR) $\sin^{-1}[\sin\left(\frac{13\pi}{7}\right)] = \sin^{-1}[\sin\left(2\pi - \frac{\pi}{7}\right)]$ $= \sin^{-1}[\sin\left(-\frac{\pi}{7}\right)] = -\frac{\pi}{7}$	1 1 1 1 1
22	$\vec{a} + \vec{b} = 4i + (2 + \mu)j + 12k$ $\vec{a} - \vec{b} = 2i + (2 - \mu)j + 6k$ $8 + 4 - \mu^2 + 72 = 0$ $\mu = \pm 2\sqrt{21}$	(1/2) (1/2) (1/2) (1/2)
23	$\hat{n} = \frac{i - 2j + 2k}{3}$ Required vector = $5(i - 2j + 2k)$	1 1
24	$\log y = \sin x \cdot \log(\sin x)$ $\frac{1}{y} \frac{dy}{dx} = \cos \cos x [1 + \log(\sin \sin x)]$ $\frac{dy}{dx} = (\sin \sin x)^{\sin \sin x} \cdot \cos \cos x [1 + \log(\sin \sin x)]$	(1/2) 1 (1/2)
25	$R(x) = 26x + 26$ $M(x) = 26(7) + 26$ $= 208$	1 (1/2) (1/2)
26	$\int \frac{dx}{\sqrt{-(x^2+2x-3)}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{2}\right) + C$ [$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$]	(1+1) (1)

27



3 lines
(1.5)
Corner
points
(0.5)

1

Minimum value of z is 60 at (5,5)

28

$$\begin{aligned} 1. \quad \frac{x^2+1}{x^2-5x+6} &= 1 + \frac{5x-5}{(x-2)(x-3)} \\ &= 1 - \frac{5}{x-2} + \frac{10}{x-3} \end{aligned}$$

$$\therefore I = x - 5 \log(x-2) + 10 \log(x-3) + C$$

1

1

1

29

$$\text{Let } I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots (1)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}}{\sqrt{\sin(\frac{\pi}{6} + \frac{\pi}{3} - x)} + \sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}} dx$$

(1)

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots (ii).$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

(1)

$$2I = \int_{\pi/6}^{\pi/3} dx$$

(0.5)

$$= [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

(0.5)

$$\text{Hence, } I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \frac{\pi}{12}$$

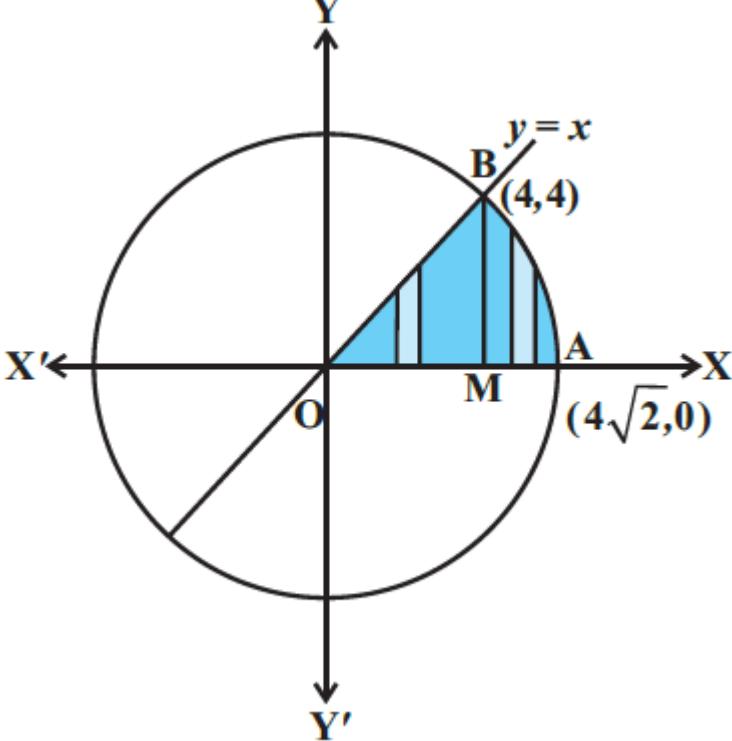
(0.5)

(0.5)

30.

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin \frac{y}{x}}$$

	$\text{put } \frac{y}{x} = v$ $v + x \cdot \frac{dv}{dx} = v - \frac{1}{\sin v}$ $-\int \sin v \, dv = \int \frac{1}{x} \, dx$ $\cos v = \log x + c$ $\cos(\frac{y}{x}) = \log x + c$ (OR) $ydx + (x - y^2)dy = 0$ Reducing the given differential equation to the form $\frac{dx}{dy} + Px = Q$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$ I.F = $e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$ The general solution is given by $x \cdot I.F = \int Q \cdot I.F dy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution	(0.5) (0.5) (0.5) (0.5) (0.5) (1) (1) (0.5)
31	A: both are girls B: older is a girl $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$ $= \frac{\frac{1}{4}}{\frac{2}{4}}$ $= \frac{1}{2}$ (OR) $\Sigma P(X) = 1$ $\Rightarrow 10k^2 + 9k - 1 = 0$ $k = \frac{1}{10}$	(0.5) (1) (1) (0.5) (0.5) (1.5) (1)
32	$ A = 10$ $A^{-1} = \frac{1}{10}(-6 25 - 24 - 12 40 - 38 10 - 40 40)$ $x = 1, y = \frac{1}{2}, z = -1$ (OR) $AB = 6I$ $A^{-1} = \frac{1}{6} B$	(1) (3) (1) (2) (1)

	$X = \frac{1}{6}(22 - 4 - 42 - 42 - 15)(3177)$ $x = 2, y = -1, z = 4$	(1) (1)
33	$\vec{a}_2 - \hat{\vec{a}}_1 = \hat{i} - \hat{k}$ $\vec{b}_1 \vec{x} \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$ $ \vec{b}_1 \vec{x} \vec{b}_2 = \sqrt{59}$ $(\vec{b}_1 \vec{x} \vec{b}_2) \cdot (\vec{a}_2 - \hat{\vec{a}}_1) = 10$ $d = \frac{10}{\sqrt{59}}$ (OR) $\vec{m} = ijk 3 - 16738 - 5 $ $= 24i + 36j + 72k$ Required equation is $(i + 2j - 4k) + \mu(-2i + 3j + 6k)$	(1) (1) (1) (1) (2) (2) (1)
34	Proving reflexive, symmetric and transitive Elements related to 1 are {1,5,9}	(4.5) (0.5)
35	 <p>To find point of intersection as (4,4)</p> <p>Required area = $\int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx.$</p> $= 8 + (4\pi - 8)$	(1) (1) (1/2 + 1/2) (1/2 + 1) (1/2)

	= 4π sq. units										
36	<p>(1) $f(x)$ being a polynomial function, it is differentiable in $(0, 12)$</p> <p>(2) $f(x) = -0.2x + 1.2 = -0.2(x - 6)$</p> $f(x) = 0 \Rightarrow x = 6$ <p>(3).</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>In the Interval</th> <th>$f'(x)$</th> <th>Conclusion</th> </tr> </thead> <tbody> <tr> <td>$(0, 6)$</td> <td>+ve</td> <td>f is strictly increasing in $[0, 6]$</td> </tr> <tr> <td>$(6, 12)$</td> <td>-ve</td> <td>f is strictly decreasing in $[6, 12]$</td> </tr> </tbody> </table>	In the Interval	$f'(x)$	Conclusion	$(0, 6)$	+ve	f is strictly increasing in $[0, 6]$	$(6, 12)$	-ve	f is strictly decreasing in $[6, 12]$	(1) (0.5) (0.5)
In the Interval	$f'(x)$	Conclusion									
$(0, 6)$	+ve	f is strictly increasing in $[0, 6]$									
$(6, 12)$	-ve	f is strictly decreasing in $[6, 12]$									
37	<p>(1) $y = \sqrt{4a^2 - x^2}$</p> <p>(2) $A = \frac{x\sqrt{4a^2 - x^2}}{2}$</p> <p>(3) $\frac{dy}{dx} = \frac{-x^2 + 2a^2}{\sqrt{4a^2 - x^2}}$</p> <p>(4) $x = y = \sqrt{2}a$ and $\frac{d^2y}{dx^2} < 0$ when $x = \sqrt{2}a$</p>	(1) (1) (1) (1)									
38.	<p>(1) $P(E_1) = \frac{3}{5}$</p> <p>(2) $P(E/E_1) = 1$</p> <p>(3) $P(E_1/E) = \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$ $= \frac{9}{11}$</p>	(1) (1) (1) (1)									